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ON  
APPLIED MECHANICS

NINTH EDITION  
*THOROUGHLY REVISED*

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By PROFESSOR JAMIESON, M.Inst.C.E., M.Inst.E.E., F.R.S.E., Formerly Professor of Engineering in the Glasgow and West of Scotland Technical College; Consulting Engineer and Electrician, 16 Rosslyn Terrace, Kelvinside, Glasgow.

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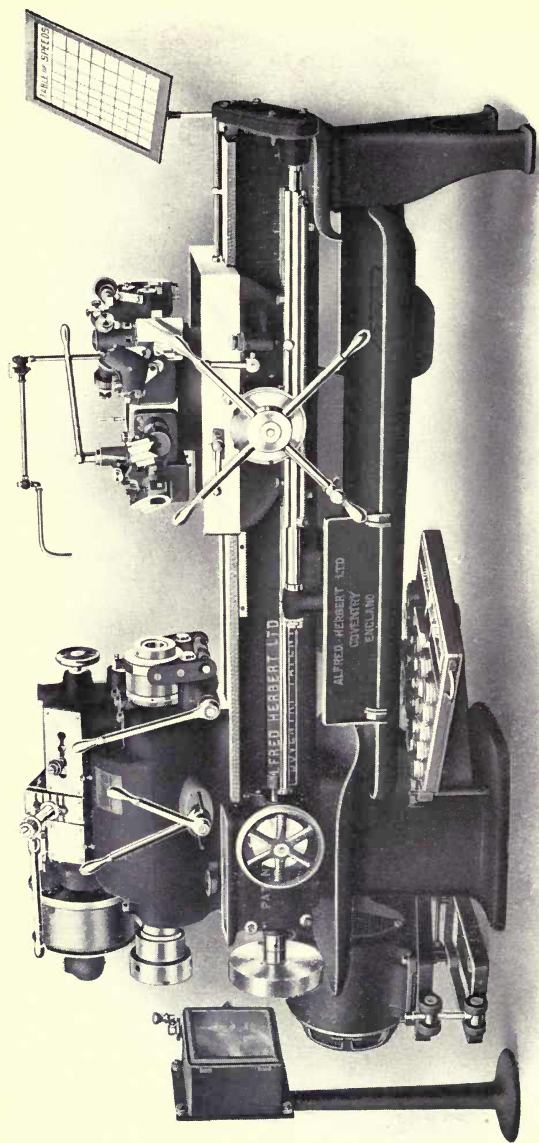
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BY

ANDREW JAMIESON, M. INST. C.E.

FORMERLY PROFESSOR OF ENGINEERING, THE GLASGOW TECHNICAL COLLEGE  
FELLOW OF THE ROYAL SOCIETY, EDINBURGH; MEMBER OF THE INSTITUTION OF  
ELECTRICAL ENGINEERS; AUTHOR OF "TEXT-BOOKS ON STEAM AND STEAM  
ENGINES," "ADVANCED APPLIED MECHANICS," "MAGNETISM AND  
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Numerous Diagrams, Arithmetical Examples, Examination  
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GENERAL

## PREFACE TO NINTH EDITION

THIS Elementary Manual on Applied Mechanics has been carefully revised, whilst several important alterations and additions have been made to it.

I have added a drawing and description to the end of Lecture VIII of Butters Brothers and Co.'s new 1909 "Jib Crane Arrester."

A new Frontis Plate, as well as a new plate for Lecture XXI, showing the latest 1909 improvements in "Alfred Herbert's Electrically Driven Turret Lathe," together with a description thereof, have been duly inserted at the proper places.

Owing to a very clear and useful hint which was given in *The Electrician*, by the reviewer of the previous Edition, I have transferred the former Lectures numbered XXIV and XXV, upon "Bending and Shearing Stresses in Beams and Metal Structures," &c., to my new and more advanced book, Volume III, which deals entirely with the "Theory of Structures." This change has enabled me not only to reduce the total number of Lectures in the book from twenty-eight to twenty-six, but to re-arrange and considerably extend the last Lecture. To it, I have added an account of "Sir Joseph Whitworth's Early Realisations of Mechanical Accuracy"; as well as a detailed description of one of his "Millionth Measuring Machines."

Further, I have described by aid of six very clear photo-process diagrams, a "New Set of English Standard Gauges." I have specially to thank Mr. H. M. Budgett, of the Crown Works, Chelmsford, England, not only for supplying these good figures, but also for permitting me to copy the Certificate given to him last autumn by The National Physical Laboratory of their very careful tests of these British-made gauges.

Special attention has been paid to the Appendices A, B, C, and D, where Teachers and Stage I Students will find, not only the latest General Instructions of the Board of Education (B. of E.) and the City and Guilds of London (C. & G.); but also the concise abstract from the Rules and Syllabus for the examination of Students of The Institution of Civil Engineers (Stud. I.C.E.). Further, all the questions given by these three well-known and respected Examining Bodies, up to and including the year 1908, have been collected and arranged under the numbers of the various

Lectures to which they naturally belong. Finally, the entire examination papers for 1909 and 1910 have been given as issued, with the view that students should test their knowledge of this book by trying these papers under *exact* Examination conditions. At all events, I find that this is one of the best methods of preparing my C.E. and other Engineering *Correspondence Students* for these and several other Examinations and Appointments.

I have again to thank Mr. John Ramsay, Assoc. M. Inst. C.E., for his assistance with this book.

ANDREW JAMIESON.

*Consulting Engineer and Electrician.*

16 ROSSLYN TERRACE,  
KELVINSIDE, GLASGOW.  
*August 1910.*

## PREFACE TO EIGHTH EDITION

I HAVE taken advantage of the working out of many Questions at the end of the several Lectures, and of various Examination Papers in the Appendices, by my "Science Correspondence Students," to obtain a number of checked Answers. These Answers have been inserted at their Questions up to Lecture XXIV; but thereafter, they have been printed just before the Index, at the end of Appendix C.

The 1907 Examination Papers for Stage I of the Board of Education (B. of E.), South Kensington; the City and Guilds (C. and G.) of London Institute, Ordinary Grade, Part I. Sections A and B on Mechanical Engineering; and the Entrance or Students' Examinations of The Institution of Civil Engineers (Stud. I.C.E.) in Elementary Mechanics for 1907 and Feb. 1908 will be found in Appendix B, along with those of previous years, the Rules, and an Abstract of the Syllabus for these Examinations.

These Answers and new up-to date Examination Questions, which have been taken from the best sources, should prove of great interest and assistance to Teachers and Students of Applied Mechanics.

At the end of Lecture I a new page has been added. It deals with the terms and definitions of words used in measuring Pressures, as well as a discussion of the recent endeavour by some authors to introduce the words *Thrust* and *Resultant Thrust*. I see no necessity for these terms and shall feel obliged if practical Engineering Reviewers, Teachers and Students will give me their opinion after carefully studying my remarks.

At the end of Lecture XVIII a part of a new page has been inserted, wherein are given concise distinctions between solids, liquids and gases, with definitions of perfect viscous and elastic fluids.

A more detailed and clearer explanation of the application and uses of Limit Gauges as well as of the construction and uses of Berridge's Tangentometer have been given in Lecture XXVIII. Students should study and apply this instrument to find the sines, cosines, tangents, &c., of different angles and then check their results by the Tables near the end of Appendix C, on "Functions of Angles," before they finish Lecture IV.

Owing to the necessity for young engineers becoming early acquainted with Electrical Engineering terms and units of measurement, I have added to appendix C what is believed to be the latest and most correct set of definitions for the Nomenclature with their Abbreviations or Symbols of the C.G.S. and of the authorised Practical Electrical Units.

Finally, the whole book has been most carefully revised and brought up to date.

I have much pleasure in thanking Forrest Sutherland, B.Sc., Science and Technical Educationist, Natal, South Africa, for his useful suggestions, and John Ramsay, Assoc. M. Inst. C.E., for the interest which he has taken and the help he has given with this new Edition. The Author hopes that this eighth edition will be found to be an advance upon previous issues. He also desires reviewers, teachers and students to kindly let him know in what way they think that he can still further improve this "First Year Manual on Applied Mechanics."

<sup>a</sup>  
r

ANDREW JAMIESON,

*Consulting Engineer and Electrician.*

16 ROSSLYN TERRACE,  
KELVINSIDE, GLASGOW.

## INSTRUCTIONS TO BE FOLLOWED IN THE WRITING OF HOME EXERCISES.

1. Put the date of handing in each exercise at the right-hand top corner.
2. Leave a margin an inch wide on the left-hand side of each page; and in the margin place the number of the question, and nothing more.
3. Leave a space of at least three lines between your answers for remarks or corrections.
4. Be sure you understand *exactly* what the question requires you to answer, then give *all* it requires, but *no more*. If unable to answer any question, write down its number and the reason why.
5. Make your answers concise, clear, and exact; and accompany them, whenever practicable, by an *illustrative sketch*.
6. Make all sketches large, open, and in the centre of the page, and do not crowd any writing about them.

NOTE.—The character of the sketches will be considered in awarding the marks to the several questions. Neat sketches and an "Index to Parts," with the first letter of name of Part, will always receive more marks than a bare written description.

7. Every sketch must be accompanied by an "Index to Parts" written immediately beneath it, and must accompany the answer it is designed to illustrate.

NOTE.—The initial letter or letters of the name of the Part must be used, and not A, B, C, or 1, 2, 3, &c.

8. Unless otherwise specially requested by the question, every sketch must be accompanied by a *concise* written description.
9. Every answer which receives less than *half* of the full marks awarded to it, must be re-written correctly for next evening, before the usual class work, and headed "*Re-written*."

REMARKS.—Students are strongly recommended to write out each answer in scroll first, and then to compare it with the question. After committing it to their book, they should then read it over a second time, to correct any errors they may discover. Reasonable and easily intelligible contractions are permitted. Students are invited to ask questions and explanations regarding anything they do not understand. Except in special cases, arrears of Home work *will not receive marks*.

N.B.—Students who from any cause have been absent from a lecture, must send a post-card or note of explanation to the teacher. If they miss any exercise or exercises, they must state the reason (in red ink, or underlined) in their exercise books when handing them in next night. If these rules are not complied with, then marks will be deducted.

# CONTENTS.

PAGE

## LECTURE I.

Definition of Applied Meehanics—Force—Matter—Unit of Force —The Elements of a Force—Graphic Representation of Forces—Forces in Equilibrium—Action and Reaction— Resultant and Components—Resultant of Forces acting in a Straight Line—Terms and Definitions used in Measur- ing or Calculating Pressures and their Effects—Definitions of Scalar, Vector and Rotor Quantities—Engineering Cal- culations . . . . .	1-56
--	------

## LECTURE II.

Work—Unit of Work—Examples I. II. III. IV.—Work done against a Variable Resistance—Example V.—Diagrams of Work—With Uniform Resistance—With a Uniformly In- creasing Resistance—With a Uniformly Decreasing Resis- tance—With a Combination of Uniform and Variable Loads—Example VI.—Power or Activity—Units of Power —The Horse-power Unit—To find the Horse-power of any working Agent—Example VII.—Uses of Squared Paper— Clark's Adjustable Curve—Example VIII.—Questions .	6-20
--	------

## LECTURE III.

The Moment of a Force—Principle of Moments applied to the Lever—Experiments I. II. III.—Pressure on and Reaction from the Fulcrum—Equilibrant and Resultant of two Parallel Forces—Couples—Centre of Parallel Forces or Position of Equilibrant and Resultant—Centre of Gravity —Examples of Centre of Gravity—The Lever when its Weight is taken into Account—Examples I. II.—Position of the Fulcrum—Example III.—Questions . . . . .	21-34
--	-------

## LECTURE IV.

Practical Applications of the Lever—The Steelyard, or Roman Balance—Graduation of the Steelyard—The Lever Safety Valve—Example I.—Lever Machine for Testing Tensile	
---	--

<b>Strength of Materials—Straight Levers acted on by Inclined Forces—Bent Levers—The Bell Crank Lever—Bent Lever Balance—Duplex Bent Lever, or Lumberer's Tongs—Turks, or Pincers—Examples II. and III.—Toggle Joints—Questions . . . . .</b>	<b>35-51</b>
---	--------------

**LECTURE V.**

<b>The Principle of Work—Work put in, Work lost, Useful Work—Efficiency of a Machine—Principle of Work applied to the Lever—Experiments I. II.—Wheel and Axle—The Principle of Moments applied to the Wheel and Axle—The Principle of Work applied to the Wheel and Axle—Experiment III.—The Winch Barrel—Example I.—Ship's Capstan The Fusee—Questions . . . . .</b>	<b>52-62</b>
---	--------------

**LECTURE VI.**

<b>Pulleys—Snatch Block—Block and Tackle—Theoretical Advantage—Velocity Ratio—The Principle of Work applied to the Block and Tackle—Actual or Working Advantage—Work put in—Work got out—Efficiency—Percentage Efficiency—Example I.—Questions . . . . .</b>	<b>63-71</b>
--	--------------

**LECTURE VII.**

<b>The Wheel and Compound Axle, or Chinese Windlass—The Principle of Moments applied to the Wheel and Compound Axle—The Principle of Work applied to the Wheel and Compound Axle—Examples I. II.—Weston's Differential Pulley Block—The Principle of Work applied to Weston's Differential Pulley Block—Experiment I.—Cause of the Load not overhauling the Chain—Questions . . . . .</b>	<b>72-79</b>
---	--------------

**LECTURE VIII.**

<b>Graphic Demonstration of Three Forces in Equilibrium—Parallelogram of Forces—Triangle of Forces—Three Equal Forces in Equilibrium—Two Forces acting at Right Angles—Resolution of a Force into Two Components at Right Angles—Resultant of Two Forces acting at any Angle on a Point—Resultant of any number of Forces acting at a Point—Example I.—Stresses in Jib Cranes—Jib Arrester—Example II., III.—Stresses on a Simple Roof—Example IV.—Questions . . . . .</b>	<b>80-92</b>
--	--------------

LECTURE IX.

PAGES

Inclined Planes—The Inclined Plane without Friction—When the Force acts Parallel to the Plane—Example I.—When the Force acts Parallel to the Base—Example II.—When the Force acts at any Angle to the Inclined Plane—Example III.—The Principle of Work applied to the Inclined Plane—Example IV.—Questions . . . . .	93-100
---	--------

LECTURE X.

Friction—Heat is Developed when Force overcomes Friction—Laws of Friction—Apparatus for Demonstrating First and Second Laws of Friction—Experiment I.—Example I.—Angle of Repose or Angle of Friction—Experiment II.—Diagram of Angles of Repose—Limiting Angle of Resistance—Experiment III.—Apparatus for Demonstration of the Third Law of Friction—Experiment IV.—Lubrication—Anti-Friction Wheel Ball—Bearings—Work done on Inclines, including Friction—Example II.—Questions . . . . .	101-115
---	---------

LECTURE XI.

Difference of Tension in the Leading and Following Parts of a Driving Belt—Brake Horse-Power transmitted by Belts—Examples I. II.—Velocity Ratios in Belt Gearing—Examples III. IV.—Open and Crossed Belts—Fast and Loose Pulleys—Belt Gearing Reversing Motions—Stepped Speed Cones with Starting and Stopping Gear—Driving and Following Pulleys in Different Planes—Shape of Pulley Face—Questions . . . . .	116-129
---	---------

LECTURE XII.

Velocity Ratio of Two Friction Circular Discs—Pitch Surfaces and Pitch Circles—Pitch of Teeth in Wheel Gearing—Rack and Pinion Velocity Ratio in Wheel Gearing—Example I.—Principle of Work applied to Wheel Gearing—Examples II. III.—Questions . . . . .	130-139
--	---------

LECTURE XIII.

Single-purchase Winch or Crab—Example I.—Double-purchase Winch or Crab—Example II.—Wheel Gearing in Jib-Cranes—Questions . . . . .	140-147
--	---------

## LECTURE XIV.

PAGES

<b>Screws</b> —The Spiral, Helix, or Ideal Line of a Screw Thread—The Screw viewed as an Inclined Plane—Characteristics and Conditions to be fulfilled by Screw Threads—Different Forms of Screw Threads—Whitworth's V-Threads—Whitworth's Tables of Standard V-Threads, Nuts and Bolt Heads—Seller's V-Thread—The Square Thread—The Rounded Thread—The Buttress Thread—Right and Left-hand Screws—The Screw Coupling for Railway Carriages—Single, Double and Treble Threaded Screws—Backlash in Wheel and Screw Gearings—Questions . . . . .	148-159
--	---------

## LECTURE XV.

<b>Efficiency, &amp;c., of a Combined Lever, Screw and Pulley Gear</b> —Example I.—Bottle Screw-Jack—Example II.—Traversing Screw-Jack—Screw Press for Bales—Screw Bench Vice—Example III.—Endless Screw and Worm-Wheel—Combined Pulley, Worm, Worm-Wheel and Winch-Drum—Worm-Wheel Lifting Gear—Example IV.—Questions . . . . .	160-173
--	---------

## LECTURE XVI.

<b>General Idea of the Mechanism in a Screw-cutting Lathe</b> —Motions of the Saddle and Slide Rest—Velocity Ratio of the Change Wheels—Rules for Calculating the Required Number of Teeth in Change Wheels—Examples I. II.—Movable Head-stock for a Common Lathe—Descriptions of a Screw-cutting Lathe and of an Electrically Driven Hexagon Turret Lathe, with Frontis-plates and complete sets of Detail Drawings—Questions . . . . .	174-206
--	---------

## LECTURE XVII.

<b>Hydraulics</b> —Definition of a Liquid—Axioms relating to a Liquid at Rest—Transmission of Pressure by Liquids—Pascal's Law—"Head" or Pressure of a Liquid at Different Depths—Total Pressure on a Horizontal Plane immersed in a Liquid—Lord Kelvin's Wire-testing Machine—Total Pressure on any Surface immersed in a Liquid—Examples I. II.—Questions . . . . .	207-213
---	---------

## LECTURE XVIII.

## PAGES

Useful Data regarding Fresh and Salt Water—Examples I. II. III.	
IV.—Centre of Pressure—Immersion of Solids—Law of Archimedes—Floating Bodies—Example V.—Atmospheric Pressure—The Mercurial Barometer—Example VI.—Low Pressure and Vacuum Water Gauges—Example VII.—The Siphon—Distinction between Solids, Liquids and Gases—Definitions of perfect, viscous, and elastic Fluids—Cohesion—Questions . . . . .	214-226

## LECTURE XIX.

Hydraulic Machines—The Common Suction Pump—Example I.—The Plunger, or Single-acting Force Pump—Example II.—Force Pump with Air Vessel—Continuous-delivery Single-acting Force Pump without Air Vessel—Combined Plunger and Bucket Pump—Double-acting Force Pump—Example III.—Centrifugal Pumps—Example IV.—Questions . . . . .	227-240
--	---------

## LECTURE XX.

Bramah's Hydraulic Press—Bramah's Leather Collar Packing—Examples I. II.—Large Hydraulic Press for Flanging Boiler Plates—The Hydraulic Jack—Weem's Compound Screw and Hydraulic Jack—Example III.—The Hydraulic Bear or Portable Punching Machine—The Hydraulic Accumulator—Example IV.—Questions . . . . .	241-258
--	---------

## LECTURE XXI.

Motion and Velocity—Uniform, Variable, Linear, and Angular Velocity—Unit of Velocity—Acceleration—Unit of Acceleration—Acceleration due to Gravity—Graphic Representation of Velocities—Composition and Resolution of Velocities—Newton's Laws of Motion—Formulae for Falling Bodies—Formulae for Linear Velocity—with Uniform Acceleration—Atwood's Machine with Experiments—Results and Formulae—Galileo's and Kater's Pendulum Experiments—The Path of a Projected Body—Centrifugal Force due to Motion in a Circle—Centrifugal Force Machine—Experiments I. II. III.—Example I.—Balancing High-speed Machinery—Centrifugal Stress in the Arms of a Fly-wheel—Example II.—Energy—Potential Energy—Kinetic Energy—Accumulated Work—Accumulated Work in a Rotating Body—
---

The Fly-wheel—Radius of Gyration—Example III.—The Fly Press—Example IV.—The Energy Stored in a Rotating Fly-wheel—Motion on Bicycle and Railway Curves—Momentum—Examples VI. to IX.—Questions . . .	259-296
---	---------

## LECTURE XXII.

Some Properties of Materials employed by Mechanics—Essential Properties—Extension—Impenetrability—Contingent Properties—Divisibility—Porosity—Density—Cohesion—Compressibility and Dilatability—Rigidity—Tenacity—Malleability—Ductility—Elasticity—Fusibility—Load, Stress, and Strain—Total Stress, and Intensity of Stress—Tensile Stress and Stress—Example I.—Compressive Stress and Strain—Example II.—Limiting Stress or Ultimate Strength—Safe Loads and Elasticity—Limit of Elasticity—Hooke's Law—Factors of Safety—Modulus of Elasticity—Ratio of Stress to Strain—Examples III.-V.—Resilience or Work Done in Extending or Compressing a Bar within the Elastic Limit—Examples VI.-IX.—Single Riveted Lap Joints—Example X.—Questions . . .	297-315
---	---------

## LECTURE XXIII.

Stresses in Chains—Shearing Stress and Strain—Example I.—Torque or Twisting Moment—Torsion of wires—Table giving the strength, moduli of Elasticity and Rigidity of various materials—Strength of Solid Round Shafts—Example II.—Table giving the Horse-Power which steel shafting will transmit at various speeds—Strength of Hollow Round Shafts—Relation between the Twisting Moment and Horse-Power transmitted by Shafting, as well as the Diameter necessary to transmit a given Horse-Power—Examples III. IV.—Questions . . .	316-328
--	---------

## LECTURE XXIV.

Hooke's Coupling or Universal Joint—Double Hooke's Joint—Sun and Planet Wheels—Cams—Heart Wheel or Heart-shaped Cam—Cam for Intermittent Motion—Quick Return Cam—Example—Pawl and Ratchet Wheel—Reversible Pawl—Masked Ratchet—Silent Feed—Watt's Parallel Motion—Parallel Motion—Questions . . .	329-342
---	---------

## LECTURE XXV.

Reversing Motions—Planing Machine—Reversing by Friction Cones and Bevel Wheels—Whitworth's Reversing Gear— Quick Return Reversing Motion—Whitworth's Quick Return Motion—Whitworth's Slotting Machine—Common Quick Return—Horizontal Shaping Machine—Quick Re- turn with Elliptic Wheels—Vertical Slotting Machine— Questions . . . . .	343-354
---	---------

## LECTURE XXVI.

Measuring Tools and Gauges—Limit Gauges—Micrometer Screw Gauge—Sir Joseph Whitworth's Early Realisation of Mechanical Accuracy—Improved Equivalents Micrometer Gauge—A New Set of English Gauges—Whitworth's Millionth Measuring Machine—Whitworth's Standard Workshop Measuring Machine—Questions . . . . .	355-366
---	---------

---

APPENDICES, Rules and Examination Papers of The Board of Education, City and Guilds of London, and Institution of Civil Engineers for Admission of Students—C.G.S. Sys- tem of Units—Practical Electrical Units—Examination Tables of Useful Constants, Logarithms, Antilogarithms, and Functions of Angles . . . . .	367-419
--	---------

INDEX . . . . .	421-430
-----------------	---------





# ELEMENTARY MANUAL

ON

## APPLIED MECHANICS.

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### LECTURE I.

**CONTENTS.**—Definition of Applied Mechanics—Force—Matter—Unit of Force—The Elements of a Force—Graphic Representation of Forces—Forces in Equilibrium—Action and Reaction—Resultant and Components—Resultant of Forces acting in a Straight Line—Terms and Definitions used in Measuring or Calculating Pressures and their Effects—Definitions of Scalar, Vector and Rotor Quantities—Engineering Calculations.

Applied Mechanics is that branch of applied science which not only explains the principles upon which machines are designed, made and act, but also describes their construction and applications, as well as how to calculate and test their strength and efficiency.

Before a student can successfully master any science, he must thoroughly understand the units of measurement that have been adopted in calculating results, and he should also have a clear conception of the exact meaning of the various terms employed. Consequently, we shall commence the study of Elementary Applied Mechanics with definitions and with units of force, work and power.\*

*Force is any cause which produces, or tends to produce, motion or change of motion in the matter upon which it acts.*

*Matter is anything which can be perceived by one or more of the senses, and which can be acted on by force.*

Matter exists under three conditions: (1) Solids, (2) Liquids, (3) Gases. For example, pieces of wood and of iron are solids; water and mercury are liquids; whilst air and oxygen are gases.

\* For the units of length, surface and cubic measure, and for the mensuration of areas and solids, the student is referred to Lectures I. II. and III. of Author's "Elementary Manual on Steam and the Steam Engine," issued by the publishers of this book.

Bodies are therefore limited portions of matter. When the resistance to motion of a body is equal to or greater than the force applied, so that no motion takes place, the body is said to be subjected to *pressure*.

Solids do not yield readily to pressure, for they tend to retain their original shape and size, whereas liquids and gases yield to a very slight pressure, and consequently possess no definite shape. A gas differs from a liquid since it possesses the property of indefinite expansion. A liquid has therefore a definite size, but not a definite shape, whilst a gas has neither definite shape nor definite size.

**Unit of Force.**—The British unit of force is the pound avoirdupois, or GRAVITATION UNIT or ENGINEER'S UNIT. The magnitude of a force is therefore reckoned by the number of pounds of matter which the force would support against gravity. For example, a force of 1 lb. means that force which would just lift the weight of a fixed mass at a fixed place if acted on by gravity alone. But the force of gravity varies at different parts of the earth's surface, being slightly greater at the Poles than at the Equator. Consequently, our engineer's unit of force is only an absolute or invariable one at a standard place, such as at Greenwich sea level.

**The Elements of a Force.**—When a force acts upon a body, then, in order to fully determine its effect we must know the three following elements:—(1) The point or place of application of the force. (2) The direction in which the force acts. (3) The magnitude of the force.

(1) *Place of Application.*—In the case of the force of gravity acting on a body, the place of application may be considered to be the whole mass of the body, or we may estimate the whole weight of the body as concentrated at one point, termed the centre of

\* Where great accuracy of measurement is required an *absolute* or *invariable unit of force* must be selected. An *absolute unit of force* may be defined as *that force which, acting for unit time on unit mass, will produce unit change of velocity*. If the units of time, mass, and velocity be the second, pound, and foot per second respectively, then we may define the *absolute unit of force* (called the *poundal*) as *that force which, acting for one second on a mass of one pound would produce a change in velocity of one foot per second*. It has been determined experimentally that if a body be let fall freely in vacuo, near the earth's surface, the attractive force of the earth will produce a change of velocity every second of  $g$  ( $=32.2$  nearly) feet per second. Clearly, then, the gravitation unit is  $g$  times the absolute unit. Hence the following relation between the gravitation and absolute units of force:—A force of one pound  $= g$  poundals, or a force of one poundal  $= 1/g$  pound.

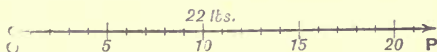
In this book the gravitation or the engineer's unit of force and work will be used.

gravity of the body. When an extended surface is subjected to pressure, as in the case of a tank containing a liquid, or the piston of an engine subjected to the pressure of a gas, the whole area under pressure may be considered as the place of application. When a body is pulled by means of a rope, or pushed by means of a rod, or supported on a small area, then we consider the force as acting at a point.

(2) *Direction*.—The direction of a force is the line or path in which it tends to move the body on which it acts.

(3) *Magnitude*.—The magnitude of the force is the pounds pull or pressure which the force exerts upon the body on which it acts.

**Graphic Representation of Forces.**—When a force acts on a body at a point, its three elements may be conveniently represented as follows:—



SCALE DIAGRAM OF A FORCE.

Where O represents the *point of application*, the straight line OP (with the arrow-head), shows the *direction* in which the force acts, and the length of the divided line OP indicates to scale the *magnitude* of the force.\* (See end of this lecture for further definitions.)

**Forces in Equilibrium.**—(1) When any number of forces acting upon a body neutralize each other's effects (*i.e.*, leave the body in the same condition as to rest or motion as before the application of the forces), these forces are said to be in equilibrium.

(2) Forces which are in equilibrium may be applied to or removed from a rigid body without altering its condition as to rest or motion.

(3) Two equal and opposite forces destroy each other's effects; and, conversely, no two forces can destroy each other's effects unless they are equal and opposite.

(4) A force will have the same effect at whatever point in its own direction it may be supposed to act; and, conversely, if a force have the same effect whether it act at one or other of two given points, then the straight line joining these points (with the suitably directed arrowhead) will be the direction of the force.

**Action and Reaction.**—(1) Whenever a fixed rigid body is

\* In the case of the above figure the force is represented as equal to 22 lbs. Students will find it convenient to plot down the representation of forces in their exercise books to a scale of  $\frac{1}{10}$  of an inch to a pound, or hundredweight, or ton, according to the values of the forces.

acted on by a force, then naturally there is at once set up in that body a secondary force, or a force of reaction, equal and opposite in direction to the primary force.

(2) Hence action and reaction are equal and opposite, and neutralize each other's effects.

For Example.—Suppose a weight is placed on a rigid horizontal table. In the table there is set up an opposing force or upward reaction which exactly counterbalances the downward force of the weight. If this were not the case, then motion would take place, and either the table would give way, or the weight would sink through the table!

**Resultant and Components.**—(1) If any number of forces acting upon a body be replaced by a single force which shall have the same effect, then this force is termed the *resultant* of these forces, and the forces are called the *components* of their resultant.

(2) The operation of finding the resultant of any number of forces is called the *composition* of forces; and finding the components is termed the *resolution* of forces.

**Resultant of Forces acting in a Straight Line.**—(1) The resultant of any number of forces acting in the one direction along one straight line is equal to their sum, and acts in that direction.

For Example.—Let  $P_1, P_2, P_3, P_4$  be any four forces acting in one direction along one straight line, then their resultant—

$$R = P_1 + P_2 + P_3 + P_4$$

(2) If the forces do not all act in one direction, then the resultant is equal to the difference between the resultant of those acting in one direction and the resultant of those which act in the opposite direction, and has the direction of the greater of the two resultants.

For Example.—Let  $P_1, P_2, P_3, P_4$  be any four forces acting along one straight line to the right hand or in a positive direction; and  $Q_1, Q_2, Q_3$  be any three forces acting along the same straight line, but in an opposite or left-hand or negative direction, and let

$$Q_1 + Q_2 + Q_3 \text{ be less than } P_1 + P_2 + P_3 + P_4$$

Then the resultant,

$$R = (P_1 + P_2 + P_3 + P_4) - (Q_1 + Q_2 + Q_3)$$

and acts in the same direction as  $P_1, P_2, P_3, P_4$ , and along the same straight line.

If equilibrium existed between these two sets of oppositely directed forces, then their algebraical sum would be zero, or the resultant would vanish; i.e.,

$$(P_1 + P_2 + P_3 + P_4) - (Q_1 + Q_2 + Q_3) = R = 0$$

A familiar illustration of the above reasoning is the game of "the tug of war," when, say, a batch of sailors are pitted against a corresponding number of soldiers, each batch pulling their utmost at the opposite ends of a rope, and in opposite directions, with the view of obtaining a resultant.

**Terms and Definitions used in Measuring or in Calculating the Values of Pressures and their Effects.**—The only terms (except in special cases) which are herein used for the above purposes, are:—

(1) *Pressure*; (2) *Total Pressure*; (3) *Intensity of Pressure*; (4) *Resultant Pressure*; (5) *Centre of Pressure*.

**DEFINITIONS.**—(1) *Pressure* is a general term for the value or amount of force acting between bodies. In certain cases the words push or pull may be used.

*Note.*—The general term *Pressure* has hitherto been loosely used to mean either the *Total Pressure* ( $P$ ) or to indicate the *Intensity of Pressure* ( $p$ ). Hence it is desirable to clearly state in all problems and writings which of these is meant when the term pressure is alone used.

(2) *Total Pressure* ( $P$ ) means the *whole* force acting between bodies. It is usual to measure or estimate *total pressure* in lbs., thus,  $P = pA$  lbs. But, when the values are great, it may be given or found in cwts., or in tons.

(3) *Pressure* or, more exactly, *Intensity of Pressure* ( $p$ ) is the pressure per unit area; (e.g.) lbs. per square inch or lbs. per square foot. *Ex.* Let  $p = 10$  lbs. per sq. in.;  $A = 100$  sq. ins. Then  $P = pA = 10 \times 100 = 1000$  lbs.

(4) *Resultant Pressure* ( $P_R$ ) is the mean pressure per unit area ( $p$ ) multiplied by the total area ( $A$ ) under consideration. Here,  $P_R = pA$ .

(5) *Centre of Pressure* ( $P_C$ ) is the point at which a *single force* will balance the *total pressure*. Or, it may be defined as the point at which the resultant pressure acts.

**Differences in Nomenclature.**—Students may find in "Examination Questions," or in papers read before various Institutions, and in books by different authors, other terms than those just defined. For example, the word *Thrust* has been recently introduced to mean the *total pressure* ( $P$ ); and the simple word *pressure* to indicate the *Intensity of Pressure* ( $p$ ), or *Pressure in lbs. per square inch* ( $p$ ), as defined above.

Moreover, to show the necessity for a more definite distinction between total and intensity of pressure, if you ask an Engineer or Manufacturer the pressure of steam in his boiler, he will say (e.g.) 100 lbs.; when it is actually 100 lbs. *per square inch* by the steam gauge, i.e., where  $p = 100$  lbs.

*Resultant Thrust* is sometimes used for the aforementioned *Resultant Pressure* or *Pressure Resultant*.

The Author believes that these new terms only confuse students. It has hitherto been the custom amongst Mechanical Engineers and Naval Architects to chiefly *confine* the words *Thrust* and *Resultant Thrust* to problems and cases dealing with the screw propellers and the thrust blocks in connection with the screw shafts of steamships.

**NOTE FOR PAGE 3.**—The word *Vector* was used in defining the conditions of equilibrium in frames, consequently it may be as well to define the following terms here :—

**Scalar.**—A quantity which has no relation to definite direction in space, or which is considered apart from such direction, is called a “Scalar” or “Scalar-Quantity.”

**Vector.**—A geometrical quantity which is related to a definite direction in space is called a “Vector” or “Vector-Quantity.”

**Vector-Quantity.**—This requires for its complete determination (1) the magnitude, (2) the direction, and (3) the *sense* to be given.

A vector-quantity may be geometrically represented by a line, if—

- (1) The length of the line represents to scale the magnitude of the quantity.
- (2) The line be placed in the proper direction.
- (3) The proper sense or way be given to the line.

The sense is usually indicated by an arrowhead on the line.

The line itself with its direction and sense is called a *Vector*→

Suppose that a force of known magnitude acts along a line from P to →Q; then, the Vector is written down as  $\overrightarrow{PQ}$ , with a bar-line over the two letters P and Q.

Any quantity, whether *scalar* or *vector* (considered as occupying a definite position in space), is said to be *localised*. Thus the mass of a body in a given position is a *localised scalar*, and a force acting on a body at a definite point is a *localised vector*.

**Vector Sum.**—The sum of a number of vectors is often called the Resultant Vector, and in relation to this resultant the other Vectors are called Components.

To add a number of vectors, place the first anywhere, the beginning of the second to the end of the first, and so on, then the vector from the beginning of the first to the end of the last is the SUM OF THE GIVEN VECTORS (*Henrici and Turner*).

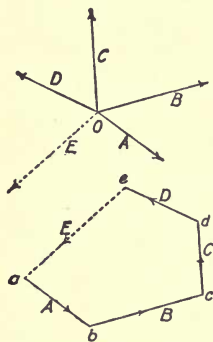
**Rotor.**—A localised vector is called a *Rotor* (*Clifford*).

**EXAMPLE.**—If A, B, C, D denote four vectors acting at the point O to find the sum or resultant.

From any convenient point *a*, draw *ab* parallel in direction and containing as many units of length as there are units in the vector A. From the end *b*, draw *bc* parallel to B, and equal in magnitude to it. Similarly, *cd* parallel and equal to C, and *de* equal and parallel to D. The resultant in direction and magnitude of the four vectors is the dotted line joining the initial point *a* to the final point *e*.

If a vector E, equal in magnitude but in the contrary direction, acts at O, then the five vectors A, B, C, D, E will have a sum equal to zero, or the five vectors are in equilibrium.

The polygon *abcdea*, when applied to forces is called the polygon of forces. Hence, if any number of vectors act at a point and can be represented by the sides of a closed polygon taken in order, the vectors are in equilibrium.



**SUM OF VECTORS  
ACTING AT A POINT.**

We shall return to the graphic representation of forces, &c., when we come to deal with the parallelogram and triangle of forces and their application to ascertaining the stresses on simple structures.\*

**Note Regarding Engineering Calculations.**—Engineering students should clearly understand, that there is no necessity or advantage to be gained in working out their arithmetical results to a greater nicety, than the tools, rules, gauges, and instruments placed at their disposal will enable them to measure with accuracy.

For example, a skilled mechanic who is furnished with a steel footrule and callipers may express his ideas of length to the  $\frac{1}{16}$ , or at best to the  $\frac{1}{128}$  (.01) of an inch, which is equivalent to 1 part in 1200. It would be ridiculous, therefore, to ask him to calculate such lengths to the third decimal place.

A carpenter who uses the well-known 3 ft. rule may be perfectly satisfied if he measures to the  $\frac{1}{16}$  of an inch, i.e., to 1 part in 500.

A mason will usually be satisfied if he can measure to within  $\frac{1}{2}$  of an inch ! The captain of a sailing vessel could not be expected to spot the position of his ship at sea, to within a couple of nautical miles ; so it is no use to ask him to place his boat on a particular meridian !

**Note on Questions in Proportion.**—When dealing with all such questions, it is best for the student to ask himself—

- (1) What is required ? Then to put the corresponding given value in the 3rd term, with  $x$ ,  $y$ , or  $z$  in the 4th for the value to be found.
- (2) Will the answer be greater or less than the value in the 3rd term ? Then to put the *greater* or the *less* given value (according to this answer) into the 2nd term, and the remaining known quantity in the 1st term.

The answer for  $x$ ,  $y$ , or  $z$  is then equal to the product of the 2nd and 3rd terms, divided by the 1st.

EXAMPLES:—

$$\begin{array}{l} \text{1st} : \text{2nd} :: \text{3rd} : \text{4th (Terms).} \\ \text{(a) If,} \quad 100 : 100 :: 1000 : x. \end{array}$$

$$\text{Then,} \quad x = \frac{100 \times 1000}{100} = 10,000.$$

$$\text{(b) If,} \quad 100 : 10 :: 1000 : y.$$

$$\text{Then,} \quad y = \frac{10 \times 1000}{100} = 100.$$

This method is more convenient to the elementary student than dealing directly with fractional ratios.

\* We have intentionally made this Lecture a short one, and have not appended any questions, because at the first meeting of a session the Lecturer has to give a series of general instructions to his students, and the class is seldom so completely formed as to make it worth while setting any home work until the second meeting.

## LECTURE II.

**CONTENTS.**—Work—Unit of Work—Examples I. II. III. IV.—Work done against a Variable Resistance—Example V.—Diagrams of Work—With Uniform Resistance—With a Uniformly Increasing Resistance—With a Uniformly Decreasing Resistance—With a Combination of Uniform and Variable Loads—Example VI.—Power or Activity—Units of Power—The Horse-power Unit—To find the Horse-power of any working Agent—Example VII.—Uses of Squared Paper—Clark's Adjustable Curve—Example VIII.—Questions.

**Work.**—If a force acts upon a body and causes that body to move through a distance, then the force is said to have done work. It does not matter how long the operation takes, whether a second, a minute, an hour, or a day, or even a year, the same amount of work is done by the force acting through the distance. Time, therefore, does not come into the question of estimating work done, but we must have a force overcoming a resistance through a definite distance. If the force applied be inadequate to overcome the resistance of the body to motion, then no work is done. The amount of work done therefore depends *solely* upon the product of the force applied (or the resistance overcome) and the distance through which it acts in its own direction.

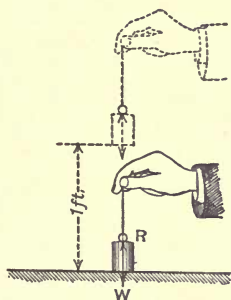
Or,  $\text{Work} = \text{Force} \times \text{Distance}.$

**Unit of Work.\***—*The unit of work, is the work done in overcoming unit force through unit distance.* Now, since the British unit of force is the pound, and unit distance the foot, the British unit of work is called the *foot-pound*, and is therefore the work done when a resistance of 1 lb. is overcome through a distance of 1 foot.

**EXAMPLE I.**—If a weight of 1 lb. be elevated a vertical distance of 1 ft. against the force of gravity, then 1 foot-pound of

\* In the case of heavy work the unit *foot-ton* is sometimes used in this country. A foot-ton simply means one ton raised one foot high against gravity, or a force of one ton exerted through a distance of one foot, or a resistance of one ton overcome for a distance of one foot. In Electrical Engineering the unit of work is the work done in overcoming a resistance of one *dynes* through a distance of one *centimetre*. It is called the *Erg*. Since the weight of 1 gramme is = 981 dynes, the work done in raising 1 gramme through a vertical height of 1 centimetre against the force of gravity is 981 ergs or (g) ergs. One foot-pound =  $1.356 \times 10^7$  ergs.

work has been performed. If 10 pounds be elevated vertically through a distance of 10 ft., then result is  $(10 \times 10) = 100$  ft.-lbs. of work.



UNIT OF WORK.

$W = 1$  lb. weight.

$R = 1$  lb. reaction.

EXAMPLE II.—If a body offers a constant resistance to motion in *any direction* of  $P$  lbs., and if it be forced along a distance of  $L$  ft., in that direction, then the work done is  $P \times L$  ft.-lbs.

$$\begin{array}{lcl} \text{Or, Work done} & = & \text{Force} \times \text{Distance} \\ \text{i.e. Foot-pounds} & = & P \text{ lbs.} \times L \text{ feet.} \end{array}$$

Suppose a cart with its load weighs  $W$  lbs. and offers a constant resistance of  $P$  lbs. to traction along a road, and that it is pulled through a distance of  $L$  feet; then,

$$\text{The work done} = P \times L \text{ (ft.-lbs.)}$$

EXAMPLE III.—In drawing a loaded cart along a level road, a horse has to exert a constant pull of 100 lbs.; how much work will be done in 10 minutes supposing the horse to walk at the rate of 6000 yards an hour?

$$\left. \begin{array}{l} \text{Distance in feet through} \\ \text{which the resistance of} \\ \text{100 lbs. is overcome in 10} \\ \text{minutes.} \end{array} \right\} = \frac{6000 \text{ (yds.)} \times 10 \text{ (m.)} \times 3 \text{ (ft.)}}{60 \text{ (m.)}}$$

$$\text{,, ,,} = 3000 \text{ ft.}$$

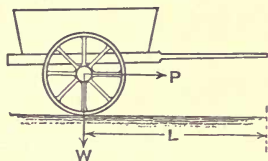
$$\text{Work done in 10 minutes} = P \times L.$$

$$\text{,, ,,} = 100 \times 3000.$$

$$\text{,, ,,} = 300,000 \text{ ft.-lbs.}$$

EXAMPLE IV.—A traction engine is employed to draw a loaded waggon along a level road where the resistance to be overcome is

Fig. for Example III.



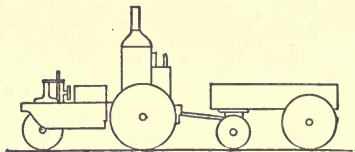
ILLUSTRATING WORK DONE.

$W =$  Weight in lbs.

$P =$  Pull in lbs.

$L =$  Length in feet.

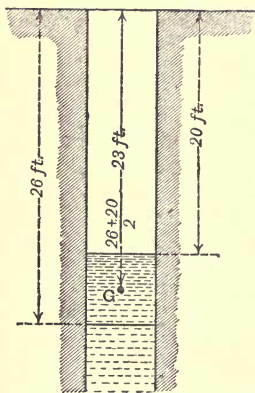
100 lbs. per ton. How many foot-pounds of work are expended in drawing 10 tons over 100 yards?



TRACTION ENGINE AND LOAD.

1. Tractive force = 100 lbs. per ton.
  2. Total pull,  $P$ , = 100 (lbs.)  $\times$  10 (tons)
  3. Distance,  $L$ , = 100 (yds.)  $\times$  3 (ft.)
  4. Work done =  $P \times L$ .
- " " =  $100 \times 10 \times 100 \times 3$ .  
 " " = **300,000 ft.-lbs.**

**Work Done against a Variable Resistance.**—If the resist-



WORK VARYING  
UNIFORMLY.

ance varies whilst the force overcoming it acts through a known distance, then the work done will be measured by the product of the average resistance and the distance. If the resistance varies uniformly, its average can be found by adding its values at the commencement and end of the motion, and dividing by two.

**EXAMPLE V.**—Explain the method of estimating the work done by a force, and define the unit of work. The surface of the water in a well is at a depth of 20 feet, and when 500 gallons have been pumped out, the surface is lowered to 26 feet. Find the number of units of work done in the operation, the weight of a gallon of water being 10 lbs. (S. and A. Exam. 1887.)

For an answer to the first part of this question refer to the previous part of this lecture.

1. Weight of water raised = weight of 500 gallons.
- " " " =  $500 \times 10$  lbs.

Or, . . .  $P$ , = 5000 lbs.

2. Mean height water is lifted = { Distance through which the centre of gravity,  $G$  (of raised water), has been elevated.

$$= \frac{20 + 26}{2} \text{ ft.}$$

Or,  $L = 23 \text{ ft.}$

3. Work done  $= P \times L.$

$$= 5000 \times 23.$$

$$= 115,000 \text{ ft.-lbs.}$$

**Diagrams of Work.**—(1) *Against a Uniform Resistance.*—If the resistance overcome is uniform, then the work done may be graphically represented by the area of a rectangle.

To find the work done in overcoming a uniform resistance of 5 lbs. through a distance of 10 ft.: Plot down a vertical line to any convenient scale to represent  $P$  (or 5 lbs.) and a horizontal line to the same scale to represent  $L$  (or 10 ft.)

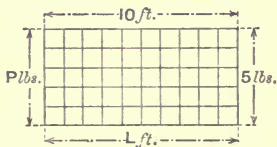


DIAGRAM OF UNIFORM WORK.

Then complete the *rectangle*.

The area  $P \times L$  or  $5 \times 10 = 50 \text{ ft.-lbs.}$  of work.

In the accompanying figure a scale of  $\frac{1}{10}$  inch has been used to represent both 1 lb. and 1 ft., consequently each of the small squares represents to scale one foot-pound of work.

(2) *With a Uniformly Increasing Resistance.*—If the resistance uniformly increases—for example, in the raising of a length of rope or chain vertically by one end from the ground, then the work done may be graphically represented by the area of a right-angled triangle, where  $P$  represents the total weight of chain in lbs., and  $L$  its total length in feet.

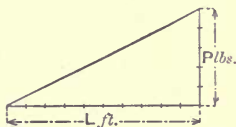


DIAGRAM OF WORK FOR AN INCREASING RESISTANCE.

$$\therefore \text{The Total Work done} = \frac{P \times L}{2} \text{ ft.-lbs.}$$

Here the work done per foot of length of chain lifted, **uniformly** increases from a minimum to a maximum, until the whole rope or chain is off the ground. When any known length,  $l$ , has been lifted, then the area enclosed by the triangle whose horizontal side is  $l$ , and vertical side  $p$  represents the work done.

- (3) *With a Uniformly Decreasing Resistance.*—If the resistance uniformly decreases, as in the case of winding a rope or chain upon the barrel of a winch or crane, then the work done will also be represented graphically by the area of a right-angled triangle, where  $P$  represents the total weight of rope or chain in pounds being lifted at the start, and  $L$  its length in feet.

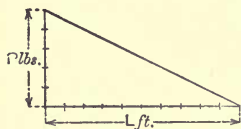


DIAGRAM OF WORK FOR A DECREASING RESISTANCE.

$$\therefore \text{The Total Work done} = \frac{P \times L}{2} \text{ ft.-lbs.}$$

Here the work done per foot of length of chain lifted, gradually diminishes from a maximum at the start to a minimum, when the last foot is being lifted.

As in Case (2), you can at any time know the work done or still to be done from the scale diagram, if you know the length of chain lifted or to be lifted.

For example, if  $l$  feet have still to come on to the barrel, then the vertical ordinate  $p$  on the scale diagram will represent the pull being exerted at the time, and consequently  $\frac{p \times l}{2}$  represents the work still to be done.

Or, generally, with any gradually increasing or decreasing resistance the work done is equal to the mean of the average resistance in lbs.  $\times$  the distance through which it acts in feet.

- (4) *With a Combination of Uniform and Variable Loads.*—When one part of a load is uniform and another part variable, as in the case of lifting a weight with a chain, by winding the chain on the barrel of a winch or crane, the diagram of work for the uniform load is naturally a rectangle, and for the chain a triangle if the chain is completely wound on to the barrel, or a trapezoid if there is still some portion of it to be lifted.\*

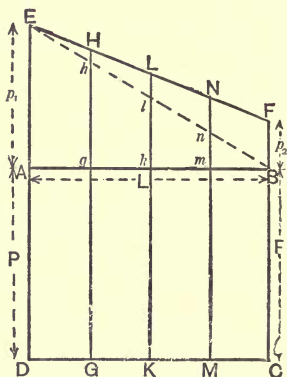


DIAGRAM OF WORK FOR A COMBINATION OF UNIFORM AND VARIABLE LOADS.

\* See p. 5 of the Author's "Elementary Manual on Steam and the Steam Engine" for how to find the Area of a Trapezoid.

Let  $P$  . = the uniform pull required to lift the load or overcome the uniform resistance.

$L$  . = the distance the weight is lifted.

$p_1, p_2$  = the weights of chain hanging at the commencement and at the finish of the lift.

Work done in lifting the uniform load  $= P \times L$

Work done in lifting the variable load  $= \frac{p_1 + p_2}{2} \times L$

∴ Whole work  $= P \times L + \frac{p_1 + p_2}{2} \times L = (P + \frac{p_1 + p_2}{2})L$

„ „ = Area of the figure, DEFC.

The diagram DEFC represents the work done and also the variation of the resistance during the lift. The rectangle ABCD represents the work done in overcoming the uniform load, and the trapezoid ABFE the work done in overcoming the variable load. The resistance at any instant of the lift will be represented by the vertical line drawn from the horizontal base DC to the inclined line EF through the point on DC or AB which represents the position of the load at that instant. The part of this vertical line intercepted between AB and EF will represent the resistance offered by the *variable part of the load* at the instant considered. Thus, at the commencement of the lift the total resistance is  $P + p_1$  and represented by DE, at the end of the lift the total resistance is  $P + p_2$  and represented by CF. At  $\frac{1}{4}$ ,  $\frac{1}{2}$ , and  $\frac{3}{4}$  of the lift the total resistances are represented by the vertical lines GH, KL, and MN respectively, while the resistances due to the variable part of the load at these points are represented by the lengths  $gH$ ,  $kL$ , and  $mN$  respectively. If the final resistance due to the variable part of the load was zero (as would be the case if the whole of the chain were wound on to the barrel) then the diagram of work for this part of the load would be the triangle AEB.

EXAMPLE VI.—Explain fully the mode of measuring the work done by a force. What unit is adopted? A weight of 2 cwts. is drawn from a mine, 30 fathoms deep, by a chain weighing 1 lb. per linear foot; find the number of units of work done. (S. and A. Exam. 1893.) Also find the resistance offered when the weight has been raised through  $\frac{1}{4}$ ,  $\frac{1}{2}$ , and  $\frac{3}{4}$  of the whole depth respectively.

ANSWER.—(1) The work done by a force is measured by the product of the force into the distance through which that force moves in its own direction. If  $P$  be the force in pounds, and  $L$  the distance in feet through which it moves in its own direction, then

Work done  $= P \times L$  ft. lbs,

(2) The unit of work adopted in this country is the work done when a force of one pound is moved through a distance of 1 foot, and is called the foot-pound (ft.-lb.).

(3) Referring to the previous figure, make AB to represent  $30 \times 6 = 180$  ft., the depth of the mine, AD to represent  $2 \times 112 = 224$  lbs. the weight of material raised, AE to represent  $1 \times 180 = 180$  lbs. the weight of chain at beginning of lift. Then assuming the whole of the chain to be wound up, complete the rectangle ABCD, and join E and B. The area of the figure DEBC then represents the work done.

$\therefore$  Work done = area DEBC =  $\frac{1}{2} (DE + CB) \times AB$ . But  $DE = DA + AE = 224 + 180 = 404$  lbs.

$CB = DA = 224$  lbs.,  $AB = 180$  ft.

$\therefore$  Work done =  $\frac{1}{2} (404 + 224) \times 180$  ft.-lbs. = 56,520 ft.-lbs.

(4) The resistance at  $\frac{1}{4}$  lift, or when the weight has been raised 45 ft., is  $Gh = Gg + gh = 224 + \frac{3}{4} \times 180 = 359$  lbs.

At  $\frac{1}{2}$  lift the resistance is  $Kl = Kk + kl = 224 + \frac{1}{2} \times 180 = 314$  lbs.

At  $\frac{3}{4}$  lift the resistance is  $Mn = Mm + mn = 224 + \frac{3}{4} \times 180 = 269$  lbs.

**Power or Activity is the rate of doing work.\***—In estimating or testing the power of any agent the time in which the work is done must be noted and taken into account. Consequently, we speak of the activity or power of a man, of a horse, or of an engine, as capable of doing so many foot-pounds of work per minute.

**Units of Power.†**—The unit of power adopted in this country is called the *horse-power*. It is the rate of doing work at 33,000 ft.-lbs. per min., or 550 ft.-lbs. per sec., or 1,980,000 ft.-lbs per hour.

The Horse-power Unit was introduced by James Watt, the great improver of the steam engine, for the purpose of reckoning the power developed by his engines. He had ascertained by experiment that an average cart-horse could develop 22,000 foot-pounds of work per minute, and being anxious to give good value to the purchasers of his engines, he added 50 per cent. to this amount, thus obtaining  $(22,000 + 11,000)$  the 33,000 foot-pounds per minute unit, by which the power of steam and other engines has ever since been estimated.

\* The word *power* is very frequently misapplied by writers and students, for they often call the mere pull, pressure, or force exercised on or by an agent the power. Students should strenuously avoid this misuse of the word power, and never employ it in any other sense than as expressing a *rate of doing work, or activity*.

† In Electrical Engineering the Unit of Power is called the *Watt*, and it equals 10<sup>7</sup> ergs per second, or 746 Watts = 1 horse-power.

To find the Horse-power of any Working Agent.—*Divide the number of foot-pounds of work which it does in one minute by 33,000.*

Let  $P$  = Pull exerted or resistance overcome in pounds.

$L$  = Length or distance through which  $P$  acts.

$M$  = Minutes the agent is at work.

H.P. = Horse-power.

Then,

$$\text{H.P.} = \frac{P \times L}{33000 \times M}; P = \frac{\text{H.P.} \times 33000 \times M}{L}; L = \frac{\text{H.P.} \times 33000 \times M}{P}.$$

EXAMPLE VII.—In what way is the rate of doing work measured in horse-power?

If 40 cubic feet of water be raised per minute through 330 feet, what horse-power of engine will be required, supposing that there is no loss of friction or other resistances? *Note.*—1 cubic foot of water weighs  $62\frac{1}{2}$  lbs. (S. and A. Exam. 1892).

ANSWER.—The rate of doing work, as measured in horse-power, is equivalent to 33,000 foot-pounds of work done per minute.

1st. 1 cubic foot of water weighs  $62\frac{1}{2}$  lbs.

∴ 40 cubic feet of water weigh  $40 \times 62\frac{1}{2} = 2500$  lbs.

$$2\text{nd. Work done per minute} = \frac{P \times L}{M} = \frac{2500 \text{ (lbs.)} \times 330 \text{ ft.}}{1}$$

$$3\text{rd. } \therefore \text{H.P.} = \frac{P \times L}{33000 \times M} = \frac{2500 \times 330}{33000 \times 1} = \frac{825000}{33000} = 25.$$

*Note.*—Students will find it a great advantage, as well as a saving of time not to multiply figures together until the last stage of the answer has been reached, and then to cancel all common factors in numerator and denominator. For example, in the answer to the above question we might proceed thus—

1st. 40 cubic feet of water =  $40 \times 62\frac{1}{2}$  lbs.

2nd. Work done per minute =  $40 \times 62\frac{1}{2} \times 330$  ft.-lbs.

$$3\text{rd. } \therefore \text{Horse-power of engine} = \frac{40 \times 62\frac{1}{2} \times 330}{33000} = \frac{4 \times 62\frac{1}{2} \times 33}{1000} = \frac{4 \times 62\frac{1}{2} \times 33}{1000}$$

$$\therefore \text{H.P.} = \frac{4 \times 62\frac{1}{2} \times 33}{1000} = \frac{250}{10} = 25.$$

The process consists in this—the factor, 330, can be cancelled from numerator and denominator, leaving 100 as the denominator. The factor, 10, can then be divided out of 40 in the numerator and from the 100 in the denominator, thus leaving  $4 \times 62\frac{1}{2}$  as the numerator and 10 as the denominator. The remainder of the work is evident.

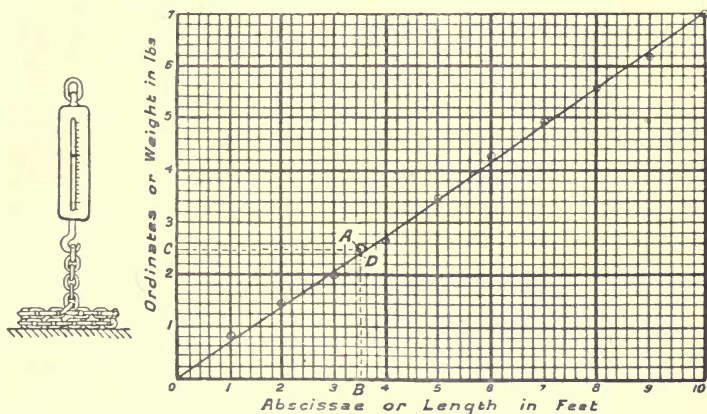
**Uses of Squared Paper.**—Squared paper is made by drawing a number of equally spaced horizontal lines and crossing these by vertical ones at the same distance apart. The paper is consequently covered with a large number of little squares, the sides of which are usually one-tenth of an inch in length. In order to facilitate the measurement of distances, every tenth line, and sometimes every fifth, is heavier or of a different colour to the others.

By the aid of squared paper we can graphically represent how two varying quantities depend upon each other. For example, take the case of a chain being gradually lifted from the ground, as already considered in connection with diagrams of work and shown by the first of the following figures. The length and weight of chain lifted will alter as the upper end is raised; but the suspended weight will always depend upon the length which hangs freely from the spring balance. In fact, any change in the length lifted will produce a corresponding change in the load registered by the balance. If we note the pull indicated by the balance for different lengths of hanging chain, we shall be able to obtain a line or curve which will show to the eye how these two quantities depend upon one another. Suppose we obtain the following results:—

Length of chain lifted in feet. }	1	2	3	3·5	4	5	6	7	8	9	10
Weight of chain lifted in lbs. }	0·8	1·4	2·0	2·5	2·7	3·5	4·3	5·0	5·6	6·2	7·0
Corrected values in lbs. }	0·7	1·4	2·1	2·45	2·8	3·5	4·2	4·9	5·6	6·3	7·0

We shall represent the lengths of the hanging chain by horizontal distances which are termed *abscissæ*, and the corresponding weights by vertical distances called *ordinates*. We choose such a scale for these quantities as will enable us to get them all upon the squared paper; at the same time we keep the scales as large as possible. In this case we have chosen five divisions horizontally to represent one foot, and five divisions vertically for one pound. It is, however, not necessary to adopt equal scales for *abscissæ* and *ordinates*, but we should select the most convenient scale for each according to circumstances.

To find the point corresponding to the fourth column in the table, take B at 3.5 on the horizontal scale, and C at 2.5 on the vertical one. Draw BA vertically and CA horizontally. Then, the intersection of these two lines is the point required. In practice, these lines are not actually drawn, but the point is found by the eye with the assistance of the lines on the paper, and a  $\times$  or  $\odot$  is placed to mark its position. When all the points have been thus plotted from the table, we draw a mean line or curve between them. In this case, it is a straight line passing through



HANGING CHAIN.

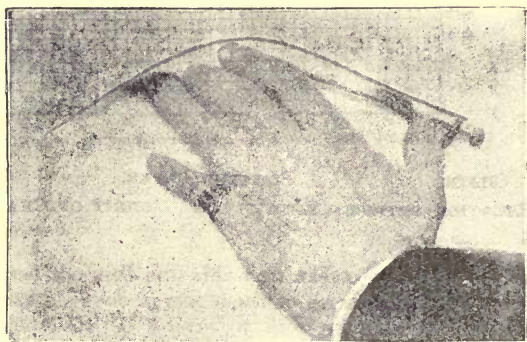
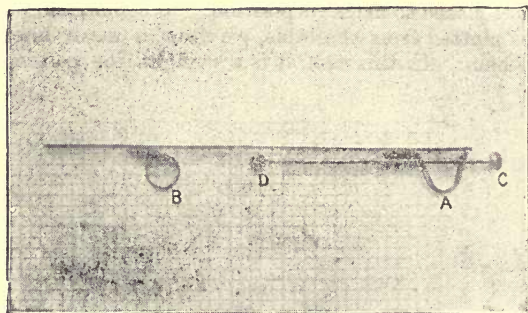
SQUARED PAPER.

RELATION BETWEEN LENGTH AND WEIGHT OF CHAIN.

the *origin* O. It will be seen that this line does not pass through all the points, but that some of them are on one side and some on the other. This may be due to errors of observation or to irregularities in the chain. If we know that the chain is uniform, the points ought to lie along the straight line we have drawn, and we can correct for errors of observation. Thus, the point A should have been at D, and the correct weight for that length of chain is represented by BD, which is 2.45 lbs. We can correct the other values in the same way, and so obtain the numbers shown in the third line of the table.

When the points lie approximately in a straight line, the nearest mean straight line is best found with the help of a fine thread which is stretched and moved among the points until it

lies most evenly amongst them. The positions of its ends are then marked and a line drawn with a straight-edge through these marks. When the points do not lie near a straight line a smooth curve may be drawn through them, either freehand, or by aid of French curves, a thin strip of wood or steel, or Clark's patent



CLARK'S PATENT ADJUSTABLE CURVE.

adjustable curve. This consists of a flexible strip of celluloid with a brass loop A, for the thumb of the left hand, and another B, for the second or third finger, as shown by the accompanying figures. Now, if these two loops are drawn together the celluloid will be formed into some curve, the shape of which can be adjusted by moving the sliding rod C with the right hand and fixing this curve by means of a cord (not shown in the figure) joining B with a V-shaped groove in A. It may also be used as

a set curve for the purpose of transferring a curve from one drawing to another.

When we represent distances along one scale of the squared paper and forces along the other, then the areas such as  $O D B$  indicate to scale the work done in raising the length of chain  $O B$ . This is true, whether the line  $O D$  is straight or curved. We can, however, represent any two quantities which depend on one another by a curve on squared paper.

EXAMPLE VIII.—A body is being acted upon by a variable lifting force. When the body is lifted  $x$  feet the force  $F$  lb. is observed.

$x$	0	15	25	50	70	100	125	150	180	210
$F$	530	525	516	490	425	300	210	160	110	90

Plot on squared paper and find the average value of  $F$  from  $x=0$  to  $x=210$ . What is the work done by  $F$  when the body has been lifted 210 feet? (B. of E. 1904.)

ANSWER—

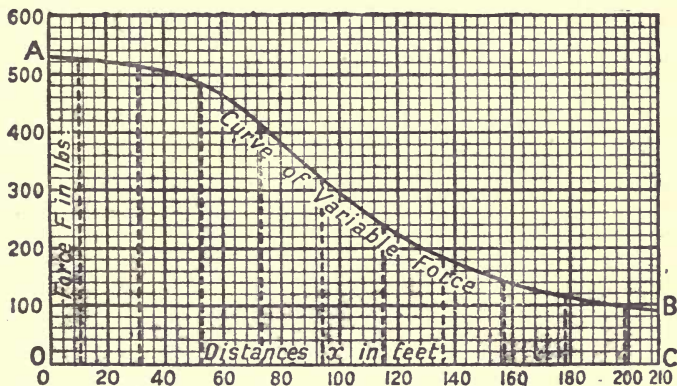


DIAGRAM OF WORK DONE BY A VARIABLE FORCE.

In order that we may obtain the *average* value of  $F$ , we must use squared paper since the lifting force is *variable*. Mark off the distances  $x$  in feet (to scale) along the abscissa  $O C$ , and at each of the values of  $x$ , plot ordinates corresponding to the values of  $F$  in lbs. Join all the points by a curve as shown in the above diagram. The area of this diagram below the curved line  $A B$ , represents the *total work done by the variable force*  $F$  lbs., in passing through a distance of 210 feet.

We can obtain the area of the diagram  $O A B C$  either by aid of a planimeter or by finding the average value of  $F$  and multiplying this by the distance

through which  $F$  has moved, viz., 210 feet. But the ordinary process used by engineers for finding the area of a diagram such as O A B C is to divide O C into 10 equal parts and then to measure the value of  $F$  at the centre of each part. The sum of these values divided by 10 gives us the *average value* of  $F$ . If this number be now multiplied by the value of O C the final result will be the area of the diagram in foot-pounds.

Thus, by the latter method we find the *average value* of  $F = 305$  lbs.

Hence the total work = average value of  $F \times$  distance it acts,

Or, the total work done =  $305 \times 210 = 64,050$  ft.-lbs.

## LECTURE II.—QUESTIONS.

1. Define the unit of work. What name is given to this unit? In drawing a load a horse exerts a constant pull of 120 lbs.; how much work will be done in 15 minutes, supposing the horse to walk at the rate of 3 miles an hour? *Ans.* 475,200 ft.-lbs.

2. How is the work done by a force measured? The resistance to traction on a level road is 150 lbs. per ton of weight moved; how many foot-pounds of work are expended in drawing 6 tons through a distance of 150 yards? *Ans.* 405,000 ft.-lbs.

3. Distinguish between *force* and *work done* by a force. How is each respectively measured? A traction engine draws a load of 20 tons along a level road, the tractive force on the load being 150 lbs. per ton. Find the work done upon the load in drawing it through a distance of 500 yards. *Ans.* 4,500,000 ft.-lbs.

4. Find the number of units of mechanical work expended in raising 136 cubic feet of water to a height of 20 yards. The weight of a cubic foot of water is  $62\frac{1}{2}$  lbs. *Ans.* 510,000 ft.-lbs.

5. A weight of 4 tons is raised from a depth of 222 yards in a period of 45 seconds; calculate the amount of work done. *Ans.* 5,967,360 ft.-lbs.

6. A hole is punched through a plate of wrought-iron one-half inch in thickness, and the pressure actuating the punch is estimated at 36 tons. Assuming that the resistance to the punch is uniform, find the number of foot-pounds of work done. *Ans.* 3360 ft.-lbs.

7. How is work done by a force measured? Give some examples. Set out a diagram of the work done in drawing a body weighing 10 lbs. up a smooth incline 4 feet high, marking dimensions.

8. A train of 12 coal waggons weighing 133 tons is lifted by hydraulic power (two waggons being raised at a time) through 20 feet in 12 minutes. Estimate the work done in foot-tons. Taking the average of work done, how many foot-pounds are done per minute? *Ans.* 2660 ft.-tons; 496,533 $\frac{1}{3}$  ft.-lbs. per minute.

9. The plunger of a force-pump is  $8\frac{1}{2}$  inches in diameter, the length of the stroke is 2 feet 6 inches, and the pressure of the water is 50 lbs. per square inch; find the number of units of work done in one stroke, and plot out a diagram of work to scale. *Ans.* 7517 ft.-lbs.

10. A chain 30 feet long, and weighing 100 pounds per yard, lies coiled on the ground. Find by calculation and by a scale diagram of work how many units of work would be expended in just raising it by the top end from the ground. *Ans.* 15,000 ft.-lbs.

11. A chain, weighing 30 lbs. to the fathom, is employed to lift 1 ton to a height of 30 ft. by winding the chain on a barrel. Find by calculation and by a scale diagram of work, how many units of work will be expended—(a) when the outer end of the chain is brought home to the barrel; (b) when 18 feet of it are still hanging free with the weight at the end of it. *Ans.* (a) 69,450 ft.-lbs.; (b) 28,320 ft.-lbs.

12. Define the following mechanical terms:—Force, work, unit of work, power, activity, and horse-power. A horse drawing a cart at the rate of 2 miles per hour exerts a traction of 156 lbs.; find the number of units of work done in one minute. *Ans.* 27,456 ft.-lbs.

13. In what way is the rate of doing work measured in horse-power? If 100 cubic feet of water be raised per minute through 330 feet, what horse-power of engine will be required, supposing that there is no loss by friction or other resistances? *Ans.* 62 $\frac{1}{2}$  h.p.

14. If a horse, walking at the rate of  $2\frac{1}{2}$  miles per hour, draws 104 lbs. out of a well by means of a cord going over a wheel, how many units of work would he perform in one minute? *Ans.* 22,880 ft.-lbs.

15. What unit do you employ in measuring force, and what unit in measuring the work done by a force? A horse exerting a pull of 40 lbs. per ton draws a load of 15 cwt. along a level road; how far will the horse travel in 10 minutes if he does work on the load at the rate of  $\frac{1}{2}$  horse-power? *Ans.* 5500 ft.

16. Distinguish between the expressions "foot-pound" and "horse-power" by giving a clear definition of each. A bucket when filled with water weighs 180 lbs., and is raised at a uniform rate from a depth of 150 feet in eight minutes. Find the work done in one minute. *Ans.* 3375 ft.-lbs.

17. What work in foot-pounds is done in raising the materials for building a brick wall 50' high, 12' long, and 2' 3" in thickness, if one cubic foot of brickwork weighs 112 lbs.? *Ans.* 3,780,000 ft.-lbs.

18. A man of 150 lbs. climbs a hill regularly 1200' vertically per hour; at another time he climbs a staircase at  $2\frac{1}{2}$ ' per second. Find in each case the horse-power expended by the man. *Ans.* .09 h.-p.; .68 h.p.

19. An express train going at 40 miles per hour weighs 150 tons; the average pull on it is 12 lbs. per ton, what is the horse-power exerted? This power is only 40 per cent. of the total indicated power of the engine; find the indicated power. *Ans.* 192 h.-p.; I. H. P. = 480.

20. Water at 750 lbs. per square inch pressure acts on a piston one square foot in area, through a stroke of 1 foot; what is the work that such water does per cubic foot? and per gallon? If an hydraulic company charges 18 pence for a thousand gallons of such water, how much work is given for each penny? *Ans.* 108,000 ft.-lbs.; 17,280 ft.-lbs.; 960,000 ft.-lbs.

21. Explain how squared paper is used, and mention a few of the purposes to which it is applied.

22. Plot out a curve from the following data showing the pressure on a piston at various distances from the commencement of the stroke:—

Distance in feet.	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
Pressure in lbs. per square inch. }	20	21	21	20	19	18.5	18	13.5	9	4.5	0

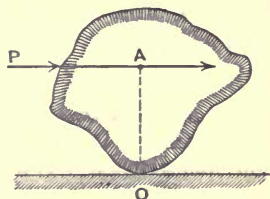
23. A chain weighing 10 lbs. per foot of its length is 240 feet long, and hangs vertically; what work is done in winding the chain upon a drum? *Ans.* 288,000 ft.-lbs.

## LECTURE III.

CONTENTS.—The Moment of a Force—Principle of Moments applied to the Lever—Experiments I. II. III.—Pressure on and Reaction from the Fulcrum—Equilibrant and Resultant of two Parallel Forces—Couples—Centre of Parallel Forces or Position of Equilibrant and Resultant—Centre of Gravity—Examples of Centre of Gravity—The Lever when its weight is taken into Account—Examples I. II.—Position of the Fulcrum—Example III.—Questions.

*The Moment of a Force, with respect to a point, is equal to the force multiplied by the perpendicular distance from the point to its line of action.*

For example, suppose a body to be resting on the point O, and a force, P, to be applied to the body in the direction PA. Then, if the perpendicular distance from O to the line of action of the force be OA, the *moment of the force P*, tending to turn the body about the point O, is  $P \times AO$ . If the force be reckoned in pounds, and the perpendicular distance in feet, the product will be in pounds-feet. The student must therefore avoid confusing the answer with ft.-lbs. of work.



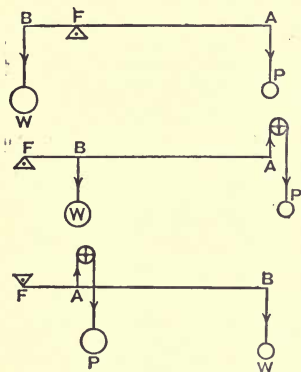
MOMENT OF A FORCE.

**Principle of Moments.**—If any number of forces act in one plane on a rigid body, and if these forces are in equilibrium; then the *principle of moments* asserts that the sum of the moments of those forces which tend to turn the body in one direction about a point, is equal to the sum of the moments of the forces which tend to turn the body in the opposite direction about the same point.

Or, to state the principle more concisely, *the opposing moments about the point are equal when equilibrium exists.*

If the moments of those forces which tend to turn the body to the right hand (i.e., in the direction of the motion of the hands of a clock) be called *positive* (+), and the moments of the remaining set of forces which tend to turn the body to the left hand (i.e., in the opposite direction to the movement of the hands of a clock) be called *negative* (−), then the *algebraical sum of the moments of the forces which act in one plane, and which are in equilibrium about a point, is zero.*

**Principle of Moments applied to the Lever.**—A lever is simply a rigid rod, bar, or beam, capable of turning about a fixed point called the fulcrum (F). Acting on the lever in one direction is a force or set of forces which we shall term the pull or pressure (P), and in the other direction there is the resistance or set of resistances to be overcome, which we shall term the weight (W). The pressure, P, and the weight, W, produce a reaction at the fulcrum, which is called the equilibrant (E).



LEVERS IN EQUILIBRIUM.

The parts of the lever between the fulcrum and the pressure and between the fulcrum and the weight are called the arms of the lever.

The accompanying three figures show three ways in which F, P and W may be arranged with a straight

lever.\* *In each case, the opposing moments about the fulcrum are equal, when the lever is in equilibrium.*

$$\text{Or,} \quad P \times AF = W \times BF$$

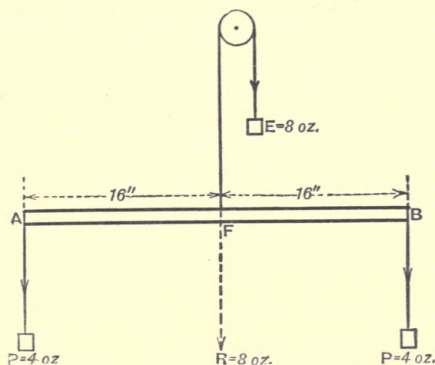
*satisfies the conditions for equilibrium in the case of a lever.*

**EXPERIMENT I.**—To prove the foregoing statements, take a rigid homogeneous bar, AB, of uniform section. Let the bar be of yellow pine, 1 inch deep,  $\frac{1}{2}$  inch broad, and 32 inches long. Attach to the ends, A and B, light flexible cords with small hooks at their lower ends, and attach to the middle of the bar at F another light flexible cord, and pass this cord over a pulley having a minimum of friction at its bearings. Fix such a weight to the free end of this middle cord as will just counterpoise the bar and cords. Test the accuracy of this preliminary adjustment by

\* The levers represented by the above three figures are assumed to be without weight. A force, P, acts through a perfectly flexible, weightless cord at A, and another force, W, acts also through an exactly similar cord at B, with the fulcrum at F in each case. In the second and third case the cord attached at A passes over a frictionless pulley in order to give the necessary direction to the force P. These three relative positions of P, W and F used to be termed the first, second and third order of levers; but there is no necessity for any such distinction, since all the student has to remember is this, that when equilibrium exists the opposing moments about the fulcrum are equal, i.e., ( $P \times AF = W \times BF$ ), or,  $\frac{P}{W} = \frac{BF}{AF}$ , or  $\frac{W}{P} = \frac{AF}{BF}$

The ratio W to P is termed the *theoretical advantage* of the lever.

Observing whether the bar hangs horizontal, and, if pulled down or up a little, whether the weight balances the bar and cords. Now affix equal weights,  $P$ , of, say, 4 oz., to the cords hanging



#### EXPERIMENT I. ON PARALLEL FORCES.

from the ends  $A$  and  $B$ , and add an equilibrating weight,  $E$ , of 8 oz. to the end of the central cord. You will find that the bar will come to rest in a horizontal position, thus proving that—

$$P \times AF = P \times BF$$

*i.e.*,  $4 \text{ (oz.)} \times 16'' = 4 \text{ (oz.)} \times 16''$

Or,  $P : P :: BF : AF$

$$\text{i.e., } \frac{P}{P} = \frac{BF}{AF} = \frac{16}{16} = \frac{4}{4} = \frac{1}{1}$$

Also, that the equilibrant,

$$E = P + P$$

$$8 \text{ oz.} = 4 \text{ oz.} + 4 \text{ oz.}$$

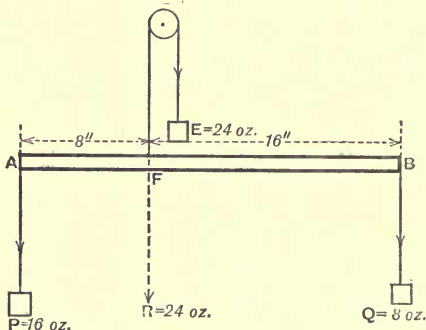
If  $P$  and  $P$  are now removed from the ends  $A$  and  $B$ , and a single weight,  $R$ , of 8 oz. be hung from  $F$  (as represented by the vertical dotted line and arrow), the result as far as the balancing of the system is concerned will be unaffected.

Consequently,  $R = E = P + P$

*i.e.*,  $8 \text{ oz.} = 8 \text{ oz.} = 4 \text{ oz.} + 4 \text{ oz.}$

Or, the resultant of two equal parallel forces acting in the same direction is equal to the sum of the two forces, and acts midway between them and parallel to them—*i.e.*, at the same point as the equilibrant, and in the same line therewith, but in the opposite direction.

EXPERIMENT II.—Take another rigid homogeneous bar, AB, of the same uniform section as the previous bar, but let its length be 24 inches. Attach cords with hooks to the ends A and B, and to a point F, say 8 inches from A and 16 inches from B. Pass this latter cord over the guide-pulley, and fix it there until you



EXPERIMENT II. ON PARALLEL FORCES.

have just added sufficient weight to the end A to balance the longer end BF; then unfix the end of the middle cord, and attach such a weight to it as will counterpoise the whole system. Now attach to the cord at A a weight  $P = 16$  oz.; to the other end, B, a weight  $Q$ ; and a weight at E, so as to again balance the whole system. It will be found that  $Q$  equals 8 oz. and  $E$  equals 24 oz., thus proving that—

$$\begin{array}{rcl} P & \times & AF \\ 16 \text{ (oz.)} & \times & 8'' = 8 \text{ (oz.)} \times 16'' \\ & & BF \end{array}$$

Or,  $P : Q :: BF : AF$

i.e.,  $\frac{P}{Q} = \frac{BF}{AF} = \frac{16}{8} = \frac{2}{1}$

Or, the point F is twice the distance from the end B that it is from the end A, and P has twice the value of Q.

Also, that the equilibrant,

For, 
$$\begin{array}{rcl} E & = & P + Q \\ 24 \text{ oz.} & = & 16 \text{ oz.} + 8 \text{ oz.} \end{array}$$

If P and Q be now removed from the cords at A and B, and a single weight, R, of 24 oz., be hung from F (as represented by the vertical dotted line and arrow), the result, as far as the balancing of the system is concerned, will be unaffected.

Consequently,  $R = E = P + Q$   
 For,  $24 \text{ oz.} = 24 \text{ oz.} = 16 \text{ oz.} + 8 \text{ oz.}$

Or, *the resultant of any two parallel forces acting in the same direction is equal to the sum of the two forces, and acts parallel to them and at a point between them, so that the ratio of the forces is inversely proportional to their distances from the point; or so that—*

$$P : Q :: BF : AF$$

**Pressure on, and Reaction from, the Fulcrum.**—You may also conclude from these two experiments, if the lever had been balanced on a knife-edge or journals, that the *pressure* on the fulcrum *due to the forces* P and Q would have been equal to and act in the direction of the *resultant* R, and that the *reaction* from the fulcrum would have been equal to and act in the direction of the *equilibrant* E.

**EXPERIMENT III.**—Supposing that in the last experiment, after adjusting the lever by placing a counterpoise weight at A, in order to bring the beam to a horizontal position, and after balancing the weight of the beam and cords by an equivalent weight at position E, you added a weight Q, of 8 oz., to the cord at B, and a weight E, of 24 oz., to the cord attached at F, the beam would turn, and would only be brought to a horizontal position by attaching a weight at A of 16 oz. Hence you observe that P acts at A as the equilibrant both in direction and magnitude to the two unequal parallel *unlike* forces Q and E. Consequently a force equal and opposite in direction to P would be the resultant of the two forces Q and E; and it would replace their combined effect on the balanced beam.

Further,  
 for,  $P = E - Q;$   
 $16 \text{ oz.} = 24 \text{ oz.} - 8 \text{ oz.}$

And the moments about the position where the equilibrant acts are equal,

for  
*i.e.*,  $Q \times BA = E \times FA$   
 $8 \text{ (oz.)} \times 24'' = 24 \text{ (oz.)} \times 8''$

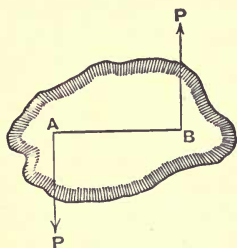
**Equilibrant and Resultant of Two Parallel Forces.**—From the above experiments you conclude that the equilibrant and the resultant of *any two like* parallel forces are equal to their sum, and any two *unlike* parallel forces are equal to their difference.

**Couples.**—When the two parallel forces are equal and act in opposite directions upon a body, they are termed a *couple*. The perpendicular distance between the two forces is termed "*the arm of the couple*," and the "*moment of the couple*" is the product of

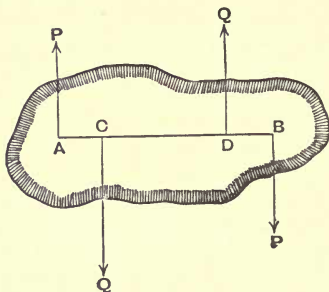
one of the forces and the arm. *A couple simply tends to cause rotation of the body upon which it acts, for it has no resultant, since*

$$R = P - P = 0.$$

One couple can, however, be equilibrated or balanced by another couple of an equal moment, acting in the same plane, and tending to



A COUPLE.



TWO BALANCING COUPLES.

turn the body in the opposite direction. In the accompanying figure the couple  $P, AB, P$  will be balanced by the couple  $Q, CD, Q$  if their moments are equal; *i.e.*, if

$$P \times AB = Q \times CD.*$$

We shall frequently have to refer to practical examples of couples, such as in a ship's capstan, the screw press used in copying manuscript, pressing bales of goods, and the fly press for punching holes in thin plates, or for stamping or embossing metals, &c.

**Centre of Parallel Forces, or Position of Equilibrant and Resultant.**—From Experiment III. and the accompanying figure to Experiment II., you conclude that the position where the equilibrant and resultant act is such, with respect to the positions where the forces act, that the moments of the forces *about that position* are equal and opposite in effect upon the lever.

$$\text{For, } Q \times BA = E \text{ (or } R) \times FA; \text{ or, } \frac{Q}{E} = \frac{FA}{BA}$$

\* Let  $P = 8$  lbs., and  $Q = 10$  lbs.;  $AB = 10$  ft. and  $CD = 8$  ft.  
 Then,  $P \times AB = Q \times CD$   
 Or,  $8 \times 10 = 10 \times 8$   
 $80 = 80$

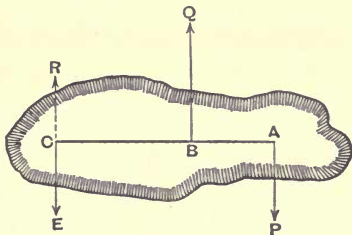
$$\text{i.e., } 8 \text{ (oz.)} \times 24'' = 24 \text{ (oz.)} \times 8''; \text{ or, } \frac{8}{24} = \frac{8}{24} = \frac{1}{3}$$

$$\text{And } P \times AB = E \text{ (or } R) \times FB; \text{ or, } \frac{P}{E} = \frac{FB}{AB}$$

$$\text{i.e., } 16 \text{ oz.} \times 24'' = 24 \text{ (oz.)} \times 16''; \text{ or, } \frac{16}{24} = \frac{16}{24} = \frac{2}{3}$$

The fulcrum F, where the equilibrant and resultant act, is termed the *centre* of the two parallel forces, and it is  $\frac{1}{3}$  of the length of the lever from one end, and  $\frac{2}{3}$  from the other end.

Reasoning generally from this particular case, if you have any two unequal *unlike* parallel forces, P and Q, acting on a body in the directions AP and BQ respectively, and of which Q



CENTRE OF PARALLEL UNLIKE FORCES.

is the greater force, then if the line AB be drawn perpendicular to the directions of these forces, and prolonged, a single force E, parallel to P and Q and equal to  $Q - P$ , will balance these two forces at a point C, so that the moments about C are equal and opposite; or,

$$P \times AC = Q \times BC$$

Further, a force R, equal and opposite to E, acting at C, will represent the resultant of P and Q. *This point, C, is termed the centre of the parallel forces.*

The position of the point C, which is determined by the above equation, is not affected by the directions of the forces so long as they act at the same points A and B, and have the same magnitudes.

You may imagine any number of parallel forces acting in one plane being replaced by a single force. For in the above case you have formed a resultant, R, for the two forces P and Q; consequently you could find a resultant,  $R_1$ , for R and any other parallel force—say S; and so on for any number.



*The final resultant of the whole of the forces would act at a point which would be the centre of the system of the whole of the parallel forces acting on the body.*

**Centre of Gravity.**—Since gravity attracts towards the earth each particle of matter of which a body is composed, the *weight* of a body may be considered as the sum of a system of parallel forces. *The centre of these parallel forces is called the centre of gravity of the body, and is the point where the resultant of the weights of all the particles composing the body acts.*

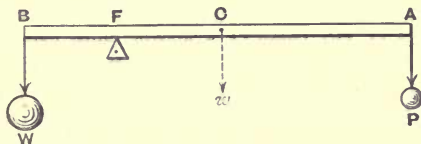
The following statements in small type, which are generally proved as propositions and corollaries in books on Elementary Theoretical Mechanics, should be remembered by the student:—

1. If a body is symmetrical, the centre of gravity (or *c.g.* of the body) coincides with the centre of the mass.
2. If a body be uniform, the *c.g.* coincides with the centre of volume.
3. In a very thin plate of uniform density the *c.g.* coincides with the centre of surface.
4. If the *c.g.* of a body be determined for any one position of the body, the same point is the *c.g.* for every other position.
5. If a body be supported on its centre of gravity, the body will balance in any position. Or, a body will balance about its *c.g.* in all positions.
6. If a body balance in all positions about a straight line through it, the *c.g.* lies in that line.
7. If the *c.g.* be vertically above or below the point of support, the body will rest in that position. Hence, if you balance or support a body from two different points, the *c.g.* lies in the intersection of the two vertical lines from the two points respectively. Or, if you balance a body on an edge, the *c.g.* is in the vertical plane passing through that edge. Balance it again on a different edge, thus finding another plane which passes through the *c.g.* Then the *c.g.* lies in the straight line constituting the intersection of the two planes. Balance the body for a third time in another position, then the point where this third vertical plane intersects with the straight line will be the *c.g.* of the body.
8. The *c.g.* of regular geometrical bodies may easily be found by mere inspection when they are of uniform density.

For Example.—The *c.g.* of a line is at the middle of the line; of a circle at its centre; of a sphere at its centre; of the surface of a uniform cylinder and of a solid cylinder at the centre of the axis; of a parallelogram at the intersection of its diagonals; of a triangle at the intersection of straight lines drawn from two of the angles to the middle points of the opposite sides—i.e., at a distance from one of the angles along one of these lines equal to  $\frac{2}{3}$  of the line; of the perimeter of a triangle (i.e., of three uniform rods forming a triangle) at the intersection of the two straight lines which bisect two of the angles of the triangle formed by joining the centres of the three uniform rods; of a polygon at the point of application of the resultant of the parallel forces represented by the areas of the respective triangles into which the polygon may be formed, and where each of these forces is considered to act at the *c.g.* of its own triangle; of a pyramid at  $\frac{3}{4}$  of the line from the vertex to the *c.g.* of the base; of a cone at  $\frac{3}{4}$  of the axis from the vertex; of the curved surface of a cone at  $\frac{3}{4}$  of the axis from the vertex; of a prism at the middle of the line connecting the *c.g.*'s of its ends.

**The Lever when its Weight is taken into Account.—**

In this case we have to add the moment due to the weight of the lever, to the moment of P or of W according as it acts along with the one force or with the other; *i.e.*, according as the *c.g.* of the lever is on the same side of the fulcrum as P or W. When the lever is of uniform section and density throughout, then the *c.g.* of the lever is at its middle point, and consequently the whole weight of the lever may be considered as concentrated and acting at that point.



WEIGHT OF LEVER CONSIDERED.

Let AB be a uniform lever, of weight  $w$ , acting at its *c.g.* or middle point C, let a weight,  $W$ , be attached to the end B, then the force P, which will have to be applied to the other end A, in order to balance the whole about the fulcrum F, will be found by taking moments about F.

$$\begin{aligned} \text{Thus,} \quad & P \times AF + w \times CF = W \times BF \\ \text{Or,*} \quad & P = \frac{W \times BF - w \times CF}{AF} \end{aligned}$$

**EXAMPLE I.**—A uniform lever, 5 ft. long, of 30 lbs. weight, is placed on a fulcrum 10 in. from one end, and has a weight of 100 lbs. attached to the short end. What force must be applied, and in what direction, in order to produce equilibrium? Also, what is the pressure on the fulcrum, and in what direction does the reaction from the fulcrum act?

1. Referring to the above figure, we find from the question that  $AB = 5 \text{ ft.} = 60 \text{ in.}$ ;  $BF = 10 \text{ in.}$   $\therefore AF = 50 \text{ in.}$  and  $CF = 20 \text{ in.}$   
 $W = 100 \text{ lbs.}$  and  $w = 30 \text{ lbs.}$

2. By the principle of moments—  
*The Opposing Moments about the Fulcrum are equal.*

$$\begin{aligned} \text{Consequently,} \quad & P \times AF + w \times CF = W \times BF \\ \therefore P = & \frac{W \times BF - w \times CF}{AF} \end{aligned}$$

Substituting the numerical values—

$$P = \frac{100 \times 10 - 30 \times 20}{50} = 8 \text{ lbs.}$$

\* If the *c.g.* of the lever was on the opposite side of the fulcrum, *i.e.*, on the side of W, then  $P \times AF = W \times BF + w \times CF$ .

3. *P* acts vertically downwards, since the moment due to the weight of the lever is not sufficient to equalise the moment due to the weight *W* about the point *F*.

4. *The pressure on the fulcrum* is evidently equal to the sum of all the forces, since all the forces act in one direction, or vertically downwards. It is therefore equal to

$$W + w + P = 100 + 30 + 8 = 138 \text{ lbs.}$$

5. *The reaction from the fulcrum* is equal and opposite in direction to this resultant. It therefore acts vertically upwards, and is the equilibrant of the whole of the forces, for a vertical force of 138 lbs. applied to the lever at *F* would counterpoise or just lift the whole bar with the attached weights *P* and *W*.

EXAMPLE II.—Suppose everything the same as in the previous example but the weight of the lever, which you may consider as now equal to 60 lbs.; what force *P* would be required, and in what direction would it have to act, in order to produce equilibrium? Also, what would be the resultant or downward pressure at *F*.

1. You observe at once that the moment of the weight of the lever is greater than the moment of *W* about the fulcrum.

$$\begin{array}{lcl} \text{For,} & w \times CF & > W \times BF \\ \text{Since,} & 60 \times 20 & > 100 \times 10 \end{array}$$

Consequently by the principle of moments *P* must act against *w*, or vertically upwards, so as to assist *W*, in order that *the opposing moments about the fulcrum may be equal*.

2. The formula therefore becomes

$$\begin{array}{lcl} & w \times CF - P \times AF & = W \times BF \\ \text{Or,} & w \times CF & = W \times BF + P \times AF \\ & \therefore \frac{w \times CF - W \times BF}{AF} & = P \end{array}$$

Substituting the numerical values, we have

$$\frac{60 \times 20 - 100 \times 10}{50} = P = 4 \text{ lbs.}$$

3. *The resultant pressure* at *F* is equal to the algebraical sum of the forces, or

$$W + w - P = 100 + 60 - 4 = 156 \text{ lbs.}$$

And acts vertically downwards. The *equilibrant* would therefore be 156 lbs. acting on the lever at *F* and vertically upwards.

*Position of the Fulcrum.*—In answering questions which give the magnitude of the forces with which they act, and require only an answer for the position of the fulcrum, the student has

simply to employ the general formula for the principle of moments, and then to substitute the known numerical values in order to get the unknown. Or, he may reason out the formula into the following shape, and then interpolate the numerical values. Referring to the last figure, suppose that the distance AF is required :

Then, *neglecting the weight of the lever*, we have by the principle of moments—

$$P \times AF = W \times BF = W (BA - AF) = W \times BA - W \times AF.$$

$$\text{Or, } P \times AF + W \times AF = AF (P + W) = W \times BA$$

$$\therefore AF = \frac{W \times BA}{P + W}$$

Now, *taking the weight of the lever into account*, we have by the principle of moments :

$$P \times AF + w \times CF = W \times BF.$$

Or,

$$P \times AF + w (AF - AC) = W (BA - AF) = W \times BA - W \times AF.$$

Or,

$$P \times AF + w \times AF + W \times AF = W \times BA + w \times \frac{BA}{2} = BA (W + \frac{1}{2}w)$$

$$\therefore AF = \frac{BA (W + \frac{1}{2}w)}{P + w + W}$$

**EXAMPLE III.**—Where should the *fulcrum* be placed under a uniform lever in order to produce equilibrium, if the lever is 5 ft. long, weighs 30 lbs., and has weights of 100 and 8 lbs. respectively hung at its ends.

From the above general equation for equilibrium—viz.:

$$P \times AF + w \times CF = W \times BF$$

We get

$$AF = \frac{BA (W + \frac{1}{2}w)}{P + w + W}$$

$$AF = \frac{60 (100 + 15)}{8 + 30 + 100} = 50 \text{ inches.}$$

Which proves the data given in Example I. to be correct.

## LECTURE III.—QUESTIONS.

1. Define what is meant by "the moment of a force," and give an example with a sketch.
2. State "the principle of moments," and apply it to the case of a simple straight lever.
3. A weight of 10 lbs. on the end of a lever 100 inches from the centre of motion is found to balance a weight of 100 lbs. at a distance of 10 inches. Explain the natural law which governs matter and motion, upon which the above mechanical fact depends. (*Answer this by giving the definition of the principle of moments.*)
4. Describe an experiment to prove the equality of the moments when the pull is between the weight and the fulcrum and acts in the opposite direction to the weight.
5. In the case of a straight lever, how would you ascertain the pressure on and the reaction from the fulcrum?
6. Three forces, of 12, 10 and 2 lbs., act along parallel lines on a rigid body; show by a sketch how they may be adjusted so as to be in equilibrium? *Ans.* The force of 12 lbs. must act as the equilibrant to the forces 2 and 10 lbs.—i.e., in a line with their resultant, but in the opposite direction.
7. Two parallel forces of 10 and 12 lbs. act in opposite directions on a rigid body, and at 2 feet apart. Where is the centre of the two forces, and what is their resultant? *Ans.* 10 feet from the force of 12 lbs., 2 lbs.
8. Define the "centre of gravity" of a body, and show how you would find it experimentally in the case of any irregular body. Give an example.
9. State the rule which applies when two unequal forces balance on opposite sides of the fulcrum of a straight lever, the weight of the lever being neglected. A uniform straight lever, 4 feet long, weighs 10 lbs., the fulcrum is at one end; find what upward force acting at the other end will keep the lever horizontal when a weight of 10 lbs. is hung at a distance of 1 foot from the fulcrum. Find also the pressure on the fulcrum and the direction in which it acts. *Ans.* 7.5 lbs.; 12.5 lbs. downwards.
10. A uniform bar, 4 feet long and weighing 4 lbs., can turn about a fulcrum at one end, and a weight of 10 lbs. is hung upon the bar at a distance of 1 foot from the fulcrum. Find the upward force at the free end which will keep the bar horizontal. *Ans.* 4.5 lbs.
11. A uniform bar of metal 10 inches long weighs 4 lbs., and a weight of 6 lbs. is hung from one end. Find the fulcrum or point upon which the bar will balance. *Ans.* 2 inches from the 6 lbs. weight.
12. Two parallel forces whose magnitudes are 8 and 12 lbs. respectively, act in the same direction on a rigid body at points 10 inches apart. Find the magnitude and line of action of the resultant of the two forces. *Ans.* 20 lbs. at a point 6 inches from the force of 8 lbs.
13. A uniform lever is 5 feet long, and weighs 10 lbs., the fulcrum being at one end. A weight of 30 lbs. is hung at a distance of 4 feet from the fulcrum; what upward force acting at the middle point of the lever will keep it in a horizontal position? *Ans.* 58 lbs.
14. Define "moment of a force." How is it measured? A bar of metal of uniform section weighs 5 lbs., and a weight of 10 lbs. hangs from one end. It is found that the bar balances on a knife edge at 9 inches from the end at which the weight hangs; what is the length of the bar? *Ans.* 4 ft. 6 in.

15. State the principle of the lever, and prove it when P and W act on opposite sides of the fulcrum. A weight of 5 lbs. is hung at one end of a uniform bar, which is balanced over a knife edge at a point 14 inches from the end at which the weight hangs. The bar weighs 30 lbs. ; find its length. *Ans.*  $32\frac{2}{3}$  inches.

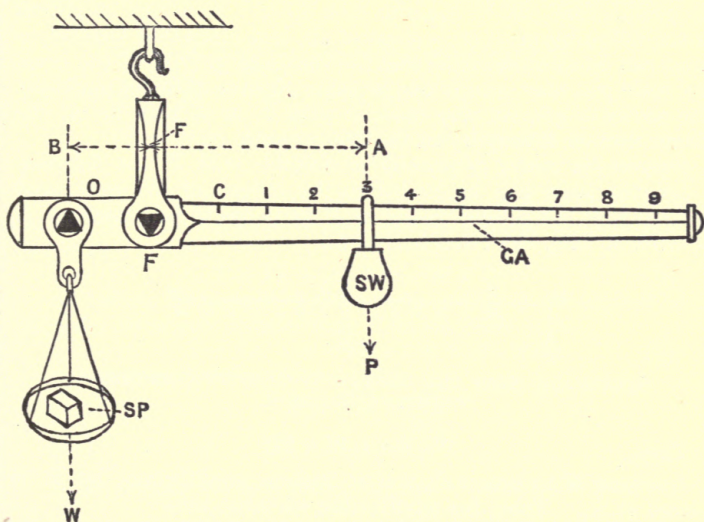
16. State the principle of the lever. A uniform straight bar, 14 inches long, weighs 4 lbs. ; it is used as a lever, and an 8 lb. weight is suspended at one end. Find the position of the fulcrum when there is equilibrium. *Ans.*  $2\frac{1}{3}$  inches from the 8 lb. weight.



## LECTURE IV.

**CONTENTS.**—Practical Applications of the Lever—The Steelyard, or Roman Balance—Graduation of the Steelyard—The Lever Safety Valve—Example I.—Lever Machine for Testing Tensile Strength of Materials—Straight Levers acted on by Inclined Forces—Bent Levers—The Bell Crank Lever—Bent Lever Balance—Duplex Bent Lever, or Lumberer's Tongs—Turkus, or Pincers—Examples II. and III.—Toggle Joints—Questions.

In this Lecture we shall give a number of examples of the application of the lever.



STEELYARD, OR ROMAN BALANCE.

## INDEX TO PARTS.

F	represents	Fulcrum.
GA	"	Graduated arm.
SW	"	Sliding weight.
P	"	Pull due to SW
SP	"	Scale pan.

W	represents	Weight in SP.
AF	"	Distance of P
		from F.
BF	"	Distance of W
		from F.

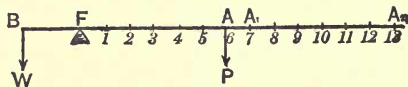
**The Steelyard, or Roman Balance,** is a straight lever with unequal arms, having a movable or sliding weight on the longer arm. It is very much used by butchers for weighing the carcasses of cattle and sheep, and in such cases it generally has two fulcra and two scales of division corresponding to them, the one set being, say, for hundredweights and the other for pounds.

**Graduation of the Steelyard.**—The practical method of graduating the steelyard is to put unit weight (say 1 lb.) into the scale pan, SP (or attach it to the hook on the shorter arm if there should be no such pan), and mark the position where the sliding weight, SW, has to be placed in order to cause equilibrium. Mark this position 1 on the scale. Then put in two units (say 2 lbs.) into SP, and adjust SW as before, marking its new position as 2 on the scale; and so on until SW is at the end of the longer arm.

In this form of steelyard, if the differences of the weights  $W$ , corresponding to successive distances. 1 to 2, to 3, &c., be the same, the graduations will be equal to each other. This may be proved in the following manner:—First of all, it is clear that the instrument can be so constructed that the centre of gravity of the beam and scale pan may occupy one or other of three different positions. The centre of gravity may coincide with  $F$ , or it may be on the longer arm, or it may be on the shorter arm.

*Suppose the centre of gravity to coincide with  $F$ , the fulcrum.*

Let the scale pan be loaded to the extent of  $W$  units, and suppose that the sliding weight of  $P$  units has to be placed at  $A$  in order to keep the beam horizontal.



STEELYARD WITH THE CENTRE OF GRAVITY COINCIDING WITH THE FULCRUM.

Then, 
$$P \times FA = W \times FB \quad . \quad . \quad (1)$$

Increase  $W$  by one unit, and to restore equilibrium, let  $P$  be placed at  $A_1$ . Then, for equilibrium we must have

$$P \times FA_1 = (W + 1) \times FB \quad . \quad (2)$$

Subtracting corresponding members of equations (1) and (2) we get

$$P(FA_1 - FA) = (W + 1 - W) \times FB,$$

Or, 
$$P \times AA_1 = FB,$$

$$\therefore AA_1 = \frac{FB}{P} \quad . \quad . \quad (3)$$

Increase  $W$  by  $n$  units, and let  $P$  occupy the position  $A_n$ . Then, for equilibrium, we must have

$$P \times FA_n = (W + n) \times FB \quad (4)$$

As before, subtract the corresponding members of (1) and (4), when we get

$$P \times AA_n = n \times FB,$$

$$\therefore AA_n = n \times \frac{FB}{P},$$

Or,  $AA_n = n \times AA_1$  by equation (3)

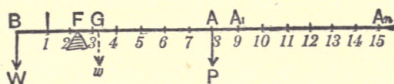
Thus we see, that the graduations are all equal for equal increments of  $W$ .

The student will readily observe that the zero of the scale is at  $F$ , and by putting  $W = 1$  in equation (1) we can fix the position of the first number on the scale

$$\text{i.e.,} \quad P \times FI = 1 \times FB,$$

$$\text{Or,} \quad FI = \frac{FB}{P}.$$

Next, suppose the centre of gravity to lie in the longer arm at  $G$ .



STEELYARD WITH THE CENTRE OF GRAVITY IN THE LONGER ARM AT  $G$ .

Let  $w$  = weight of beam and scale pan, and suppose  $P$  at  $A$  and  $w$  at  $G$  to balance  $W$  units at  $B$ . Then, for equilibrium, we have

$$P \times FA + w \times FG = W \times FB \quad (5)$$

As before, increase  $W$  by one unit, and let  $P$  be shifted to  $A_1$  in order to restore equilibrium, then we must have

$$P \times FA_1 + w \times FG = (W + 1) \times FB \quad (6)$$

Subtracting (5) from (6) we get

$$P \times AA_1 = FB,$$

$$\text{Or,} \quad AA_1 = \frac{FB}{P} \quad (7)$$

Now increase  $W$  by  $n$  units, and let  $P$  occupy the position  $A_n$ , then

$$P \times FA_n + w \times FG = (W + n) \times FB \quad (8)$$

Subtracting (5) from (8) we get

$$P \times AA_n = n \times FB,$$

$$\text{Or,} \quad AA_n = n \times \frac{FB}{P},$$

That is,  $AA_n = n \times AA_1$  by equation (7)

Thus we again see that the graduations are equal for equal increments of the weight  $W$ .

To find the zero of the scale in this case :

In equation (5) put  $W = 0$ , then

$$P \times F0 + w \times FG = 0,$$

$$\therefore F0 = -\frac{w.FG}{P}.$$

That is, the zero is in the *shorter arm* at a distance from  $F$ , represented by  $\frac{w.FG}{P}$  units of distance, the units in this case being the same as those measuring  $FG$ .

By making  $W = 1$  in equation (5) the position of the first figure on the scale can be fixed, and then the whole beam graduated, since all the divisions are of the same size.

One important point to be observed in this arrangement is, that when the centre of gravity lies in the longer arm, there is a limit to the smallness of the weight which can be weighed in the scale pan, since the sliding weight moves along the longer arm *only*.

Let  $P$  coincide with  $F$ , then the weight  $W$  which must be placed in the scale pan in order to just balance the weight of the beam and scale pan at  $G$  is

$$W \times FB = w \times FG,$$

$$\therefore W = \frac{w.FG}{FB}.$$

Any weight less than this cannot be weighed. This is not an objection to the instrument where the weights to be measured are great, as in the case of the butcher's steelyard used for weighing heavy carcasses.

*When the centre of gravity lies in the shorter arm the graduations will still be equal.* The reasoning is the same as in the last two cases. The student can also prove that the zero of the scale is on the *longer arm* at the point  $O$  given by the equation

$$F0 = \frac{w.FG}{P}.$$

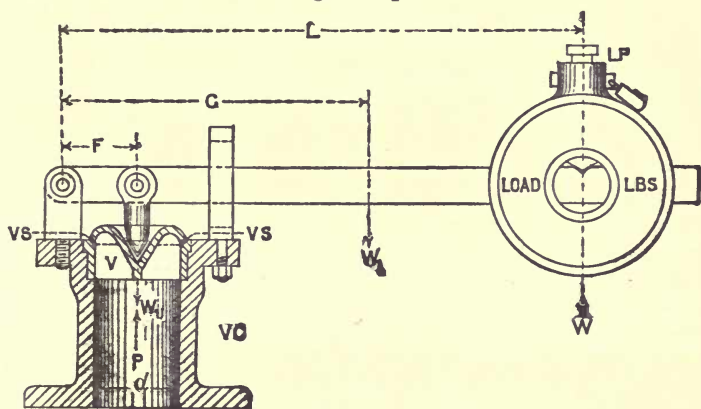
In this case all weights, however small, can be weighed.

**The Lever Safety Valve.\***—The lever safety valve is a simple

\* For a more detailed description of safety valves and their action refer to Lecture XXVII. of the author's Elementary Manual on "Steam and the Steam Engine."

contrivance fixed on the top of a boiler for the purpose of automatically preventing the steam exceeding an agreed-upon working pressure.

Referring to the next figure, VC is a cast-iron valve chest, containing a tightly-fitted gun-metal valve seat, VS, on which rests a steam-tight gun-metal valve, V. On the centre of the upper side of this valve rests a conical steel pin attached to a straight lever by an eye and bolt. One end of this lever is free to turn on a fulcrum fixed to the upper flange of the valve chest, and a lock-fast cast-iron weight is placed near the other end, so



LOCKFAST LEVER SAFETY VALVE.

#### INDEX TO PARTS.

VC represents Valve chest.  
VS       "       Valve seat.

V represents Valve.  
LP       "       Locking pin.

that the downward moment of the weight about the fulcrum balances the upward moment of the steam pressure on the valve about the same fulcrum.

Let  $L$  = length of lever in inches from fulcrum to the *c.g.* of the weight,  $W$ .

$F$  = Distance in inches from fulcrum to centre line of valve,  $V$ .

$G$  = " " " to *c.g.* of the lever.

$W$  = Weight in lbs. of the cast-iron counterpoise block.

$W_l$  = " " lever.

$W_v$  = " " valve.

$P$  = Pressure of steam in lbs. per square inch.

$d$  = Diameter of valve in inches.

$A$  = Area of valve in square inches  $= \frac{\pi}{4} d^2$ .

$P \times A$  = Total pressure in lbs. on the valve.

Then, by taking moments about the fulcrum, we find the pressure of steam per square inch which will balance the several forces.

*For the upward moment = the sum of the downward moments.*

$$\begin{aligned} (P \times A - W_v)F &= (W \times L) + (W_l \times G). \\ \text{Or, } (P \times A) \times F &= (W \times L) + (W_l \times G) + (W_v \times F) \end{aligned}$$

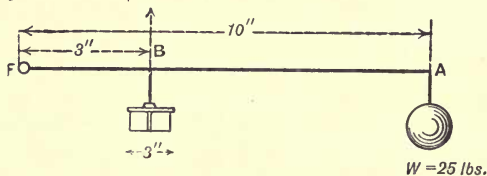
$$\therefore P = \frac{W \times L + W_l \times G + W_v \times F}{A \times F}$$

If we neglect the weight of the lever and the valve—

$$\text{Then, } (P \times A) \times F = W \times L$$

$$\text{Or, } P = \frac{W \times L}{A \times F}$$

**EXAMPLE I.**—A valve, 3 inches in diameter, is held down by a lever and weight, the length of the lever being 10 inches, and the valve spindle being 3 inches from the fulcrum. You are to disregard the weight of the lever and to find the pressure per square inch which will lift the valve when the weight hung at the end of the lever is 25 lbs.



Referring to the previous figure as well as to the accompanying one, we see from the question that

$$d = 3'' \therefore A = \frac{\pi}{4} d^2 = .7854 \times 3 \times 3 = 7.07 \text{ sq. ins. ;}$$

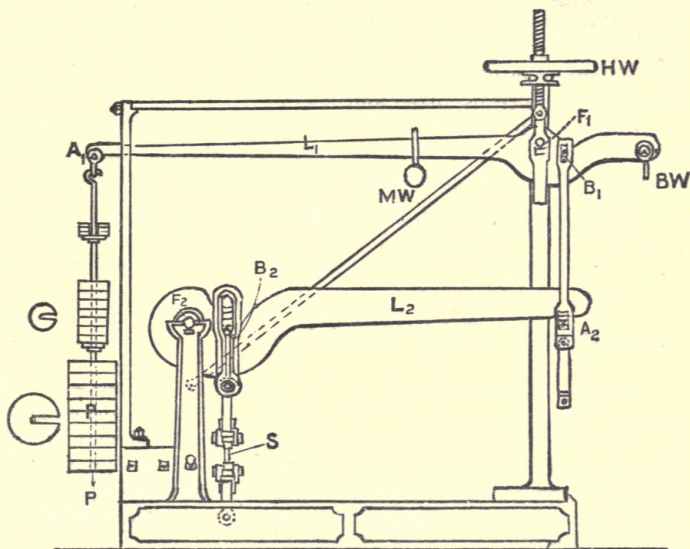
$$BF = 3', AF = 10'' \text{ and } W = 25 \text{ lbs.}$$

Taking moments about F, we get—

$$\begin{aligned} (P \times A) \times BF &= W \times AF \\ P \times 7.07 \times 3 &= 25 \times 10 \end{aligned}$$

$$P = 11.8 \text{ lbs. per square inch.}$$

**Testing Machine.**—The following figures illustrate a machine which is used for testing the tensile strength of iron, steel and such like materials. It consists of a combination of levers. After



LEVER MACHINE FOR TESTING TENSILE STRENGTH OF MATERIALS.

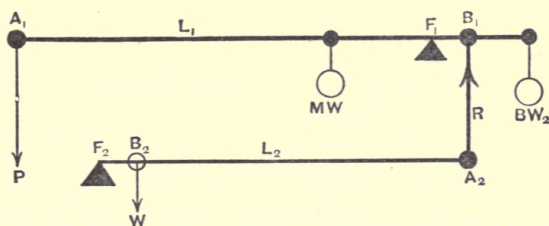


DIAGRAM OF THE LEVERS.

INDEX TO PARTS.

$L_1, L_2$	represent Levers.	$B_1, A_2$	represents Where R acts on $L_1, L_2$ .
$F_1, F_2$	" Fulcra.	$B_2$	" Where $W$ acts on S.
$P$	" Pull, or dead weights.	MW	" Movable weight.
$A_1$	" Where P acts on $L_1$ .	BW	" Balance weight.
S	" Specimen under test.	HW	" Hand-wheel and screw for elevating $F_1$ , &c.

mastering the general arrangement of the machine by comparing the index to parts with the side elevation, the student should refer to the accompanying skeleton diagram (where the same index letters have been used), from which he will readily understand how the stresses are transmitted and magnified.

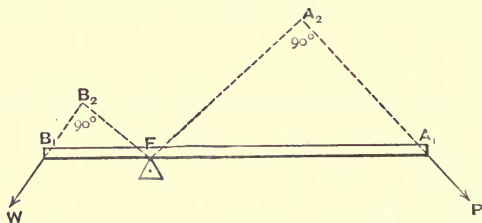
Looking at the second of the above figures, or skeleton diagram of the levers, it will be seen that when equilibrium exists between the stress  $W$  on the specimen  $S$ , and the pull  $P$ , applied at  $A_1$ ,

$$P \times A_1F_1 = R \times B_1F_1, \text{ and } R \times A_2F_2 = W \times B_2F_2,$$

$$\therefore R = \frac{P \times A_1F_1}{B_1F_1} = \frac{W \times B_2F_2}{A_2F_2}$$

$$\text{Consequently, } W = \frac{P \times A_1F_1 \times A_2F_2}{B_1F_1 \times B_2F_2}$$

**Straight Levers Acted on by Inclined Forces.**—In the previous Examples and in Lecture III. we have considered the forces  $P$  and  $W$  as acting at right angles to the straight levers. In such cases the forces had the greatest advantage, or their turning moments were a maximum. But the *principle of moments* is equally applicable to inclined forces acting on straight levers and to bent levers.



STRAIGHT LEVERS WITH INCLINED FORCES.

For, let  $A_1B_1$  be a straight lever acted on by inclined forces,  $P$  and  $W$ . Draw from the fulcrum,  $F$ , lines at right angles to the produced directions of the forces as shown by the dotted lines in the above figure.

Then, the effective arms for the forces  $P$  and  $W$  are respectively  $A_2F$  and  $B_2F$ ; and equilibrium takes place when their moments about  $F$  are equal;

*i.e.*, when

$$P \times A_2F = W \times B_2F$$

Or,

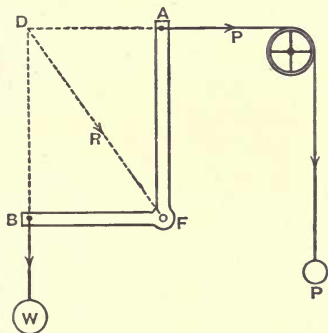
$$P : W :: B_2F : A_2F.$$

**Bent Levers.**—*The Bell Crank Lever.*—The same principle and action hold good in the case of bent levers. Take an ordi-

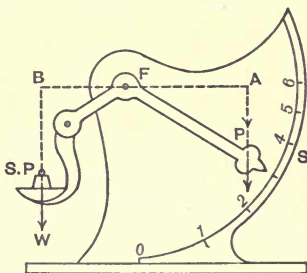
nary right-angle bell crank lever, as shown by the accompanying figure. Here the effective arms are equal to the actual arms of the lever, because the forces have been shown as acting at right angles to their respective arms, or with maximum turning moments.

Therefore,  $P \times AF = W \times BF$ .

But, if the lever be turned round through any angle by, say, an extra pull at P, then, in order to ascertain the virtual moments we should have to draw lines at right angles from F on the directions of P and W in order to calculate their effective arms.



BELL CRANK LEVER



BENT LEVER BALANCE.

**Bent Lever Balance.**—Examine an ordinary bent lever balance, such as is frequently used for weighing letters and light parcels, where the force P is a constant quantity, and the variable force W is represented by the article to be weighed. As shown by the accompanying figure, the effective arms change with each weight to be ascertained, and consequently the scale S of this balance has to be graduated by trial, or by introducing standard pounds, such as SP, or other units, and marking the values on the scale opposite the position where the end of the pointer on P comes to rest. Or, the graduation might be done by plotting the various positions of the arms and values of the forces to scale. In the illustration we have evidently got equilibrium when

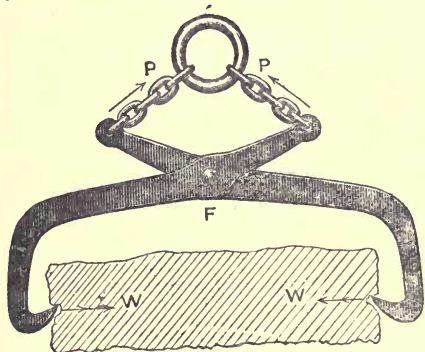
$$P \times AF = W \times BF.$$

**Duplex Bent Lever, or Lumberer's Tongs.**—The accompanying illustration shows a very useful and simple application of the bent lever, which is used at the end of a winch or crane chain,

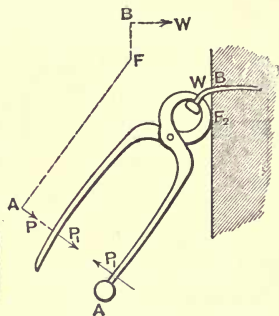
for affixing to and holding fast stones, logs of wood, blocks of ice, or other heavy articles when they have to be lifted.

$P, P$  indicate the directions of the pulling forces on the short chains between the ends of the shorter arms and the common link which is attached to the crane chain.  $F$  is the common fulcrum, and  $W, W$  show the directions of the forces with which the article is gripped. The student will be able to draw a diagram of the forces and calculate their effective moments for himself for any particular case.

**The Turkus, or Pincers.**—The ordinary carpenter's turkus, or pincers, which is frequently used for extracting nails from wood, is another familiar illustration of the duplex bent lever. As shown by the accompanying figure, the forces  $P, P_1$  represent the forces with which the pincers is gripped by the hand after the jaws have been closed on the neck of the nail, and the force  $B$



TONGS OR DUPLEX BENT LEVER.



TURKUS, OR PINCERS.

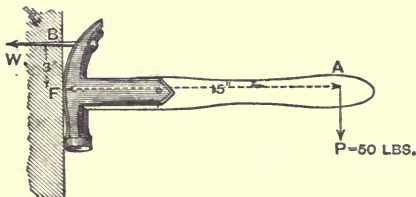
the pressure which has to be exerted by the arm and body in order to extract the nail from the wood—*i.e.*, to overcome the frictional resistance,  $W$ , between the wood and the nail. As shown by the separate diagram of forces in dotted lines, straight lines have been drawn, not from the joint of the pincers, but from a position representing the fulcrum  $F$  (or point where the nose of the pincers rests on the wood), perpendicular to the directions of the forces  $P$  and  $W$ , in order to obtain the lengths  $AF$  and  $BF$  of the effective arms of the bent lever.

$$\text{Here again, } P \times AF = W \times BF.$$

**EXAMPLE II.**—The handle of a claw-hammer is 15 inches long, and the claw is 3 inches long. What resistance of a nail would be overcome by the application of a pressure of 50 lbs. at the end of the handle?

You are required to show, by a diagram, the manner in which you arrive at your result. (S. and A. Exam. 1892.)

ANSWER.—Here we have a simple case of a bent lever, with fulcrum at F, and effective arms, AF, BF, 15 and 3 inches long



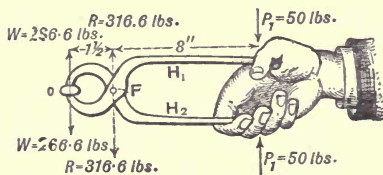
EXAMPLE OF A BENT LEVER.

respectively. Let W represent the resistance in lbs. offered by the nail at B. Then, by taking moments about F, we get

$$\begin{aligned} W \times BF &= P \times AF \\ \text{Or, } W \times 3 &= 50 \times 15 \\ \therefore W &= \frac{50 \times 15}{3} = 250 \text{ lbs.} \end{aligned}$$

EXAMPLE III.—State the mechanical law known as the *Principle of the Lever*. In a pair of pincers the jaws meet at  $1\frac{1}{2}$  inches from the pin forming the joint. The handles are grasped with a force of 50 lbs. on each handle at a distance of 8 inches from the pin. Find the compressive force on an object held between the jaws, and also the pressure upon the pin. (S. and A. Exam. 1888.)

Let P denote the force of 50 lbs. with which the handles are grasped at a distance of 8 inches from F, the pin. Let W denote



PINCERS OR NIPPERS.

the compressive force on the object O, and R the resultant reaction or pressure on the pin or fulcrum F. Although there are two levers here, each having a common fulcrum, F, it is best to con-

*Note.*—It is a mistake to speak of the “Principle of the Lever”; what is evidently meant is the Principle of Moments as applied to the lever.

sider the action of one lever only. Suppose the lower handle,  $H_2$ , to be fixed, and consider the action of the upper handle,  $H_1$ . It then becomes a simple question on the lever.

(1) To find  $W$ , take moments round  $F$ , then

$$W \times 1\frac{1}{2}'' = 50 \times 8''$$

$$\therefore W = 266\cdot6 \text{ lbs.}$$

(2) To find  $R$ , the pressure on the pin  $F$ , take moments round  $O$  then

$$R \times 1\frac{1}{2}'' = 50 \times (1\frac{1}{2} + 8'') = 50 \times 9\frac{1}{2}''$$

$$\therefore R = 316\cdot6 \text{ lbs.}$$

Or, since  $R$  must be the resultant of  $P$  and  $W$ , we get

$$R = P + W = 50 + 266\cdot6 = 316\cdot6 \text{ lbs.}$$

The "Toggle," or "Knuckle Joint," consists of a well-known combination of levers. It is characterised by its capability of exerting an enormous force through a short distance by means of very compact and simple elements. This device has been applied in many well-known cases, such as in the stone-crushing machine, certain brakes, printing and several forms of packing presses, and in the familiar frame by which carriage hoods are held in position.

The accompanying diagrams illustrate the principle of the "Toggle," and the plan of its application to a cane mill.

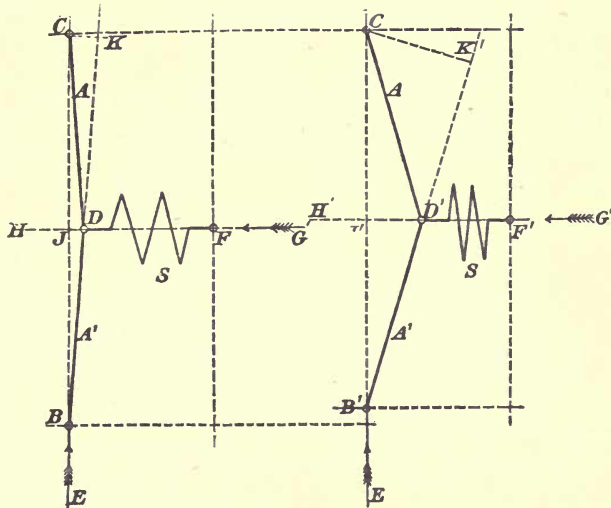
In Fig. 1,  $A A'$  are two links jointed at  $B$ ,  $C$ , and  $D$ . The point  $C$  is fixed, whilst the point  $B$  is free to move vertically along the line  $EC$ . The centre  $D$  is connected with the point  $F$  by means of a third link, formed partly of a spring  $S$ . The point  $F$  is supposed to be fixed in relation to its distance from the vertical line  $EC$ , but is free to move in a line parallel to  $EC$ . Under these conditions, a force acting in the direction of the arrow along the line  $GH$  will resist a much greater force acting along the line  $EC$ , and tending to move the point  $B$  upwards. The force along  $GH$  acts with a leverage equal to the distance from  $J$  to  $C$ , whilst the force along  $BD$  has only a leverage equal to the short length  $KC$ . The nearer the centre  $D$  approaches the vertical line, the greater will be the leverage of the force acting on the centre  $D$  in the direction  $GH$ , and the less will be the leverage of the force acting in the direction  $EC$  through the link  $A'$ . In practice, a few railway buffer springs, corresponding to  $S$  in Fig. 1, combined with simple links in a compact arrangement, occupying little space, are sufficient to resist forces amounting to several hundreds of tons. The line  $EC$  corresponds to the centre line of one of the top cover bolts of a cane mill,  $C$

corresponding to the nut at the top of the bolt, and therefore one of the fixed points in the system; whilst *B* may be taken as representing a point in the top cover itself, which is supposed to be free to move up and down.

Fig. 2 illustrates the operation of the system, and directs attention to a feature of great practical interest and value. The

FIG. 1.

FIG. 2.



THE PRINCIPLE OF THE TOGGLE JOINT.

parts in this figure correspond with those in Fig. 1, but it is supposed that a force acting in the direction *EC* has raised the point *B* from its original position, as in Fig. 1, to the position *B'*. The point *D* has consequently been moved further away from the vertical line *EC* to the position *D'*. In this position, the leverage of the force acting along *GH* is reduced, whilst the leverage of the force acting through the link *A'* is increased, as compared with the previous conditions in Fig. 1; but it will be seen that *D*, in moving to the position *D'*, has compressed the spring *S*, and therefore increased its resistance.

Hence, by the principle of moments :

$$(\text{Force along } BD) \times KC = (\text{Force along } GF) \times JC.$$

$$\therefore \text{Force along } BD = (\text{Force along } GF) \times \frac{JC}{KC}.$$

NOTE.—See the Tangentometer at end of Lecture XXVIII.

But the vertical force  $E B = (\text{Force along } B D) \times \frac{B J}{B D}$ .

„ „ „ „ „  $= (\text{Force along } G F) \times \frac{J C}{K C} \times \frac{B J}{B D}$ .

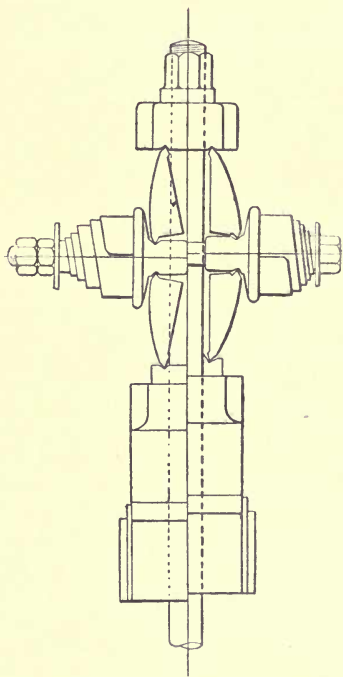
But if the links  $B D$  and  $D C$  are equal, then  $B J = J C$ .

Hence, force along  $E B = (\text{force along } G F) \times \frac{J C^2}{K C \times B D}$ .

If the movement along  $B C$  is very small compared with the length of the links  $B D$  and  $D C$ , then  $J C$  may be considered as a constant length. Hence, when the spring is so adjusted that

its resistance to compression is proportional to  $K C$ , the pressure  $E B$  upon the rollers will be nearly constant. This arrangement of the “toggle joint,” therefore, automatically permits of light or heavy feeding of the canes in a sugar mill without bringing undue stresses upon the various parts, and thus diminishes the chance of breakdowns.

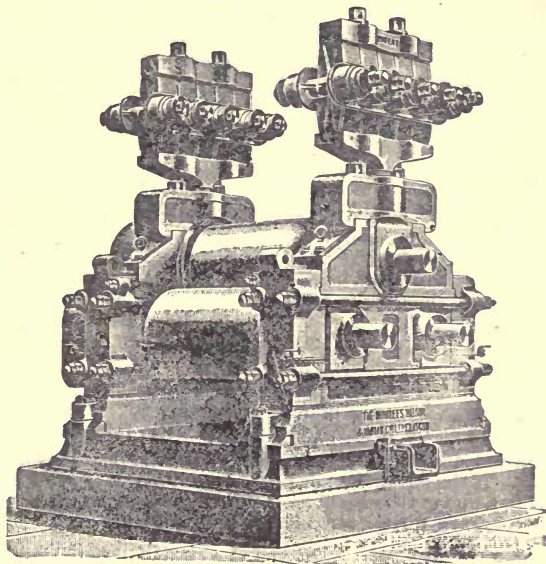
The following illustration shows the actual construction of the “Toggles” as applied to the top roller of an ordinary 3-roller cane-crushing mill. The left-hand half of the illustration shows the positions of the parts when the mill is empty. In this condition the top roller rests in its bearings and gives the minimum openings between the rollers. The right-hand half of the illustration shows the condition of the system when the mill is taking a heavy feed. It will be seen that



PATENT TOGGLE APPARATUS.

as the roller rises it lifts the top caps, which are under the control

of the "Toggles," until the upward pressure is balanced by the resistance of the "Toggles."



TOGGLE JOINT AS APPLIED TO A SUGAR-CANE MILL.

The above is an illustration of a mill made by the Messrs. Mirrlees, Watson & Co., Ltd., of Glasgow, with rollers 32 inches in diameter by 60 inches long. The arrangement of their patent "Toggles" in this case is of a special kind. The vertical bolts of the "Toggles" are formed by continuations of the top cover bolts, but the mill covers or caps are fixed by nuts screwed directly down upon them, and do not lift with the roller. The communication between the "Toggles" and the top roller is made by means of plungers formed on the under side of the bottom plates of the "Toggles," and working through the top caps. The only advantage of this arrangement is, that it permits the top cap to be used as a brace to bind the upper jaws of the mill cheek together, and thus adds in some measure to the strength of the cheek.

## LECTURE IV.—QUESTIONS.

1. Sketch and describe the steelyard, or Roman balance, and explain fully how the graduations on the scale are equal for equal differences in the weights applied to the shorter arm.

2. Sketch and describe a lockfast lever safety valve. A valve, 3 inches in diameter, is held down by a lever and weight, the length of the lever being 30 inches, and the valve spindle being 4 inches from the fulcrum. You are to disregard the weight of the lever and to find the pressure per square inch which will lift the valve when the weight hung at the end of the lever is 56 lbs. *Ans.* 59.4 lbs.

3. The diameter of a safety valve is 3", its weight  $3\frac{1}{2}$  lbs.; length of lever is 30", and its weight 16 lbs.; the distance from fulcrum to centre of valve is 3", and to *c.g.* of lever 12". Find where a weight of 50 lbs. must be placed on the lever in order that steam may just blow off at 70 lbs. per square inch by gauge. *Ans.* 25.65 inches from the fulcrum.

4. The safety valve of a boiler is required to blow off steam at 100 lbs. per square inch by gauge. The dead weight is 100 lbs., weight of lever 10 lbs., and of valve 5 lbs.; diameter of valve  $3\frac{1}{2}$ ", distance from centre of valve to fulcrum 4", from *c.g.* of lever to fulcrum 15". Where should you place the weight on the lever? *Ans.* 36.9 ins. from fulcrum.

5. Sketch and describe a lever machine for testing the tensile strength of materials. If the advantage, or ratio of resistance *R* to pull *P* in the first lever, is 56 to 1, and of the second lever 40 to 1, what stress will be produced on the test specimen when *P* = 100 lbs.? *Ans.* 100 tons.

6. A force of 100 lbs. acts at one end of a straight lever, but at an angle of  $60^\circ$  to it. What force acting at the other end of the lever, at an angle of  $45^\circ$  to it, will keep the lever in equilibrium if the fulcrum be placed half the distance from the first force that it is from the second? Draw a diagram of the forces and their effective arms. *Ans.* 61.25 lbs.

7. Sketch a bell crank lever, to convey a small movement from one line to another, cutting each other at  $60^\circ$ ; the distances moved through to be as 1 to 2.

8. The handle of a claw-hammer is 12 inches long, and the claw is 2 inches long. What resistance of a nail would be overcome by the application of a pressure of 40 lbs. at the end of the handle? Show, by a diagram, the manner in which you arrive at your result. *Ans.* 240 lbs.

9. In a pair of pincers the jaws meet at  $1\frac{1}{2}$  inches from the pin forming the joint. The handles are grasped with a force of 30 lbs. on each handle at a distance of  $7\frac{1}{2}$  inches from the pin. Find the compressive force on an object held between the jaws, and also the pressure upon the pin. Sketch the apparatus and show the direction and values of all the forces. *Ans.* 180 lbs; 210 lbs.

10. There is a contrivance for obtaining great pressure through a small distance, commonly termed the toggle or toggle joint. Will you explain it, and show wherein its peculiar action and efficiency consist?

11. Explain the mechanical advantage of the combination known as a *toggle joint*. Show its application in printing machinery, or in stone crushing machines, or in any other instances with which you are acquainted.

12. Sketch any one form of toggle joint with which you are acquainted, and point out its object. (C. & G., 1903, O., Sec. A.)

13. The figure shows a bent lever  $AOB$  with a frictionless fulcrum  $O$ .  $AO$  is 12",  $BO$  is 24". The force  $Q$  of 1000 lb. acts at  $A$ , what force  $P$  acting at  $B$  will produce a balance? Work the question graphically or in any other way. Neglect the weight of the lever. (B. of E. 1904.)  
*Ans.* Force  $P = 550$  lbs.



NOTE.—Before answering this question and any future questions involving angles, or the ratios of the sides of a right angled triangle, students should refer to the construction, action and uses of the Tangentometer at the end of Lecture XXVIII.

## LECTURE V.

**CONTENTS.**—The Principle of Work—Work put in, Work lost, Useful Work—Efficiency of a Machine—Principle of Work applied to the Lever—Experiments I. II.—Wheel and Axle—The Principle of Moments applied to the Wheel and Axle—The Principle of Work applied to the Wheel and Axle—Experiment III.—The Winch Barrel—Example I.—Ship's Capstan—The Fusee—Questions.

**The Principle of Work.\***—The principle of work is applicable to all machines, and may be stated as follows:—

*The work put into a machine is equal to the work absorbed by the machine plus the work given out by the machine.*

Or,  $\text{WORK PUT IN} = \text{LOST WORK} + \text{USEFUL WORK}.$

This is an *axiom*. But, nevertheless, many deluded would-be inventors have spent much time and money in devising "*perpetual motion*" appliances, or machines which should turn out as much work as, or even more than, was put into them!

1. When a machine is employed to perform mechanical work, a certain force must be applied to one part of it in order to move the machine and to perform work at another part.

The product of this applied force and the distance through which it acts constitute *the whole work put into the machine*.

2. Some of this work *must* be expended in merely keeping the different parts in motion, against natural resistances due to friction at the fulcrum or journals, and friction between moving parts and the air or water in the case of an hydraulic apparatus. The work so absorbed is termed *lost work*.

The mean value of the frictional resistances, multiplied by the mean distance through which they are overcome, constitute *the work lost in the mechanism*. One great object to be kept in view, in designing most machines, is to minimise this *lost work* by minimising the internal resistances to motion in the machine

\* The *Principle of Work* is usually stated as follows in books on Mechanics, but I find that engineering students *much* prefer the *above* definition. "If a system of bodies be at rest under the action of any forces, and be moved a very little, no work will be done." "*Conversely*: If no work is done during this small movement, the forces are in equilibrium."—Prof. Goodeve's "*Manual of Mechanics*," p. 73.

itself; but you must remember that these can *never be entirely disposed of*, as has only too often been conjectured by "perpetual motion" faddists.

3. The remainder goes to do the *useful work* for which the machine was designed, and therefore—

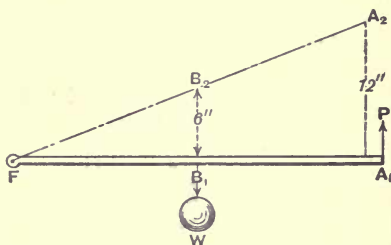
$$4. \text{ The efficiency of a machine} = \frac{\text{the work got out.}}{\text{the work put in.}}$$

To impress these facts on the mind of the student we present them in the following condensed form:—

1. *Work put in* = *force applied*  $\times$  *the distance it acts.*
2. *Work lost* = *force absorbed in overcoming internal resistances*  $\times$  *the distance it acts.*
3. *Useful work* = *force given out*  $\times$  *the distance it acts.*
4. *Efficiency* = *ratio of work got out to work put in.*
5. *Work put in* = *lost work* + *useful work.*

**Principle of Work applied to the Lever.**—In applying the above "principle of work" to the lever, we will take the liberty of neglecting the lost work. We shall therefore assume that the friction at the fulcrum is so small that it may be neglected for the purpose we have in view.

**EXPERIMENT I.** — Let  $A_1F$  be a straight lever without weight, having its fulcrum at  $F$ , a force,  $W$ , acting vertically downwards from the point  $B_1$ , and a force,  $P$ , acting vertically upwards at the end  $A_1$ , keeping  $W$  in equilibrium. Now imagine the lever elevated to the position  $A_2F$ .



PRINCIPLE OF WORK APPLIED TO A LEVER.

The *work put in* at  $A_1 = P \times$  the vertical distance from  $A_1$  to  $A_2$ .  
 The *work got out* at  $B_1 = W \times$  the vertical distance from  $B_1$  to  $B_2$ .  
 Therefore, since we neglect all frictional resistances—

$$\text{Or,} \quad \text{The work put in} = \text{the work got out}$$

$$P \times A_1A_2 = W \times B_1B_2$$

$$\text{i.e.,} \quad \frac{P}{W} = \frac{B_1B_2}{A_1A_2}$$

But by Euclid the triangles  $A_1FA_2$  and  $B_1FB_2$  are similar in every respect.

Therefore, 
$$\frac{B_1 B_2}{A_1 A_2} = \frac{B_1 F}{A_1 F}$$

Hence, 
$$\frac{P}{W} = \frac{B_1 F}{A_1 F}$$

Or, 
$$P \times A_1 F = W \times B_1 F$$

But this is the equation we proved in Lecture III. with respect to the lever as complying with the "principle of moments." Hence the "*principle of work*" and the "*principle of moments*" are in agreement.

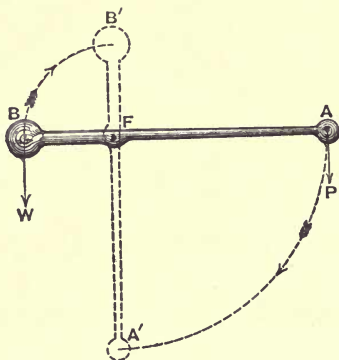
In the accompanying figure the force  $P$  has been shown as elevated through 12", and the force  $W$  as elevated through 6".

Therefore, 
$$P \times 12'' = W \times 6''$$

Or, 
$$\frac{P}{W} = \frac{6}{12} = \frac{1}{2}$$

$P$  being half the magnitude of  $W$ , it has to be elevated through double the distance in order that the same amount of work may be done in the same time.

EXPERIMENT II.—Consider the case of a simple lever, where a weight,  $W$ , at  $B$  is balanced by another weight,  $P$ , at  $A$ , around



PRINCIPLE OF WORK APPLIED TO A LEVER.

a fulcrum at  $F$ , without friction. Let the lever be turned through  $90^\circ$ , or a quarter of a revolution—i.e., from a *horizontal* position,  $AB$ , to a *vertical* position,  $A'B'$ .

Then by the definition of work—

The work put in at  $A = P \times A'F$ , and

The work got out at  $B = W \times B'F$ .

It does not matter in the slightest degree how circuitous the paths P and W take in passing from their original to their new positions in this case, since all we require to know is the vertical distances through which P is depressed and W elevated.

Consequently, by the "*Principle of Work*,"

$$P \times A'F = W \times B'F$$

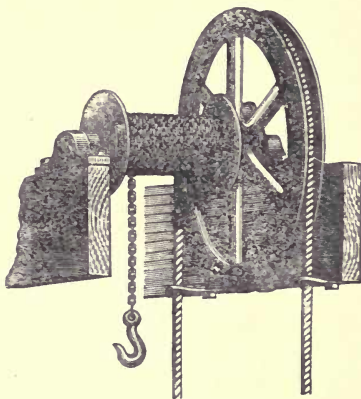
But,  $A'F = AF$ , and  $B'F = BF$ ,

$\therefore$  Substituting AF for A'F, and BF for B'F,

We get,  $P \times AF = W \times BF$

But this is the equation for the "*principle of moments*," which we have again deduced from the "*principle of work*" by another and simpler form of reasoning. We find that this latter method appeals more directly to the minds of young engineering students than the proofs usually found in books on Mechanics.

**The Wheel and Axle.**—The wheel and axle has been used for centuries for drawing water by a bucket from a well. It is used by the navy for lifting the material which he excavates from the earth, by the mason for raising stones, bricks and mortar, and by many other tradesmen for a variety of purposes; as well as by the quartermaster as a steering-gear, and the able seaman as a capstan. The accompanying illustration shows the form it takes when used for elevating goods in a store or mill.\* It is simply a practical arrangement for continuing the action of the lever as long as required. So long as a sufficient pull is applied to the rope, which fits into the grooved wheel, to overcome the resistance of the load attached to the chain hook, the weight will be raised. The wheel and axle is therefore a form of lever by which a weight may be raised through any desired height.

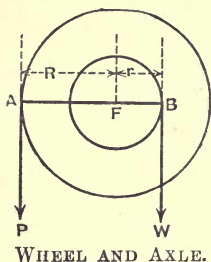


WHEEL AND AXLE.

**The Principle of Moments applied to the Wheel and Axle.**—In the diagram let the larger circle represent the circumference of a wheel of radius, R, to the periphery of which a force,

\* The above figure represents a wheel and axle as supplied by Messrs. P. & W. MacLellan, of Glasgow.

$P$ , is applied. Let the smaller circle represent the circumference of the axle or barrel of radius,  $r$ , to the periphery of which is applied a resistance  $W$ . Let the forces  $P$  and  $W$  act in the same direction and vertically downwards. Join the points where the lines of action of the forces are tangents to the wheel and axle by a straight line,  $AB$ . Then,  $AB$  passes through the common centre of the circles—i.e., through their common centre of motion or fulcrum  $F$ , and  $AF$  is the effective arm for the force  $P$ , whilst  $BF$  is the effective arm for the force  $W$ . In fact,  $AFB$  is a straight lever in equilibrium, with the fulcrum at  $F$ .



WHEEL AND AXLE.

Therefore, taking moments about  $F$ , we have—

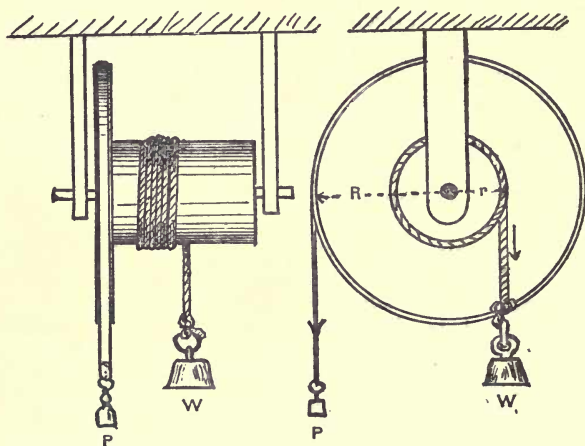
$$P \times AF = W \times BF$$

Or,

$$P \times R = W \times r.$$

**The Principle of Work applied to the Wheel and Axle.**

**EXPERIMENT III.**—Take a model of the wheel and axle as illustrated by the accompanying figure. Let forces,  $P$  and  $W$ , act in equilibrium, as in the previous case, at radii  $R$  and  $r$  respectively.



MODEL TO TEST THE PRINCIPLE OF WORK APPLIED TO THE  
WHEEL AND AXLE.

Now mark carefully with a piece of coloured chalk or ink the exact positions where the tape supporting  $P$  is a tangent to the wheel,

and where the cord supporting  $W$  is a tangent to the barrel. Pull  $P$  until the wheel and barrel have just made one complete revolution. Then, neglecting any force required to overcome friction at the bearings of the spindle—

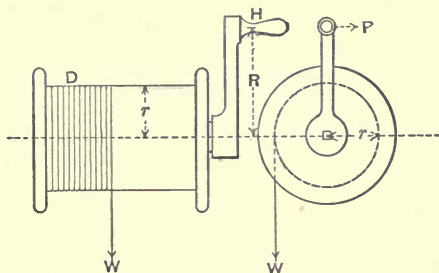
$$\begin{aligned} \text{The work put in by } P &= P \times 2\pi R \\ \text{The work got out in raising } W &= W \times 2\pi r \\ \text{But the work put in} &= \text{the work got out} \\ \therefore P \times 2\pi R &= W \times 2\pi r \end{aligned}$$

Cancelling  $2\pi$  from each side of the equation—

$$\text{We have} \quad P \times R = W \times r.$$

But this is the same equation as we obtained above by applying the “*principle of moments*.” Therefore, we see that the “*principle of moments*” and the “*principle of work*” harmonise.

**The Winch Barrel.**—The wheel may be replaced by a handle  $H$ , and the mere axle by a barrel or drum  $D$ , of any desired size.



SIDE VIEW.

END VIEW.

WINCH BARREL AND HANDLE.

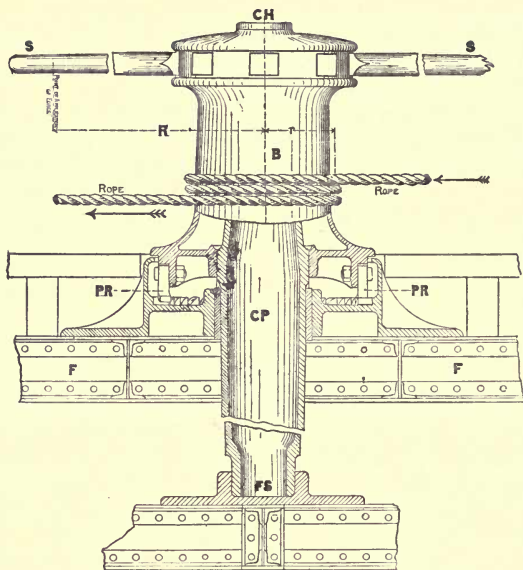
**EXAMPLE I.**—A man exerts a constant force of 30 lbs. on a winch handle of 15" radius; what weight will he be able to lift attached to a rope hanging from a barrel of 5" radius?

By the principles of moments and of work; and interpolating the numerical values—

$$\begin{aligned} P \times R &= W \times r \\ 30 \times 15 &= W \times 5 \\ \therefore W &= \frac{30 \times 15}{5} = 90 \text{ lbs.} \end{aligned}$$

**Ship's Capstan.**—A partly sectional, partly outside view of this useful machine is illustrated by the following figure:

A capstan is generally fixed upon the forecastle of a ship, or near to the side of a quay or dock, for the purposes of warping and



SHIP'S CAPSTAN.

## INDEX TO PARTS.

CH represents Capstan head.  
 SS     "     Spokes or arms.  
 R     "     Radius of S.  
 B     "     Barrel.  
 r     "     Radius of B.

PR represents Pall and Ratchet.  
 F     "     Frame.  
 CP   "     Capstan pillar.  
 FS   "     Footstep of CP.

berthing the vessels. The above illustration shows a capstan as built into a forecastle, where the round turned footstep, FS, of the vertical cast-iron capstan pillar, CP, bears in a cast-iron or cast-steel shoe fitted upon the steel or wrought beams of the main deck. The frame F, which supports the casing for the pall and ratchet gear, may be the beams of the upper or foreccastle deck. A strong rope made fast on shore is passed several times round the capstan barrel B, and the slack end of the rope is coiled on deck. The addition of the rope to the barrel increases the effective arm or radius  $r$ , at which the resistance of the ship acts by half the diameter of the rope. Eight or any desired less number of wooden spokes, S, S, having their inner ends squared and tapered, are fixed into hollow square holes in the cast-iron capstan head CH. Then, just as many sailors as may be required to

overcome the resistance of the ship apply themselves to the outer rounded ends of the spokes, and push away as hard as they can.

It will be observed that, calling,  $p$ , the force applied by each sailor at radius  $R$ ; then, when we have two sailors acting on diametrically opposite spokes  $p$ ,  $2R$ ,  $p$  forms a couple tending to cause rotation of the capstan in one direction. Consequently from the property of couples (as we showed in Lecture III.) this couple can only be balanced by another couple acting in the opposite direction and having an equal moment. Such another couple exists, when the resistance of the ship,  $W$ , acting with an arm,  $r$  (equal to the distance from centre of capstan to centre of rope), balances the corresponding reaction at the centre of the capstan barrel. Hence, when the force applied by the *two* sailors is balanced by the resistance to motion of the ship, we have the one couple just balancing the other one.

Or Couple  $p$ ,  $2R$ ,  $p$  balancing couple  $W$ ,  $r$ ,  $W$   
*i.e.*,  $p \times 2R = W \times r$

In the same way, with two, three, or four pairs of sailors, each pair being supposed to act on diametrically opposite spokes, we have two, three, or four couples acting in one direction, balanced by one couple, *viz.*, the resistance of the ship into the distance from the centre of the capstan barrel and the reaction from that centre.\*

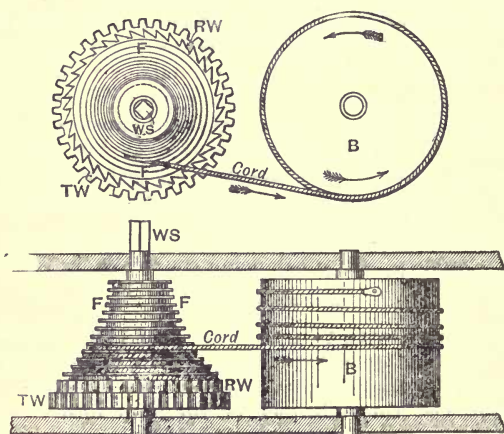
In the case of four sailors just being able to move the ship, two couples,  $p$ ,  $2R$ ,  $p + p$ ,  $2R$ ,  $p$ , balance one couple,  $W$ ,  $r$ ,  $W$ ;

*i.e.*,  $p \times 2R + p \times 2R = W \times r$

**The Fusee.**—As an illustration of the lever action and of work put into and got out of a machine, we cannot do better than finish this lecture by a description of the construction and action of the simple yet most ingenious contrivance termed the fusee. In good watches and clocks, where the elastic force of a coiled spring is used to drive the works, the fusee is used for the purpose of compensating the gradually diminishing pull of the uncoiling spring. The driving of the works at a constant rate is the object for which a watch or clock is designed. This naturally entails a *constant* resistance to be overcome, but since one of the most compact and convenient forms of mechanism into

\* The student should draw a plan of the capstan barrel, and show radial lines to indicate one, two, or more pairs of diametrically opposite spokes with forces,  $p$ , acting at their ends, all tending to turn the barrel in *one direction*. He will then see that a couple formed by resistance to the stress on the rope, and an equal reaction from the centre of motion, will be required to act in the other direction in order that equilibrium may take place.

which mechanical force can be stored is that of a coiled spring, and since the very nature of the spring is such that its force decreases as it uncoils, we must employ some compensating device between this variable driving force and the constant resistance. The fusee does this in a most accurate and complete



THE FUSEE FOR A CLOCK OR WATCH.

INDEX TO PARTS.

F represents Fusee.  
 B     "     Barrel.  
 RW    "     Ratchet wheel.

TW represents Toothed wheel.  
 WS    "     Winding square.

manner. Looking at the accompanying figures and index to parts, we see that the barrel B, which contains the watch or clock spring, is of uniform diameter, and that between the outside of this barrel and the fusee, or spirally grooved cone, there passes a cord or chain. When the winding key is applied to the winding square WS, and turned in the proper direction, a tension is applied to the cord, and it is wound upon the spiral cone, thus coiling up the spring inside the barrel B; for the outer end of this spring is fixed to the periphery of the barrel, and the inner end to its spindle or axle, is in direct gear with the works of the clock. When the spring is fully wound up it has the greatest force, but it acts with the least advantage, since then the cord is on the smallest groove of the cone pulley. When the spring is almost uncoiled it acts with the greatest advantage, for then the cord is on the largest groove of the cone. Consequently the radii

of the grooves of this cone are made to increase in proportion as the force applied to the cord decreases in order that there shall be a constant turning effort on the works of the clock.

The work put in when winding up the coiled spring, is given up by it in overcoming the frictional resistances of the different parts of the mechanism.

Or the work put in = lost work, for the whole of the work put in is devoted to simply keeping the parts of the machine in motion, thus leaving nothing for other work, unless the clock is used to strike a bell or do some other kind of work.

## LECTURE V.—QUESTIONS.

1. State the "Principle of Work," and explain the manner in which it is applied in determining the relation of a  $P$  to  $W$  in the lever. A lever, centred at one end, is 15 feet long, and a weight of  $W$  lbs. hangs from the opposite end. The weight  $W$  is supported by an upward pressure of 28,270 lbs. at 13 feet from the fulcrum. Find  $W$ . *Ans.* 24,500.6 lbs.
2. Define work put in, lost in, and got out of a machine, and prove that the work put in = lost work plus the useful work. How are the "advantage" and the efficiency of a machine reckoned?
3. Sketch and describe the wheel and axle. Apply both the "principle of moments" and the "principle of work" to find the relation between the force applied and the weight raised by aid of this machine. A wheel and axle is required so that the force applied at the circumference of the wheel in moving through a distance of 10 feet shall raise a weight of 4 cwts. through a height of 2 feet. If the diameter of the axle is 10 inches, find the force applied in lbs., and the radius of the wheel in feet. *Ans.* 89.6 lbs.; 2 feet 1 inch.
4. The crank or handle which turns a windlass is 14 inches in length; what must be the diameter of the axle when a man exerting a force of 60 lbs. upon the handle raises a tub of coals weighing 2 cwt.? *Ans.* 7½ inches.
5. In a windlass the barrel is 8 inches diameter, the rope is 1½ inches diameter, and the crank handle 15½ inches long. What force must be applied at the handle to raise 2 cwt.? Also, what weight would be raised by a constant force of 30 lbs. applied at the handle? *Ans.* 66.8 lbs.; 100.5 lbs.
6. A capstan is worked by four men; each man exerts a constant force of 30 lbs. at a distance of 4 feet from the axis. A rope of ¾-inch diameter is wound round the drum, of 5½ inches radius. Find the pull on the rope which balances the pressure on handles. Make a diagram showing the action of the forces, and find the pressure on the central shaft of the capstan. *Ans.* 921.6 lbs.; 921.6 lbs.
7. Describe, with a sketch, the spring-barrel and fusee of a clock or of a watch. Explain its action by reference to the principle of moments.
8. A ship's capstan has a ratchet-wheel with two detents or pawls, arranged so that when one is engaged with a tooth of the wheel the point of the other is midway between two teeth. Sketch the arrangement, and say why in this case two detents or pawls are better than one.
9. Find the average horse-power exerted by a winding engine to lift 3 tons from a pit ¾ mile deep at a uniform speed in two minutes, supposing that 30 per cent. of the total work done is lost in friction.  
(O. & G., 1905, O., Sec. A.) *Ans.* 576 H.P.

## LECTURE VI.

CONTENTS.—Pulleys—Snatch Block—Block and Tackle—Theoretical Advantage—Velocity Ratio—The Principle of Work applied to the Block and Tackle—Actual or Working Advantage—Work put in—Work got out—Efficiency—Percentage Efficiency—Example I.—Questions.

**Pulleys.**—Suppose you had to elevate a sack of flour from the ground to an upper storey of a mill or store, you might place it upon your back and carry it up the stairs. In doing so, you would expend so many foot-pounds of work. Let the sack of flour be 100 lbs., your own weight 150 lbs., and the height to which it is raised be 30 feet. Then the

Work done in elevating the flour = 100 lbs.  $\times$  30' = 3000 ft.-lbs.

" " yourself = 150 "  $\times$  30' = 4500 "

Total work done = 250 "  $\times$  30' = 7500 "

And your efficiency as a machine would be found thus—

Mechanical efficiency =  $\frac{\text{useful work}}{\text{total work}}$  ; or,  $\frac{\text{work got out}}{\text{work put in}} = \frac{3000 \text{ ft.-lbs.}}{7500 \text{ ft.-lbs.}} = .4$

Or, your percentage efficiency would be found from the proportion—

$$7500 : 3000 :: 100 : x$$

$$x = \frac{3000 \times 100}{7500} = 40 \%$$

In other words, 60 per cent. of the total work done is *lost* work, and only 40 per cent. is *useful* work.

If instead of carrying the sack upstairs, you found ready to hand a long rope (with its two ends close to the ground) that had been passed over a smooth iron hook fixed to the outside wall above an outside landing for the particular storey of the building, and, if you attached one end of this rope to the sack and found that by pulling with all your strength (or say with a force of 150 lbs., i.e., equal to your weight) on the other end, you could just lift the sack. Then, if by this means you elevated the sack to the landing, you would have expended less work than by the former method ; for,

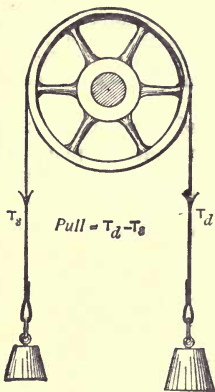
$$\begin{aligned}
 \text{Work done in elevating flour} &= 100 \text{ lbs.} \times 30' = 3000 \text{ ft.-lbs.} \\
 \text{,, against friction, \&c.} &= 50 \text{ ,,} \times 30' = 1500 \text{ ,,} \\
 \text{Total work done} &= 150 \text{ ,,} \times 30' = 4500 \text{ ,,}
 \end{aligned}$$

$$\therefore \text{Mechanical efficiency} = \frac{\text{useful work}}{\text{total work}}; \text{ or, } \frac{\text{work got out}}{\text{work put in}} = \frac{3000}{4500} = .6$$

And the percentage efficiency is therefore 66.6.

$$\text{For, } 4500 : 3000 :: 100 : x$$

$$x = \frac{3000 \times 100}{4500} = 66.6 \%$$



PULLEY AND WEIGHTS.

Hence 33.3 per cent., or  $\frac{1}{3}$  of the total work put in by you in pulling at one side of the rope, is spent in overcoming the friction between the rope and the hook and bending the rope over the hook, whilst only 66.6 per cent., or  $\frac{2}{3}$ , remain for elevating the sack of flour.

If, instead of the iron hook you had found a double-flanged deep V-grooved pulley with a rope over it, as in the accompanying illustration, and that this pulley revolved so easily on its bearings that you had only to pull with a constant force of 110 lbs. in order to lift the sack of flour from the ground up to the 30-foot level, then—

$$\begin{aligned}
 \text{Work done in elevating flour} &= 100 \text{ lbs.} \times 30' = 3000 \text{ ft.-lbs.} \\
 \text{,, against friction, \&c.} &= 10 \text{ ,,} \times 30' = 300 \text{ ,,} \\
 \text{Total work done} &= 110 \text{ ,,} \times 30' = 3300 \text{ ,,}
 \end{aligned}$$

$$\therefore \text{Mechanical efficiency} = \frac{\text{useful work}}{\text{total work}}; \text{ or, } \frac{\text{work got out}}{\text{work put in}} = \frac{3000}{3300} = .909$$

And the percentage efficiency is 90.9

$$\text{For } 3300 : 3000 :: 100 : x$$

$$x = \frac{3000 \times 100}{3300} = 90.9 \%$$

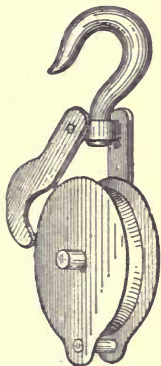
Hence only 9.1 per cent. of the total work put in is *lost* work in overcoming friction at the pulley bearing and in bending the rope over the pulley.

You see, therefore, what a useful machine a pulley is, not only for enabling you to change the direction of a force, but also for the saving of labour.

A pulley is simply a wheel and axle wherein their radii are one and the same, or a lever with equal arms. Hence the principles of moments and of work may be applied to it in the same way as we applied them to the lever and to the wheel and axle.

**Snatch Block.**—If you should require to put the bend of a rope on a pulley, and at the same time prevent the possibility of the rope coming out of the groove, without having to reeve the end of the rope between its cheeks, you would use what is called a snatch block. One form of snatch block is illus-

trated by the accompanying figure, where on the side of one cheek there is a sneck or snatch, which is turned to one side, to enable the bend of the rope to be placed around the U groove of the pulley. The snatch then falls down and closes upon the central pin. Another form has a hinged snatch which can be lifted up at right angles to the face of the cheek, and after the rope has been put on the pulley the snatch is closed down and locked by a pin attached to a short chain fixed to the side of the cheek, just like an ordinary front hinge for closing a chest. The single movable pulley, which is used for supporting the load to be lifted by a Chinese windlass or by a jib crane, is sometimes called a snatch block (see the illustration of the wheel and compound axle in next Lecture, and of jib cranes in Lectures VIII. and XIII.). In the latter case the chain passes from the barrel of the crane over the pulley at the point of the jib, then vertically down, underneath the snatch-block pulley, and vertically upwards to a point on the under side of the jib where it is fixed by an eye-shackle with a bolt and nut. If the load, including the weight of the snatch-block, be  $W$ , then, neglecting friction, the



SNATCH BLOCK.

pull  $P$  on the chain will be  $\frac{W}{2}$ ; for  $W$  is supported by two vertical or parallel parts of the chain, each part carrying half the load, or  $W = 2P$ . If the load be elevated any distance  $L$ , then the chain will have to be pulled in on the barrel a distance of  $2L$ , for by the principle of work

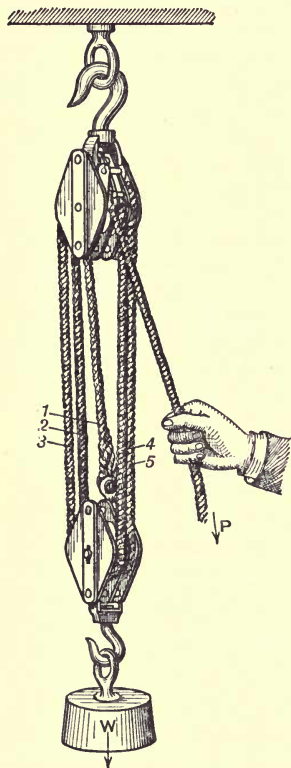
*The pull  $\times$  its distance = the load  $\times$  its distance.*

Or,  $P \times 2L = W \times L$ .

The theoretical advantage is therefore 2 to 1, or a certain force would lift double the weight, neglecting friction.

**Block and Tackle.**—Passing over the various arrangements of pulleys for lifting weights which are treated of in theoretical

mechanics, we come to this well-known and useful contrivance. As will be seen from the accompanying sketch, it consists of a number of pulleys (or sheaves as they are technically termed) free



BLOCK AND TACKLE.

to run round on a turned central iron or steel spindle, and inserted in a block, having their iron divisions between each pulley, and strong iron cheeks fixed to a swivel joint terminating in an iron hook hung from an eye bolt. Three sheaves are shown in this block, but the number may range from one upwards, according to the size and work to be done. There is a similarly constructed block with two sheaves, to which the weight to be raised, or the body to be pulled, is attached, and this is called the movable block, whereas the upper or home one is termed the fixed block. Around the pulleys of both blocks there is reeved a rope with the inner end made fast to an eye on the movable block, whilst the free end hangs from one of the outside sheaves; but this arrangement is frequently reversed, for the inner end of the rope may be attached to an eye on the fixed block, and the free end may spring from the other one (see the figure in connection with Example I. of this Lecture). The free end of the rope is then ready to be pulled by the hands or by aid of a winch.

Now, neglecting friction, and supposing the rope to be perfectly flexible, a force,  $P$ , applied to the free end of the rope would be transmitted throughout it to the other end at the movable block. Hence the effect of this force in overcoming a resistance,  $W$ , is multiplied by the number,  $n$ , of parts of the rope which spring from the movable block.

Or,

$$W = nP$$

$$\text{And (1) The theoretical advantage} = \frac{W}{P} = \frac{n}{1}$$

(2) *The velocity ratio, or ratio of the distance through which P acts, to that through which W is overcome in the same time.*

Or, 
$$\text{Velocity ratio} = \frac{\text{P's distance}}{\text{W's distance}} = \frac{n}{1}$$

In the figure there are shown three pulleys in the upper block and two in the lower, with five parts of rope springing from the latter; therefore in this case  $n = 5$ .

Here 
$$W = nP = 5P; \text{ or, } P = \frac{W}{n} = \frac{W}{5}$$

since P must pass through five times the distance that W does in the same time.

$$\text{The velocity ratio} = \frac{\text{P's distance}}{\text{W's distance}} = \frac{n}{1} = 5$$

So that the theoretical advantage and the velocity ratio have the same algebraical expression and numerical value. (See note, p. 68.)

**The Principle of Work applied to the Block and Tackle.**  
—Using the very kind of block and tackle represented by the previous figure, attach a light Salter's spring balance by its hook to the rope where the hand is shown. Fix such a weight to the lower block that the weight of rope between the blocks, the movable block, and the load are 60 lbs. Call this W. Now pull the ring of the spring balance until the load rises slowly and uniformly, and note the reading on the balance; let it be 18 lbs., and let the weight of this balance and the hanging free end of the rope, which is assisting the arm, be 2 lbs. Call this total pull of 20 lbs. P; then:

(3) *The actual or working advantage* 
$$= \frac{\text{weight raised}}{\text{pull applied}} = \frac{W}{P} = \frac{60 \text{ lbs.}}{20 \text{ lbs.}} = 3$$

Lift W up through one foot exactly, and measure the length of rope which you have pulled out from the upper block, and you will find that it is five feet; hence,\*

(4) *The work put in* 
$$= P \times n = 20 \text{ lbs.} \times 5 \text{ ft.} = 100 \text{ ft.-lbs.}$$

(5) *The work got out* 
$$= W \times 1 = 60 \text{ lbs.} \times 1 \text{ ft.} = 60 \text{ ft.-lbs.}$$

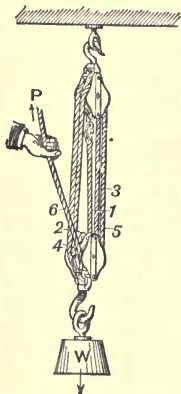
(6) *The efficiency* 
$$= \frac{\text{Work got out}}{\text{Work put in}} = \frac{60 \text{ ft.-lbs.}}{100 \text{ ft.-lbs.}} = .6$$

(7) *The percentage efficiency* 
$$= .6 \times 100 = 60 \%$$

In the same way the efficiency of any other block and tackle may be found, and the student should carry out a series of

\* The above results were obtained by the Author from a block and tackle of the same kind as that shown by the previous figure, at a demonstration in his Junior Applied Mechanics class.

experiments in a laboratory or workshop so as to impress the various measurements and the results on his memory. He will find that if the efficiency is over 50 per cent. a comparatively small load will run down and overhaul the free end of the rope, unless it has some restraining force applied to it, or be fixed to some rigid body. It is for this reason that sailors, who work very much with ordinary block and tackle, always "belay" the free end of the rope when they have adjusted their sails or have heaved up a body to the required height.



BLOCK AND TACKLE.  
2ND CASE, EXAMPLE I.

it will be apparent that the weight  $W$  is supported by *five* parts of the rope, or  $n = 5$ .

$$\therefore P_1 = \frac{W}{n} = \frac{W}{5} = \frac{600}{5} = 120 \text{ lbs.}$$

*Second Case.*—Here the system is inverted, so that the block with the three pulleys is lowermost, as shown by the accompanying figure. In this case it is evident that there are *six* parts of the rope supporting  $W$ , or  $n = 6$ .

$$\therefore P_2 = \frac{W}{n} = \frac{W}{6} = \frac{600}{6} = 100 \text{ lbs.}$$

*Note.*—If a machine be supposed to work without friction, then the ratio of the resistance overcome to the effort applied is termed the theoretical or hypothetical mechanical advantage. If, however, friction be taken into account and an effort  $P$  be able to overcome a resistance  $W$ , then the ratio  $\frac{W}{P}$  is termed the mechanical advantage.

## LECTURE VI.—QUESTIONS.

1. Suppose that your weight is 10 stone 10 lbs., and that you lift a weight of  $\frac{1}{2}$  cwt. on your shoulder, and walk upstairs with it to a height of 20 ft. ; what work have you expended, and what will be your efficiency as a machine ? *Ans.* 4120 ft.-lbs. ; 27 per cent.

2. Suppose that you had a rope passed round a beam of wood, and that you attached  $\frac{1}{2}$  cwt. to one end and pulled with a force of 84 lbs. on the other end and then elevated it 10 ft. : (a) what work have you put in ? (b) what is the percentage efficiency of the arrangement ? (c) what is the percentage of lost work ? *Ans.* (a) 840 ft.-lbs. ; (b) 66·6 ; 33·3.

3. Suppose that a weight of  $\frac{1}{2}$  cwt. is attached to one end of a rope passed round a pulley, and that you lift it 10 ft. by pulling on the other end of the rope with a force of 70 lbs. : what percentage of the work done is *lost* in overcoming the friction at the pulley ? *Ans.* 20 per cent.

4. What will be the difference, and why, in the tension on the chain of a crane when a *snatch-block* is used, and when the weight is lifted directly Sketch a snatch-block, and describe its construction and action.

5. In a rope and pulley lifting block with three sheaves in the upper block, and two sheaves in the lower block, find the theoretical advantage gained. Give the reason for your answer, and sketch the arrangement, showing where the rope is to be attached. *Ans.* W : P :: 5 : 1.

6. Sketch an arrangement of 5 equal pulley sheaves for lifting a weight of 1 ton. What force is exerted on the rope in your arrangement ? Explain the mode of arriving at this numerical result by the principle of work. *Ans.* With 3 pulleys in upper block and 2 in lower block, P = 448 lbs.

7. A tackle is formed of two blocks, each weighing 15 lbs., the lower one being a single movable pulley, and the upper or fixed block having two sheaves ; the parts of the cord are vertical, and the standing end is fixed to the movable block ; what pull on the cord will support 200 lbs. hung from the movable block, and what will then be the pressure on the point of support of the upper block ? Give a sketch. *Ans.* 71·6 lbs. ; 301·6 lbs.

8. A weight of 400 lbs. is being raised by a pair of pulley blocks, each having two sheaves. The standing part of the rope is fixed to the upper block, and the parts of the rope, whose weight may be disregarded, are considered to be vertical. Each block weighs 10 lbs. ; what is the pressure at the point from which the upper block hangs ? *Ans.* 522·5.

9. A tackle, consisting of an ordinary double and treble block, is employed for lifting a weight of 1000 lbs. attached to the double block. What force is required, neglecting friction ? If the tackle is reversed, so that the weight is attached to the treble block, the free end of the rope being pulled upwards, what force would now be required to lift the weight ? Sketch the two arrangements. *Ans.* 200 lbs. ; 166·6 lbs.

10. Apply the "principle of work" to find the relation between the force applied and the weight raised by an ordinary set of block and tackle. State what is meant by the following terms :—(1) velocity-ratio ; (2) theoretical mechanical advantage ; (3) actual or working advantage ; (4) work put in ; (5) work got out ; (6) efficiency of an apparatus or machine ; (7) percentage efficiency.

11. With an ordinary block and tackle having 3 pulleys in upper block and 2 in lower block—i.e., 5 ropes attached to lower block—it is found that a pull of 50 lbs. is required to raise a weight of 165 lbs. Find—(1) Theoretical advantage and velocity ratio = 5 : 1 ; (2) Actual advantage = 3·3 : 1 ; (3) Efficiency of apparatus = 66 ; (4) Percentage efficiency of apparatus = 66.

12. If the upper block of a set of pulleys and tackle has four equal sheaves, and the lower block three equal sheaves, and if a weight of one ton is hung on the lower block, one end of the rope being fixed to the ground and the other end free, what pull upon the free end will raise the weight, and what distance will the weight rise for every yard of increase of length in the free end? If the rope be fastened to the lower block instead of to the ground, what pull will raise the weight?

*Ans.* 373 lbs. ; 6 inches ; and 320 lbs.

13. A machine is concealed from sight except that there are two vertical ropes ; when one of these is pulled down the other rises. How would you find the efficiency of this lifting machine? What do you mean by *velocity ratio*, and by *mechanical advantage*? (S. E. B. 1901.)

14. In a lifting machine an effort of 26·6 lbs. just raised a load of 2260 lbs. ; what is the mechanical advantage? If the efficiency is 0·755, what is the velocity ratio? *Ans.* 85 ; 113. (B. of E., 1902.)

15. Distinguish between *force*, *work* and *rate of work*. Find the pull on the draw bar exerted by a locomotive which develops 600 horse-power when travelling at 60 miles an hour—the mechanical efficiency of the locomotive being taken as 60 per cent. *Ans.* 2250 lbs. (C. & G., 1903, O.)

16. Define the terms *mechanical advantage*, *velocity ratio*, and *efficiency*, as applied to lifting tackle.

In a lifting machine an effort of 26 lbs. just raises a load of 2200 lbs., and the efficiency is 0·75. Find the values of the mechanical advantage and velocity ratio. If, with the same machine, a load of 12 lbs. lifts a load of 600 lbs., what is the new efficiency? (C. & G., 1905, O., Sec. A. *Ans* Mech. Adv. = 84·6 ; Vel. Ratio = 112·5 : 1 ; Efficiency = ·44.

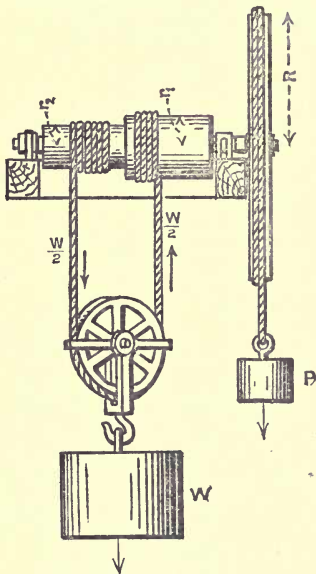


## LECTURE VII.

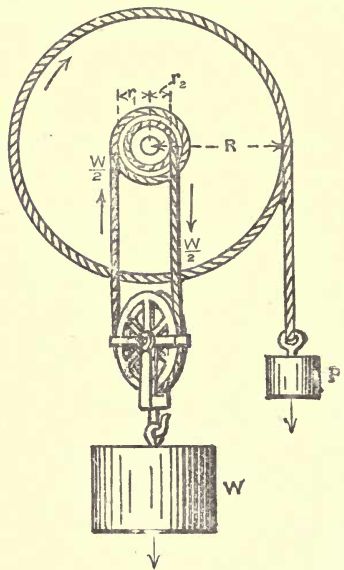
**CONTENTS.**—The Wheel and Compound Axle, or Chinese Windlass—The Principle of Moments applied to the Wheel and Compound Axle—The Principle of Work applied to the Wheel and Compound Axle—Examples I. II.—Weston's Differential Pulley Block—The Principle of Work applied to Weston's Differential Pulley Block—Experiment I.—Cause of the Load not overhauling the Chain—Questions.

**The Wheel and Compound Axle, or Chinese Windlass.**  
—This ingenious contrivance was first devised by the Chinese for the purpose of lifting weights. The theoretical mechanical advantage is very great, but it possesses the disadvantage of requiring a long length of rope to lift the weight a small height.

Its construction and action will be easily understood from the accompanying side and end views, which are taken from a model



SIDE VIEW.

END VIEW  
(Without End Bearing).

THE WHEEL AND COMPOUND AXLE.

made in the author's engineering workshop for the purpose of demonstrating its action and efficiency to his students.

**The Principle of Moments applied to the Wheel and Compound Axle.**—Taking moments about the axle, we have, when there is equilibrium between P and W,

$$P \times R + \frac{W}{2} \times r_1 = \frac{W}{2} \times r_1$$

$$P \times R = \frac{W}{2}(r_1 - r_2)$$

$$\therefore P = \frac{W(r_1 - r_2)}{2R}$$

**The Principle of Work applied to the Wheel and Compound Axle.**—Neglecting friction, and supposing the rope to be perfectly flexible, cause the wheel to make one complete revolution in the direction shown by the arrow near its circumference on the end view.

Then, by the principle of work,

The work put in = the work got out.

Or,  $P \times \text{its distance} = W \times \text{its distance}; *$

i.e.,  $P \times \text{circumference of wheel} = W \times \frac{1}{2} \text{ of the difference of the circumferences of the larger and smaller axles.}^*$

Or,  $P \times 2\pi R = W \times \frac{1}{2}(2\pi r_1 - 2\pi r_2)$

(Dividing both sides of the equation by  $2\pi$ )—

$$P \times R = \frac{W}{2}(r_1 - r_2)$$

$$\therefore P = \frac{W(r_1 - r_2)}{2R}$$

Which is the same result as the one above; consequently the principle of moments and the principle of work agree.

**EXAMPLE I.**—In a compound wheel and axle, where the weight hangs on a single movable pulley, the diameters of the two portions of the axle are 3 and 2 inches respectively, and the lever handle which rotates the axle is 12 inches in length. If a force

\* If  $\frac{W}{2}$  is raised the circumference of the larger circle on one side, then  $\frac{W}{2}$  is lowered at the same time on the other side, the circumference of the smaller axle; consequently W will be elevated a distance equal to half the difference of the circumferences of two axles, or  $= \frac{1}{2}(2\pi r_1 - 2\pi r_2)$ .

of 10 lbs. be applied to the end of the lever handle, what weight can be raised?

ANSWER.—Here  $P = 10$  lbs. ;  $R = 12''$  ;  $r_1 = 1.5''$  and  $r_2 = 1''$ .

*By the principles of moments and of work—*

$$P \times R = \frac{W}{2} (r_1 - r_2)$$

$$10 \times 12 = W \times \frac{1}{2}(1.5 - 1) = W \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}W$$

$$\therefore W = 10 \times 12 \times 4 = 480 \text{ lbs.}$$

EXAMPLE II.—In a compound wheel and axle, let the diameter of the large axle be 6 inches, and that of the smaller axle 4 inches, and the length of the handle 20 inches ; find the ratio of the velocity of the handle to that of the weight raised.

ANSWER.—Here  $R = 20''$  ;  $r_1 = 3''$  ;  $r_2 = 2''$ .

*By the principles of moments and of work—*

$$P \times R = \frac{W}{2}(r_1 - r_2)$$

$$\therefore \frac{P}{W} = \frac{\frac{1}{2}(r_1 - r_2)}{R}$$

$$\frac{P}{W} = \frac{\frac{1}{2}(3 - 2)}{20} = \frac{1}{40}$$

*But by the principle of work—*

$$P \times \text{its distance} = W \times \text{its distance}$$

$$1 \times P\text{'s distance} = 40 \times W\text{'s distance}$$

$\therefore$  The velocity ratio,

$$\text{Or, } \frac{P\text{'s distance}}{W\text{'s distance}} = \frac{40}{1}$$

**Weston's Differential Pulley Block.**—This practical application of the Chinese windlass is simply a compound axle without the wheel. Or, where  $R = r_1$ .

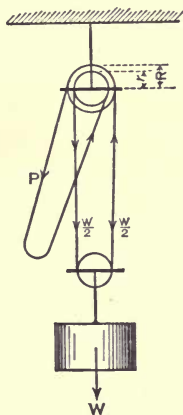
Hence, 
$$P \times R = \frac{W}{2} (R - r)$$

where  $R$  is the radius of the larger axle or pulley, and  $r$  the radius of the smaller one. After describing Weston's differential pulley block, we will deduce this formula from the "principle of work" by the same kind of reasoning as we adopted in the case of the wheel and compound axle. We leave the student, however, to apply the "principle of moments," whereby he should get the same results.

As will be gathered from an inspection of the accompanying outside view and the small diagram showing the directions of the forces and their arms, it will be seen that the apparatus consists of three parts—(1) an upper block; (2) an endless chain; (3) a movable lower block or snatch-block. The upper block has a hook with swivel joint, from which the iron frame is suspended. In the centre of this frame is a turned steel axle on which rotates a couple of pulleys cast in *one piece*, and therefore rigidly connected together.

The one pulley is slightly larger than the other, and both pulleys have V-grooved peripheries with side ridges or teeth cast on the inner sides of the grooves, so as to fit the pitch of the links of the chain, which passes over them and thereby prevent it slipping over the surface of the pulleys. The lower or movable pulley is simply an ordinary smooth V-grooved pulley with swivel and hook like that already described under the heading "Snatch Block."

The endless chain is an ordinary open-linked chain of uniform pitch and size of link. It passes from the position where the hand or pull, *P*, is applied, over the larger pulley of the upper block, underneath the lower pulley, over the smaller of the upper block pulleys, and back to the starting-point. (See also the small figure.) When a pull, *P*, is applied at this part of the chain (*if there were no friction*), it would be transmitted with undiminished value throughout its whole length where the tension



SKELETON FIGURE OF  
WESTON'S DIFFERENTIAL  
PULLEY BLOCK.



WESTON'S DIFFERENTIAL  
PULLEY BLOCK.  
(BY HOLT & WILLETT.)

It passes from the position where the hand or pull, *P*, is applied, over the larger pulley of the upper block, underneath the lower pulley, over the smaller of the upper block pulleys, and back to the starting-point. (See also the small figure.) When a pull, *P*, is applied at this part of the chain (*if there were no friction*), it would be transmitted with undiminished value throughout its whole length where the tension

can act ; but, as we shall see afterwards, a large proportion of this force is absorbed in overcoming friction. The stress due to the load  $W$  is divided equally between the two vertical parts of the chain connected to the lower block, and if  $W$  is moved through any distance, the stress  $\frac{W}{2}$  must act through double that distance.

**The Principle of Work applied to Weston's Differential Pulley Block and Tackle.**—Theoretically (*i.e.*, leaving friction out of account, the weight of the hanging part of the chain and the weight of the lower block), we have by the *principle of work*, in one revolution of the upper pulleys—

$$P \times \text{its distance} = W \times \text{its distance.}$$

$$P \times \text{circumference of the larger pulley} \Big\} = \frac{W}{2} \Big\{ \text{difference of the circumferences of the larger and smaller pulleys.} \Big\}$$

$$P \times 2\pi R = \frac{W}{2} (2\pi R - 2\pi r)$$

(Dividing each side of the equation by  $2\pi$ )

$$P \times R = \frac{W}{2} (R - r)$$

$$\therefore P = \frac{W(R - r)}{2R}$$

(1) *The Theoretical Mechanical Advantage* or ratio of  $W$  to  $P$  is found directly from the above equation by simple transposition.

$$\therefore \frac{W}{P} = \frac{2R}{R - r}$$

(2) *The Velocity Ratio* (or ratio of the distance passed through by  $P$  to the distance passed through by  $W$  in the same time) is also found in the same way.

$$\therefore \frac{P's \text{ distance}}{W's \text{ distance}} = \frac{2\pi R}{\frac{1}{2}(2\pi R - 2\pi r)} = \frac{2R^*}{R - r}$$

Or, the velocity ratio has the same numerical value as the theoretical advantage.

**EXPERIMENT I.**—With a Weston's differential pulley block, having in the upper block one pulley with an effective radius of 4" (*i.e.*, from the centre of the pulley to the centre of the chain which passes round it), and a smaller pulley with an effective radius of  $3\frac{1}{2}$ ", you can just lift a total load of 100 lbs. (including the dead weight, the lower block, and the hanging parts of the chain) by a pull of 20 lbs. on the chain.

\* Dividing numerator and denominator by  $\pi$  does not alter the fraction

In this case the *theoretical advantage* and the *velocity ratio* are each equal to—

$$\frac{2R}{R-r} = \frac{2 \times 4''}{4'' - 3.5''} = \frac{8}{.5} = \frac{16}{1}$$

Or, the pull on the forward side of the chain must act through 16 ft. for every foot the load is raised.

(3) *The Actual or Working Advantage* of the machine is, however, only as—

$$\frac{W}{P} = \frac{100 \text{ lbs.}}{20 \text{ lbs.}} = \frac{5}{1}$$

(4) *The Work put in* in lifting  $W$  1 ft. is

$$P \times 16 = 20 \text{ lbs.} \times 16' = 320 \text{ ft.-lbs.}$$

(5) *The Work got out* is  $= W \times 1 = 100 \text{ lbs.} \times 1' = 100 \text{ ft.-lbs.}$

(6) *The Efficiency is*  $= \frac{\text{Work got out}}{\text{Work put in}} = \frac{100 \text{ ft. lbs.}}{320 \text{ ft. lbs.}} = .3125.$

(7) *The Percentage Efficiency is*

$$= .3125 \times 100 = 31.25 \%$$

This is a very low efficiency for a machine, but it accounts for one of the useful properties of the Weston's differential pulley block—viz., that you can lift a weight by it, then let go your hold of the chain, and the weight will remain hanging in the exact position you left it, without overhauling the chain in the slightest degree. It is therefore an extremely useful appliance in engineering workshops where, for example, a slide valve and its valve casing port face have to be scraped so as to fit each other. After rubbing the valve on the port face, you can lift the valve by aid of a Weston's block, and leave it hanging, without any fear of its overhauling the chain which supports it, until you have scraped off the high or hard parts from the port face, when you can lower it for another rub. Or, in the case of having to adjust the centres of a heavy job to be turned in a lathe, you can lift the job from the lathe by a Weston's block, and leave it hanging quite free at the most convenient height to be acted upon, until you are ready to lower it again into position. Of course, with such apparatus, although the theoretical advantage is great, the actual or working advantage is small; yet this property of not overhauling is of such importance that appliances possessing it are constantly being used in every engineering workshop.

**Cause of the Load not Overhauling the Chain.**—In the first place, the chain cannot slip round the pulleys of the upper block, because the links of the chain fit into the notches or

between the outstanding teeth or ridges cast in their grooves. In the second place, the friction between the pulleys of the upper block and their axle is so great, that more than 50 % of the "work put in" is expended in overcoming it.

To prove this, take the case of the above experiment. When the pull  $P$  and the load  $W$  are both in action, the downward pressure (due to these two forces alone) between the pulleys and their axle is 120 lbs. (100 lbs. for  $W$  + 20 lbs. for  $P$ ). Now, remove the 20 lbs. pull, and you only relieve the pressure causing friction by  $\frac{1}{2}$ ; for 100 lbs. (or  $\frac{5}{6}$  of 120 lbs.) is still there. But friction is practically proportional to the pressure in such a case, and therefore, although the pull required to lift the load be removed,  $\frac{5}{6}$  of the total friction will remain at the upper block, and the friction at the lower block is unaltered. In lifting the load of 100 lbs. 1 ft., there was put into the machine 320 ft.-lbs., or 220 ft.-lbs. was *lost work*, required solely to overcome friction. Consequently, to lower the load of 100 lbs. 1 ft. there would have to be expended *at least*  $\frac{5}{6}$  of 220 ft.-lbs., or not less than 183 ft.-lbs. But the load *can of itself* only give out 100 ft.-lbs. in descending 1 ft.; therefore it must be assisted by at least 83 ft.-lbs. (183 ft.-lbs. - 100 ft.-lbs.) put into the chain on the slack side, or where it comes down from the smaller pulley.

This principle of the weight not running down (or overhauling, as it is technically termed) is common to *all* machines wherein *more than 50 %* of the force applied is spent in *merely* overcoming frictional resistance.\*

\* The student should be most earnestly warned against such expressions as the following, which are only too common in books dealing with Applied Mechanics:—"By increasing the number of sheaves in a pair of pulley blocks, the power may be increased." Now, *power*, or the rate of doing work, *can never* be increased by any *continuously acting* mechanical device, so long as the work given out depends *directly* on the work put in. It is simply the *force* which can be augmented, whilst the distance through which it acts is diminished. Of course, in the case of a pile-driver where the weight is lifted *slowly* and let go suddenly, so that the rate of giving out work is greater than the rate of putting it in, it is true that the power is increased. But this is not a continuous acting mechanical device in the sense referred to above. The fundamental principle to be observed is, that no more work can be got out than has been put in. The term "mechanical powers" should also be avoided, and the expressions "simple machines" or "mechanical elements" used instead.

## LECTURE VII.—QUESTIONS.

1. Sketch and describe the wheel and compound axle, or Chinese windlass. Apply the "principle of moments" and the "principle of work" to find the formula for the relation between the force applied and the weight raised by this machine.

2. In a compound wheel and axle the diameter of the two parts of the axle are 5 and 6 inches respectively. The weight raised, viz.,  $W$ , hangs from a single movable pulley in the usual manner, and is supported by a pressure,  $P$ , applied perpendicularly to a lever handle 15 inches in length. Find the ratio of  $P$  to  $W$ . Sketch and describe the compound wheel and axle, and state its inconveniences. *Ans.* 1 : 60.

3. A force of 20 lbs. draws up  $W$  lbs. by means of a wheel and compound axle. The diameter of the wheel is 5 feet, and the diameters of the parts of the compound axle are 9 and 11 inches respectively; find  $W$ . *Ans.* 1200 lbs.

4. In a compound wheel and axle, let the diameter of the larger axle be 8 inches, and the radius of the smaller one 2 inches, while the force applied to the handle passes through 47.12 inches in one revolution. Find the ratio of the velocity of the handle to that of the weight being raised. *Ans.* 7.5 : 1.

5. Explain the mechanical principle upon which Weston's pulley block is constructed, and give a skeleton diagram showing the direction of all the forces at work. If the weight which is being raised is left hanging, and the pull removed, why does the weight not descend?

6. Sketch and describe Weston's differential pulley block. If the diameters of the pulleys are 4 and 4½ inches, what weight can be raised by a force of 20 lbs.? If the weight to be raised is half a ton, what force must be applied to the leading side of the chain? (Neglect friction.) *Ans.* 360 lbs.; 62½ lbs.

7. Determine the relation between  $P$  and  $W$  in Weston's differential pulley block—(1) by the "principle of moments"; (2) by the "principle of work."

8. If a weight is raised by a Weston's differential pulley block at the rate of 5 ft. per minute, and the diameters of the pulley of the compound sheaves are 7 and 8 inches respectively, at what rate must the chain be hauled? Work out answer in full from the principle of work. *Ans.* 80 ft. per minute.

9. By experiment with a Weston's differential pulley block it was found that a pull of 15 lbs. on the leading side of the chain was required to lift a weight of 60 lbs. (including the weight of the lower pulley and hook). The dimensions of the apparatus were—radius of larger pulley, 2 inches; radius of smaller pulley, 1.75 inches. Find—(1) the theoretical advantage; (2) the actual or working advantage; (3) the efficiency or modulus; (4) the percentage efficiency of the apparatus. Why does the weight remain suspended when there is no pull on the chain? *Ans.* (1) 16 : 1; (2) 4 : 1; (3) .25; (4) 25.

10. In a Weston pulley block, the diameters of the two pulleys are 8 ins. and 7½ ins. respectively, and it is found that a pull of 25 lb. is sufficient to raise a weight of 240 lb. Find the efficiency of the tackle. *Ans.* 30%.

(C. & G., 1903, O., Sec. A.)

11. Describe how you would proceed to determine experimentally (1) the velocity ratio, (2) the mechanical efficiency of a Differential Pulley Block.

(B. of E., 1904.)

## LECTURE VIII.\*

**CONTENTS.**—Graphic Demonstration of Three Forces in Equilibrium—Parallelogram of Forces—Triangle of Forces—Three Equal Forces in Equilibrium—Two Forces acting at Right Angles—Resolution of a force into Two Components at Right Angles—Resultant of Two Forces acting at any Angle on a Point—Resultant of any number of Forces acting at a Point—Example I.—Stresses in Jib Cranes—Examples II. III.—Stresses on a Simple Roof—Example IV.—Questions.

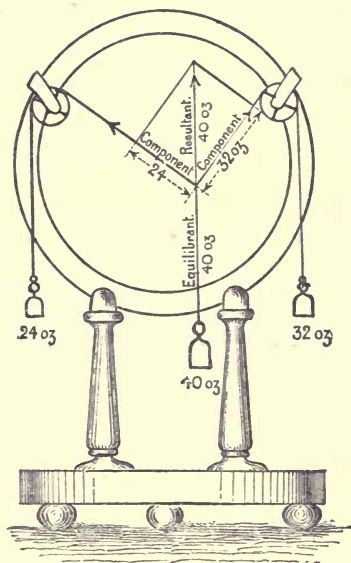
IN Lecture I. we explained and illustrated how a force may be represented by a straight line both in direction and magnitude, and we defined the terms components, equilibrant, resultant, resolution, and composition of forces. We will now discuss briefly the case of three forces in equilibrium when acting towards or from a point, as well as the parallelogram and the triangle of forces with examples, before taking up the inclined plane and friction.

**Graphic Demonstration of Three Forces in Equilibrium.**

—**EXPERIMENT I.**—Take a black board which (for convenience of handling and demonstration before a class) may be of the form shown by the accompanying figure. Select two movable clamps, each fitted with a small V-grooved pulley about 2 inches in diameter, with a minimum of friction at their bearings, and fix them to the outside of the board as indicated. Pass a very fine flexible cord over the pulleys, and attach to the ends of this cord S hooks. Hang from these hooks weights of say 24 oz. and 32 oz., and from the cord (anywhere between the pulleys) another cord with an S hook and a weight of, say, 40 oz. After a few up-and-down oscillations these three weights will come to rest in the definite position shown by the figure, and if you disturb them from this position they will invariably return to it again. Consequently, you conclude that the three forces acting from their common point of attachment are in equilibrium, and that the force 40 oz. is the *equilibrant* of the two forces 24 oz. and 32 oz.

\* This Lecture may require two meetings of a class when the students have had no previous training in Theoretical Mechanics. In any case, it will be well to spend at least one with a revisal hour before the written examination, which should now take place upon the work gone over since the beginning of the session, prior to the Christmas holidays.

With a piece of finely pointed white chalk, draw lines (from the point where the three forces act) on the black board parallel to the cords, and plot off from this point to any convenient scale (say by aid of a two-foot rule) distances along them to represent their respective magnitudes. Extend from the same point in an upward vertical direction another line, and mark it off to represent 40 oz. *This line* evidently corresponds, in point of application, direction, and magnitude, to the *resultant* of the *components* (24 oz. and 32 oz.), for it is equal and opposite in direction to their *equili-*



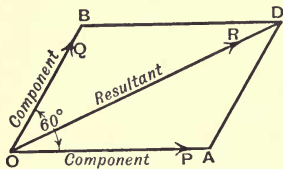
GRAPHIC REPRESENTATION OF FORCES IN EQUILIBRIUM.

*brant.* From the extremity of this resultant draw lines joining the outer ends of the components (24 oz. and 32 oz.). Then you have a parallelogram whose adjacent sides from the point of application, represent, both in direction and magnitude, the component forces, and whose diagonal represents, also both in direction and magnitude, their resultant.

If any other pair of convenient weights be selected and applied in the same way, you can find an equilibrant and resultant for them. From these experiments you conclude that a general principle, termed the "parallelogram of forces," is true without having recourse to any special mathematical reasoning.

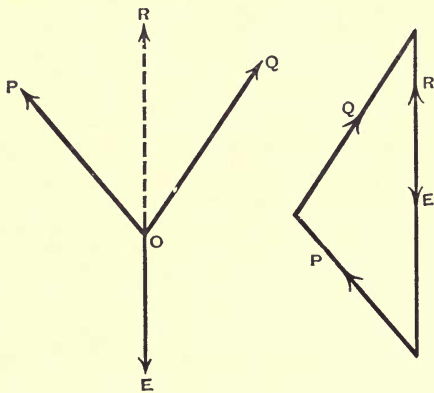
**Parallelogram of Forces.**—If two forces, acting simultaneously towards or from a point, be represented in direction and magnitude by the adjacent sides of a parallelogram, then the resultant of these forces will be represented in direction and magnitude by the diagonal of the parallelogram which passes through their point of intersection.

For example, let any two forces, P and Q, act from the point O at any convenient angle, say  $60^\circ$ , then, if OA and OB be plotted to scale to represent these forces in direction and magnitude, the diagonal OD of the parallelogram OADB will represent in direction and to the same scale their resultant R. But the resultant R is equal and opposite in direction to a force E, which would exactly balance the effect of P and Q, or to a force represented in direction and



PARALLELOGRAM OF FORCES.

in magnitude by the line DO. Further, since the side AD is equal and parallel to the side OB, it may be taken to represent Q in direction and magnitude. Hence we have the three sides of the triangle OAD taken in the order OA, AD, DO, representing in direction and magnitude three forces, P, Q, E, in equilibrium, acting from the point O. Hence we have a general proposition termed the “triangle of forces,” or a deduction from the “parallelogram of forces.”



TRIANGLE OF FORCES.

**Triangle of Forces.**—If three forces acting towards or from a point are in equilibrium, and a triangle be drawn with its sides

*respectively parallel to those forces taken in due order, then the forces will be represented to scale by the sides of the triangle.*

CONVERSELY :—*If three forces acting towards or from a point are represented in direction and to scale by the sides of a triangle taken in due order, these three forces are in equilibrium.*

For example, let the three forces P, Q and E act from the point O, and be in equilibrium. Draw a triangle with its sides, P, Q, E, respectively parallel to these forces; then the sides of this triangle, taken in that order, represent to the same scale these forces. Or, if the triangle, whose sides are respectively P, Q and E, represent in direction and to scale the three forces P, Q and E, as they act from a point O, these forces are in equilibrium. We have shown by a dotted line the resultant R, and its direction as opposed to E, by the same side of the triangle.

It is quite evident that if the forces P, Q and E acted *towards* the point O, instead of from it, the triangle P, Q, E would still represent these forces in magnitude, but the direction of all the arrows would have to be pointed the opposite way.

**SPECIAL CASES.—Three Equal Forces in Equilibrium.**—It can easily be proved by the apparatus used for Experiment I., or by construction, that if you have three equal forces in equilibrium they must act at  $120^\circ$  from each other, and that the triangle representing their directions and magnitudes will be an equilateral triangle, or a triangle whose angles are each equal to  $60^\circ$ .

**Two Forces acting at Right Angles.**—In this case it can be proved by the same apparatus, or by Euclid, Book I. Prop. 47, that any two forces P and Q, acting at right angles to each other, have a resultant R, or are balanced by a third force E, of such magnitude that—

$$E^2 = R^2 = P^2 + Q^2$$

Consequently, if you have any two forces in the proportion of 3 to 4 acting at right angles to each other, their resultant will proportionately be 5.

For, suppose  $P = 3a$ ,  $Q = 4a$ , where  $a$  is any number of units of force.

Then,

$$R^2 = P^2 + Q^2,$$

$$R^2 = 9a^2 + 16a^2,$$

$$R^2 = 25a^2,$$

$$\therefore R = \sqrt{25a^2} = 5a$$

$$\therefore P : Q : R = 3a : 4a : 5a$$

Or,

$$P : Q : R = 3 : 4 : 5.$$

Conversely, if any two forces in the proportion of 3 to 4 units are balanced by a third force proportionately of 5 units, the forces 3 and 4 must be acting at right angles to each other.

**Resolution of a Force into Two Components at Right Angles to each other.\***—Let  $R$  be the force to be resolved,  $P$  and  $Q$  the components, and let  $R$  make an angle,  $\theta$ , with the force  $Q$ .

$$\text{Then . . . . } R \cdot \cos \theta = Q ; \text{ for } \cos \theta = \frac{Q}{R}$$

$$\text{And . . . . } R \cdot \sin \theta = P ; \text{ for } \sin \theta = \frac{P}{R}$$

$$\text{Also . . . . } \frac{R \cdot \sin \theta}{R \cdot \cos \theta} = \frac{P}{Q} = \tan \theta$$

**Resultant of Two Forces acting at any Angle on a Point.**—The proof of this general case must be left to the Author's Advanced Treatise on Applied Mechanics, but the formula may be given here, viz. :

$$R^2 = P^2 + Q^2 + 2P \times Q \cos a$$

where  $P$  and  $Q$  are any two forces,  $R$  their resultant, and  $a$  the angle between the directions of the forces  $P$  and  $Q$ .

If  $P=Q$ , then—

$$R^2 = P^2 + P^2 + 2P^2 \cos a = 2P^2 + 2P^2 \cos a$$

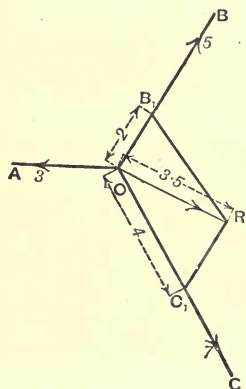
$$\text{or, } R^2 = 2P^2 (1 + \cos a) = 4P^2 \cos^2 \frac{a}{2}$$

$$\therefore R = 2P \cos \frac{a}{2} \quad (\text{Since } \cos a = 2 \cos^2 \frac{a}{2} - 1)$$

**Resultant of any Number of Forces Acting at a Point.**—Let  $P_1, P_2, P_3$ , &c., be any number of forces acting at a point ; then, by the parallelogram of forces find a resultant,  $R_1$ , for  $P_1$  and  $P_2$  ; and a resultant,  $R_2$ , for  $R_1$  and  $P_3$  ; and so on. The last resultant will be the resultant of all the forces.

**Example I.**—Forces 3, 5 and 7 units act from a central point  $O$  at equal angles. Find the resultant.

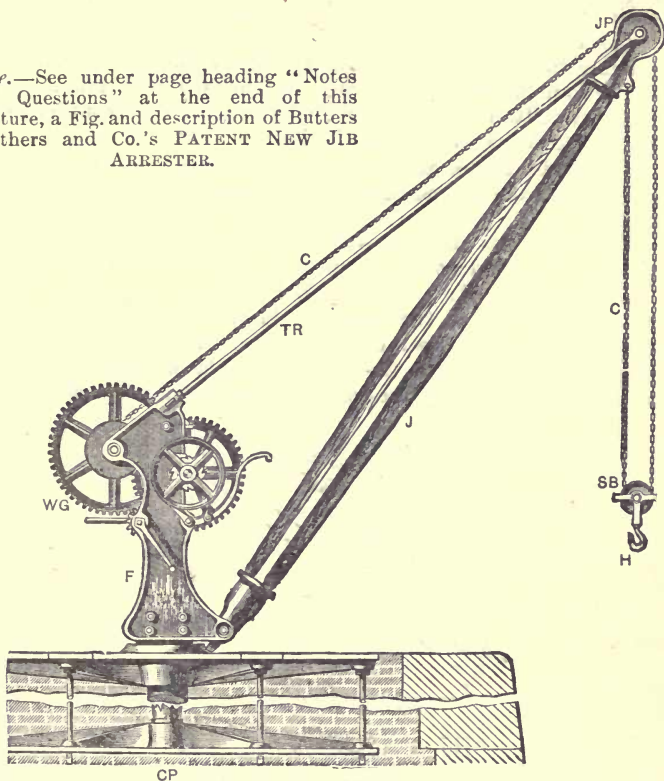
**ANSWER.**—Let  $OA, OB$  and  $OC$  represent the forces in direction and magnitude. Then you can follow out the above rule and find a resultant for, say, 3 and 5—call this  $R_1$  ; and finally find a resultant for  $R_1$  and 7. But it is obvious that you may subtract 3 units from each of them without affecting the result, since the forces are acting at equal angles from each other. This will destroy one of them, and leave  $OB_1$  to represent 2 units, and  $OC_1$  to represent 4 units. Then, by the parallelogram of forces you find the resultant  $R = 3.5$  units.



\* The reverse of this may be applied to the composition of two or more forces acting at a point in one plane, but we will leave the demonstration of such problems, as well as that of the polygon of forces, to our Advanced Book on Applied Mechanics.

**Stresses in Jib Cranes.**—As a practical example of the application of the “triangle of forces,” take the case of an ordinary hand-worked jib crane. The load is suspended from the hook H of the snatch-block SB; or, in the case of a crane for lifting light loads quickly, to a simple hook with a swivel attached directly

*Note.*—See under page heading “Notes and Questions” at the end of this Lecture, a Fig. and description of Butters Brothers and Co.’s PATENT NEW JIB ARRESTER.



HAND-WORKED JIB CRANE.

INDEX TO PARTS.

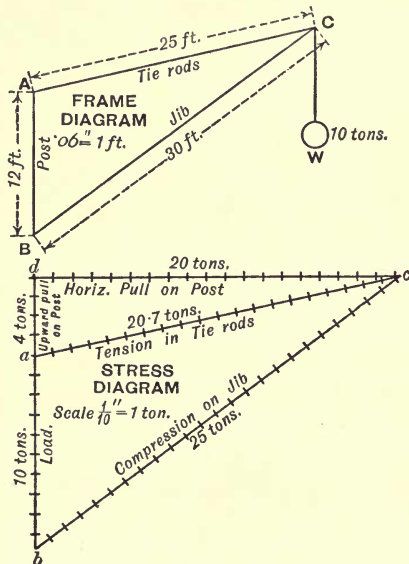
CP represents	Central post.	J represents	Jib.
F	„ Framing.	JP	„ Jib pulley.
WG	„ Wheel gear.	SB	„ Snatch-block.
C	„ Chain.	H	„ Hook.
TR	„ Tie-rods.		

to the end shackle of the chain C, as it comes down from the jib pulley JP, instead of the chain passing round a snatch-block pulley, and up to an eyebolt near the point of the jib.

- (1) The load produces a tension on the chain C.
- (2) A thrust along the jib J, from the jib pulley to the eye-bolt connecting the shoe of the jib to the bottom of the framing F.
- (3) A tension in the tie-rods from the top of the framing to their connection with the top of the jib.
- (4) This tension on the tie-rods produces a horizontal pull, tending to bend and break the crane-post CP, where it leaves the upper foundation plate-bearing and joins the framing.
- (5) It also causes an upward pull, tending to unship or lift the crane-post from its bearings in the upper and lower foundation plates.

The directions and values of these stresses will be better understood by the student after considering a particular example.

EXAMPLE II.—In a hand-worked jib crane of the form shown by the above figure, the length of the jib is 30 feet, the lengths of the tie-rods are 25 feet each, and the vertical distance between the attachments of the tie-rods and of the jib to the framing, is 12 feet. Find the stresses produced on these parts of the crane



FRAME AND STRESS DIAGRAM FOR A JIB CRANE.

by a load of 10 tons hung from the hook, neglecting all other stresses produced by the weight of the several parts of the crane.

ANSWER.—*First*, draw a “frame diagram,” or figure to scale, representing the directions and the lengths of centre lines of the various parts under stress. As shown by the frame diagram of the accompanying upper figure, AB represents the 12 feet vertical distance between the foot of the jib and inner ends of the tie-rods marked post, BC represents the 30 feet *jib*, AC the 25 feet *tie-rods*, and CW the 10-ton *load*—all to the same scale.

Now, it is evident from an inspection of this figure that the load W causes—

- (1) A vertical downward tension on the chain from C to W.
- (2) A thrust or compression along the jib from C to B, which produces an equal and opposite reaction from the framing at B along the jib from B to C.
- (3) A tension on the tie-rods from A to C.
- (4) This tension on the tie-rods may be resolved into a horizontal pull or force from A towards the direction of W, tending to bend or break the post about B.
- (5) Also, a vertical upward pull or force in the post from B towards A.

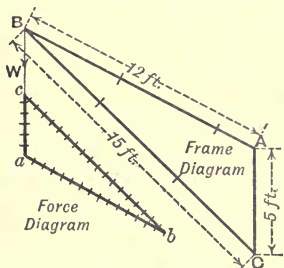
The student should mark the directions of these various forces by arrowheads on his frame diagrams.

*Second*, draw a “stress diagram,” viz., *ab*, vertical and to a convenient scale, to represent the downward force of the 10-ton load on the point C; *bc*, parallel to the *reaction* along the jib from B to C; and *ca*, parallel to the tension in the tie-rods from C to A.

Then, by “the triangle of forces,” since we have three forces acting from the point C (viz., load, reaction along jib, and tension in tie-rods) in equilibrium, and since we have drawn a triangle with its sides respectively parallel to these forces taken in due order; the forces will be represented to one scale by the sides of this triangle. Consequently, *ab* represents the load to scale; *bc* the reaction along the jib; and *ca* the tension in the tie-rods. Now, this tension on the tie-rods may be resolved into vertical and horizontal components by the method already described in this Lecture; therefore, a vertical line, *ad*, represents the vertical component or upward pull on the *post*, and *dc* the horizontal pull on the same, both in direction and to the same scale as *ab* represents the load. By applying the scale to which *ab* has been drawn to represent 10 tons (viz.,  $\frac{1}{10}$ " to 1 ton), *bc* shows 25 tons; *ca*, 20.7 tons (which would be 10.35 tons on each tie-rod if they were parallel to each other, but more if inclined from the jib-head to the outside of the framing); *cd*, 20 tons; and *ad*, 4 tons.

**EXAMPLE III.**—In a common crane the jib is 15 feet long, and the tie-rod 12 feet. The tie-rod is attached to the crane post at a point 5 feet above the foot of the jib. If a weight of 6 tons be hung from the point of the jib, find the tension in the tie-rod and the thrust in the jib.

**ANSWER.**—*First* draw the frame diagram as explained in Example II., marking the lengths of the parts by dotted lines and arrow-heads. (The student in his diagrams should also mark the directions of the stresses.)



TENSION IN THE TIE-ROD  
AND THRUST IN THE JIB  
OF A CRANE.

*Second*, on the line of action of the weight  $W$  draw  $ca$  to scale to represent the direction and magnitude of the weight, 6 tons. Then draw  $cb$  parallel to the jib, and  $ab$  parallel to the tie-rod. The triangle,  $cab$ , represents by its sides to one scale the magnitudes of the forces—viz., 14.4 tons tension in the tie-rods and 18 tons thrust or reaction in the jib.\*

**Stresses on a Simple Roof.**—

**EXAMPLE IV.**—The weight on each principal of a simple triangular roof is 1 ton. Find the stresses on the points of support and in the several members of the principal.

**ANSWER.**—*First*, draw a frame diagram of the principal, where  $AB$  and  $AC$  represent the direction and length of the rafters, and  $BC$  represents the tie-beam.

Then, the whole weight may be supposed to act in a vertically downward direction,  $AW$ , from the junction of the rafters through the middle of the tie-beam. This weight naturally produces a

pressure at  $B$  and at  $C$  of  $\frac{W}{2}$ . It also produces at these points

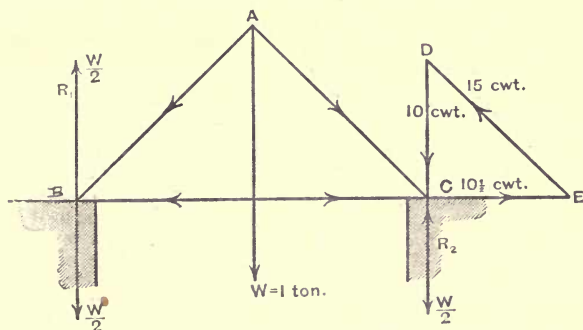
reactions  $R_1$  and  $R_2$ , each equal to  $\frac{W}{2}$ , since the whole is sym-

metrically balanced about the central vertical line  $AW$ . Further, there is a stress of compression along the rafters in the directions  $AB$  and  $AC$ , and consequently an equal and opposite reaction.

\* We have purposely used the letters  $ABC$  and  $abc$  differently placed from the previous figures in Example II., and have drawn the stress diagram in a different position, in order to teach the student that he must not depend upon his memory with regard to letters, but upon a clear understanding of the "triangle of forces." Students should draw their frame and stress diagrams to as large a scale as their exercise books will admit.

along those members from B to A, and from C to A. Also, there is an equal tension on the tie-beam from its centre towards B and towards C.

*Second*, draw the stress diagram for the three forces that are in equilibrium at the bearing C (viz., the vertical downward pressure  $\frac{W}{2}$ , the horizontal tension along the tie-beam and the reaction along the rafter from C to A) by plotting DC as a vertical line to scale to represent  $\frac{W}{2}$ , or 10 cwt., and drawing DE parallel to AC, and producing the tension on the tie-beam until it meets this line DE. Therefore, the other sides of the triangle DCE represent in



FRAME AND STRESS DIAGRAMS OF A SIMPLE ROOF.

direction and to the same scale as DC represents 10 cwt.; the tension on the tie-beam by CE, equal to  $10\frac{1}{2} \text{ cwt.}$ ; and the reaction along the rafter by ED, equal to 15 cwt.

In a precisely similar manner the stresses at the bearing B may be found by the "triangle of forces."

We will leave the more complicated questions in graphic statics to our book on the Advanced Stage of Applied Mechanics, since we believe the elementary or first year's student will find that what has been included in this Lecture is sufficient to enable him to understand what will be brought before him in the future Lectures of this book, as well as prepare him for answering the various problems which are likely to be asked of him.

## LECTURE VIII.—QUESTIONS.

1. State the principle of the parallelogram of forces, and explain how you would prove the truth thereof by experiment. A vertical force of 50 lbs. is balanced by two forces of 30 lbs. and 40 lbs. Find their directions and the angle between them.

2. Represent the point of application, the direction and the magnitude (to a scale of  $\frac{1}{10}$  inch to a pound) of the following forces:—10 lbs. acting northwards, 15 lbs. acting eastwards, 20 lbs. acting southwards, and 25 lbs. acting westwards, all from one point. Find their resultant, and its direction. *Ans.* 14·14 lbs. acting south-west.

3. State the principle of the triangle of forces. Three forces, P, Q and R, act from or towards a point, and are in equilibrium. Show graphically how you would represent their magnitude and direction by the three sides of a triangle taken in order. Explain the converse of this question.

4. Two ends of a piece of cord are fastened to two nails in a wall 8 ft. apart in a horizontal line. The cord is 10 feet in length, and has a knot 4 ft. from one end, from which point a weight of 25 lbs. is suspended. Find by construction the stresses on the nails, and indicate their direction by arrows. *Ans.* 22·5 lbs.; 17·5 lbs.

5. Show how to resolve a force into two components at right angles to each other. A force of 100 lbs. acts at (1st)  $30^\circ$ , (2nd)  $45^\circ$ , (3rd)  $60^\circ$ , (4th)  $75^\circ$  to the horizontal. Find by construction the vertical and horizontal components for each case, and prove your results by calculation. *Ans.* Vertical components:—50 lbs.; 70·7 lbs.; 86·6 lbs.; 96·6 lbs. Horizontal components:—86·6 lbs.; 70·7 lbs.; 50 lbs.; 25·8 lbs.

6. Sketch an ordinary hand-worked jib-crane. Explain its action by an index to parts, and show how the various stresses due to a load on the chain act, by aid of a frame and a stress diagram. Nine tons is hung from the end of the jib of a crane, which is inclined to the horizontal at an angle of  $60^\circ$ . If the compression on the jib is 16 tons, find by frame and stress diagrams the tension on the tie-rod. *Ans.* 9·4 tons.

7. In a crane, show the method of estimating the tension of the tie-rod and thrust on the jib when a given weight is hung from the end of the jib. If the load = 6 tons, and the tension of the tie-rod (which makes an angle of  $60^\circ$  with the vertical) = 18 tons, find by a diagram drawn to scale the thrust on the jib. *Ans.* 21·6 tons.

8. In a common crane the jib is 30 ft. long and the tie-rod 24 ft. The tie-rod is attached to the crane-post at a point 10 ft. above the foot of the jib. If a weight of 10 tons be hung from the point of the jib, find by constructing a frame and a stress diagram—(a) the tension on the tie-rod; (b) the thrust on the jib; (c) the horizontal pull on the post; (d) the upward pull on the same. *Ans.* (a) 24 tons; (b) 30 tons; (c) 21·2; (d) 11·2 tons.

9. A symmetrical pair of steps, hinged together at the top and connected together by a string at the bottom, stands on a smooth horizontal plane. If the length of each side be 3 feet 3 inches, and the string be 3 feet in length, find the tension of the string when a person of 140 lbs. in weight stands on the steps at a height of 2 feet from the ground? How is the tension of the string affected as the person ascends the steps? *Ans.* 24·8 lbs. When the person is at the very foot of the steps the

tension = 0 ; when he is at the top, the tension is a maximum of nearly 36.25 lbs.

10. A rectangular trap-door measuring 4 feet square and weighing 75 lbs. is hinged with one edge horizontal, and is supported in the horizontal position by a chain which is connected with the middle point of the outer edge of the trap-door, and with a point vertically over the middle point of the edge in which the hinges are fixed, but 7 feet above it. Sketch the arrangement, and determine the tension upon the chain and the reaction on the hinges. *Ans.*  $T = R = 43$  lbs.

(I.C.E. Feb. 1902.)

11. A load,  $W$ , of 2000 lbs. is hung from a pin,  $P$ , at which pieces  $AP$  and  $BP$ , meet like the tie and jib of a crane. The angles  $WPB$  and  $WPA$  are  $30^\circ$  and  $60^\circ$  respectively. Show by a sketch how to find the forces in  $AP$  and  $BP$ . Distinguish as to a piece being a strut or a tie. *Ans.* Strut, 3464 lbs.; tie, 2000 lbs.

12. Two pieces in a hinged structure meet at a pin, and a load is applied at the pin. Show how we find the pushing or pulling forces in the pieces. Describe an apparatus which enables your method to be illustrated.

13. The weight of a chain hanging from two points of support is 500 lbs. Its inclinations to the horizontal at the points of support are  $30^\circ$  and  $50^\circ$  respectively; what are the tensions at the points of support? *Ans.* 326.4 lbs.; 440 lbs.

14. The weight of a chain hanging from two points of support is 220 lbs.; its inclinations to the horizontal at the points of support are  $25^\circ$  and  $42^\circ$  respectively; what are the tensions in the chain at the points of support? *Ans.*  $T_1 = 177.6$  lbs.;  $T_2 = 216.6$  lbs.

15. A chain weighing 800 lb. is hung from its two ends, which are inclined to the horizontal at  $40^\circ$  and  $60^\circ$  respectively. What are the forces in the chain at the points of suspension? *Ans.* 400 and 600 lbs.

(B. of E., 1903.)

16. In a 10-ton crane the jib is 35 ft. long, the tie 30 ft. long, and the crane post 15 ft. high. Neglecting the effect of the tension in the chain, obtain the longitudinal forces in the tie and jib. *Ans.* Stress in tie = 20 tons; stress in jib =  $23\frac{1}{2}$  tons.

(C. & G., 1903, O., Sec. B.)

17. A machine 5 tons in weight is supported by two chains; one of these goes up to an eyebolt in a wall and is inclined  $20^\circ$  to the horizontal; the other goes up to a roof principal and is inclined  $73^\circ$  to the horizontal; find the pulling forces in the chains. You may use a graphical or any other method of calculation you please. *Ans.* 1.47 and 4.7 tons.

(B. of E. 1904.)

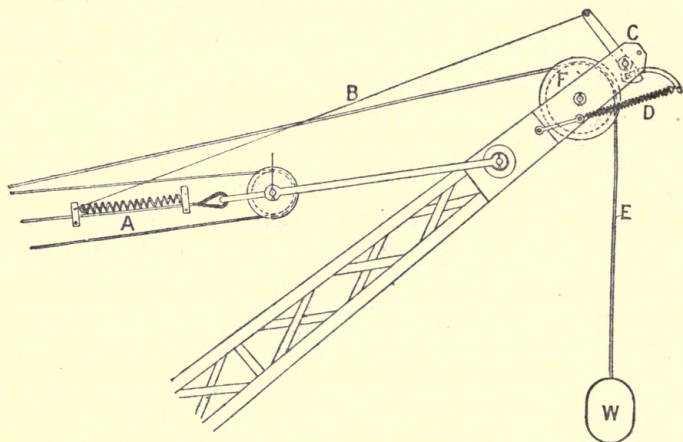
18. A cord is wrapped six times round four parallel rods, which pass through the four corners of a rectangle, and are perpendicular to the plane of the rectangle; the long sides of the rectangle are  $2\frac{1}{2}$  times the length of the short sides. The cord, while being wrapped round the rods, is kept taut with a uniform tension of  $7\frac{1}{2}$  pounds. Find graphically, or in any other way, the force exerted by the cord on any one of the rods. *Ans.* Force on any one rod = 64 lbs.

(B. of E., 1905.)

19. A roof truss consists of two rafters, equally inclined, connected by a horizontal tie at a distance of  $a$  feet below the apex. If the total span be  $l$  feet and the load carried at the apex be  $w$  tons, show that the tension in the tie rod is  $wl/4a$  tons.

(C. & G., 1904, O., Sec. B.)

Many fatal accidents have taken place due to the falling of the jibs of hand and steam worked cranes. The following simple device has been recently applied by a Glasgow firm with the object of avoiding such accidents.



1909 PATENT JIB ARRESTER, BY BUTTERS BROTHERS & Co.  
GLASGOW.

In the event of any of the jib gear or of the jib rope of the crane breaking, the spring A contracts, thus relieving the rope B, which holds the clutch C in position. The two springs at the point of the jib marked D then contract, and pull the clutch C inwards. This catches the lift rope E, on the pulley at the point of the jib F, thus holding up the jib and preventing it from falling, as well as keeping the load W in position.

## LECTURE IX.

**CONTENTS.**—Inclined Planes—The Inclined Plane without Friction—When the Force acts Parallel to the Plane—Example I.—When the Force acts Parallel to the Base—Example II.—When the Force acts at any Angle to the Inclined Plane—Example III.—The Principle of Work applied to the Inclined Plane—Example IV.—Questions.

**Inclined Planes.**—An inclined plane is a plane surface inclined to the horizontal, whereby a certain force may be used to raise a greater weight to a desired height than could be done by applying it directly to elevate the weight vertically. Inclined planes are also used for easing down weights with less retarding force than would be necessary to lower them vertically. In another form, called the wedge, inclined planes are employed for splitting bodies, or different parts of the same body, asunder, as in the case of the steel wedge used by the woodman to split up logs for firewood and other purposes. Wedges are also used for forcing bodies together, and for fixing them tightly in a desired position; or for elevating them through a small distance, as in the case of the levelling of the heavy cast-iron sole-plate of an engine. And further, as we shall have occasion to prove, the well-known screw, in whatever form it may be applied, is simply an inclined plane of a particular shape.

**The Inclined Plane without Friction.**—In the first place, we will consider the inclined plane with a body placed thereon and kept in position by a force applied to the body, *when all friction between the plane and the body is supposed to be absent or negligible*—i.e., both the plane and the body are assumed to be perfectly smooth. There are three cases of this statical problem.

(1) When the force supporting the body acts parallel to the inclined plane.

(2) When the force acts horizontally.

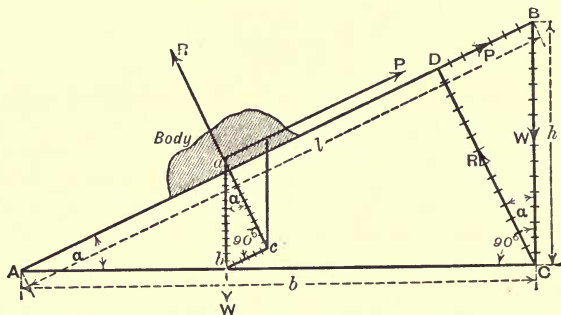
(3) When the force acts at any angle to the plane.

**Case I.**—*Let the force  $P$  act parallel to the plane*, and let the accompanying figure represent a vertical section through the plane and the c.g. of the body. Let  $a$  be the c.g. of the body;  $W$  its weight, acting vertically downwards along the line  $aW$ ;  $P$  the necessary pull (to keep the body in position) applied along the line  $aP$ ,

parallel to the plane  $AB$ ; and  $R$  the reaction from the plane (due to the weight of the body resting thereon), acting along the line  $aR$ , at right angles to the plane.

Also, let the length of the plane  $AB$  be indicated by  $l$ ; its height,  $BC$ , by  $h$ ; its base,  $AC$ , by  $b$ ; and the angle of the plane to the horizontal by  $a$ .

Now, by the "triangle of forces," since we have three forces,  $W$ ,  $P$  and  $R$ , acting at  $a$ , the  $c.g.$  of the body, and since these forces are in equilibrium, if we construct a triangle whose sides



INCLINED PLANE, CASE I.  
WHEN  $P$  ACTS PARALLEL TO PLANE  $AB$ .

are parallel to these forces, they will represent them in direction and in magnitude.

Therefore, plot off along the line  $aW$  a distance  $ab$ , to represent the weight of the body  $W$ , to any convenient scale. From  $b$ , draw a line  $bc$  parallel to  $P$ , and from  $a$ , extend the direction of  $R$  to  $c$ , by the line  $ac$ .

Then,  $W : P : R :: ab : bc : ca$

But by Euclid the triangle  $abc$  is similar to the triangle  $ABC$ .

$$\therefore ab : bc : ca :: AB : BC : CA$$

And,  $AB : BC : CA :: l : h : b$

Consequently,  $W : P : R :: l : h : b$

$$\text{Or, } \frac{P}{W} = \frac{h}{l}; \quad \frac{R}{W} = \frac{b}{l}; \quad \text{and } \frac{P}{R} = \frac{h}{b}$$

$$\text{Or, } \frac{P}{W} = \sin a; \quad \frac{R}{W} = \cos a; \quad \text{and } \frac{P}{R} = \tan a$$

Precisely the same results will be arrived at if (as shown by the right-hand side of the figure) we considered the vertical side  $BC$

of the triangle ABC as representing W, and then have drawn a line from C on the direction of AB, parallel to R. It will form a useful exercise for the student if in every case he will plot down both methods, and mark along the *sides* of the triangle of forces the respective *forces* which they respectively represent.

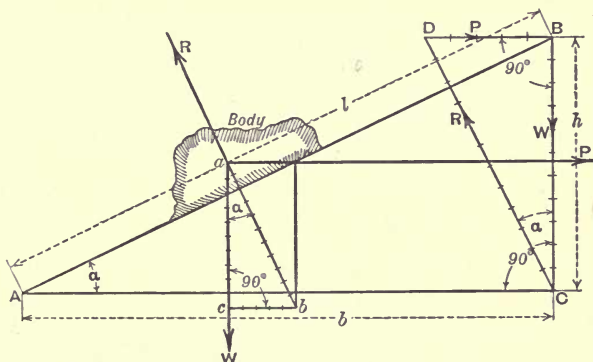
**EXAMPLE I.**—A weight of 100 lbs. is supported on a smooth inclined plane by a force P, acting parallel to the plane. If the incline be 1 in 10, find P, and give the reasoning by which you establish the result.

**ANSWER.**—Draw a figure exactly the same as that accompanying Case 1, and mark  $W = 100$  lbs.,  $l = 10$ , and  $h = 1$ . Then by the “triangle of forces”:

$$\frac{P}{W} = \frac{BC}{AB} = \frac{h}{l} = \frac{1}{10}$$

$$P = W \frac{1}{10} = \frac{100}{10} = 10 \text{ lbs.}$$

**Case 2.**—Let the force P act parallel to the base, with the same



INCLINED PLANE, CASE 2.

WHEN P ACTS PARALLEL TO BASE AC.

signification for each of the forces and parts of the inclined plane, and the same assumptions. Then plot off  $ac$ , along  $aW$ , to represent  $W$ ; draw  $cb$  parallel to  $P$ , and extend the direction of  $R$  backwards along  $ab$ , until it meets  $cb$  at the point  $b$

Then

$$W : P : R :: ac : cb : ba.$$

But by Euclid the triangle  $acb$  is similar to the triangle  $ACB$ .

$$\therefore ac : cb : ba :: AC : CB : BA$$

And,

$$AC : CB : BA :: b : h : l$$

Consequently,  $W : P : R :: b : h : l$

Or,  $\frac{P}{W} = \frac{h}{b}$ ;  $\frac{R}{W} = \frac{l}{b}$ ; and  $\frac{P}{R} = \frac{h}{l}$

Or,  $\frac{P}{W} = \tan \alpha$ ;  $\frac{R}{W} = \sec \alpha$ ; and  $\frac{P}{R} = \sin \alpha$

Precisely the same results will be arrived at if (as shown by the right-hand side of the figure) we considered the vertical side BC of the triangle ABC as representing W, and then draw a line from C parallel to R, and a line BD, parallel to P, to meet the line CD.

EXAMPLE II.—A force of 100 lbs. is supported on a smooth inclined plane by a force P acting parallel to the base. If the incline be 1 in 10, find P.

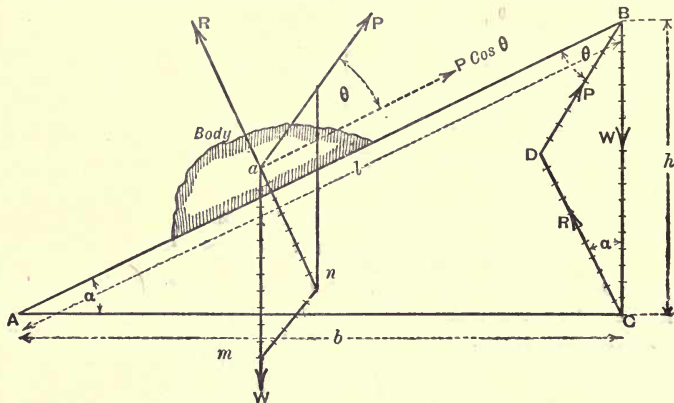
ANSWER.—Draw a figure exactly the same as that accompanying Case 2, and mark  $W = 100$  lbs.,  $l = 10$ , and  $h = 1$ .

Then, by the "triangle of forces":

$$\frac{P}{W} = \frac{CB}{AC} = \frac{h}{b} = \frac{1}{\sqrt{l^2 - h^2}} = \frac{1}{\sqrt{100 - 1}} = \frac{1}{\sqrt{99}} = \frac{1}{9.95}$$

$$\therefore P = \frac{W}{9.95} = \frac{100}{9.95} = 10.05 \text{ lbs.}$$

CASE 3.—Let the force P act at any angle  $\theta$  to the inclined plane AB. With the same signification for each of the forces and parts of the inclined plane, and the same assumptions, plot off from a,



INCLINED PLANE, CASE 3.  
WHEN P ACTS AT ANY ANGLE TO PLANE AB.

along the line  $aW$ , a distance  $am$ , to any convenient scale to represent the weight of the body  $W$ . From this point,  $m$ , draw a line  $mn$  parallel to  $P$ , and extend the direction of  $R$  backwards to meet this line. This small triangle,  $amn$ , will be a "triangle of forces," for  $W$ ,  $P$  and  $R$ , which are in equilibrium about the *e.g.* of the body at  $a$ .

But in this case the student will probably realise the proof of the problem more easily if he considers  $BC$  as representing to scale the weight  $W$ , and then draws  $CD$  parallel to  $R$ , and  $DB$  parallel to  $P$ ,

When  $W : R : P :: BC : CD : DB$ ,  
or the triangle  $BCD$  is the "triangle of forces," representing the forces  $W$ ,  $R$  and  $P$  in direction and magnitude by the sides  $BC$ ,  $CD$  and  $DB$  respectively.

Or, 
$$\frac{P}{W} = \frac{DB}{BC}; \quad \frac{R}{W} = \frac{CD}{BC}; \quad \text{and} \quad \frac{P}{R} = \frac{DB}{CD}$$

If we resolve the force  $P$  (which acts at the angle  $\theta$  to the inclined plane) parallel to the plane, then we can treat the components of  $P$  *exactly* in the same way as we did the simple force  $P$  in Case 1.

If we resolve  $P$  into the direction of  $R$ , then this component acts with  $R$ , and is evidently balanced by the resolved part of  $W$  in the same direction—*i.e.*, along the line,  $an$ .

**EXAMPLE III.**—A weight of 100 lbs. is supported on a smooth inclined plane by a force  $P$ , acting at  $60^\circ$  in an upward direction from the inclined plane. If the incline be 1 in 10, find  $P$ .

**ANSWER.**—Draw a figure exactly the same as that accompanying Case 3, and mark  $W = 100$  lbs.,  $l = 10$ ,  $h = 1$ , and  $\theta = 60^\circ$ .

Then by the "triangle of forces,"  $BC$  represents  $W$ , and  $DB$  represents  $P$  to scale. Measuring their respective lengths we get

$$P = W \frac{DB}{BC} = 100 \frac{20}{100} = 20 \text{ lbs.}$$

**Principle of Work applied to the Inclined Plane.**—Referring to the figure for Case 1, let the body, whilst under the action of the three forces  $W$ ,  $P$  and  $R$ , be moved the whole length of the incline. Therefore  $P$  acts from  $A$  to  $B$ , and at the same time  $W$  acts through a vertical height  $CB$ . Consequently, *neglecting friction* as before, we have by the "principle of work"—

The work put in = The work got out

$P \times \text{its distance} = W \times \text{its distance}$

$$P \times AB = W \times CB$$

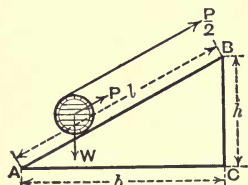
$$P \times l = W \times h$$

$$\therefore \frac{P}{W} = \frac{h}{l}; \quad \text{and} \quad P = W \frac{h}{l}.$$

But this is precisely the same result as we got by applying the principle of the "triangle of forces." Hence, the "principle of work" agrees with the "triangle of forces" in respect to the inclined plane.

Cases 2 and 3 may be treated by the student in exactly the same way, and the correct results will be the same as those found by the "triangle of forces."

EXAMPLE IV.—An inclined plane is used for withdrawing barrels from a cellar by securing two ropes to the top of the incline at B, then passing them down the incline, half round the barrel, and up to the horizontal platform at the top of the incline, where two men pull on the ropes in a direction parallel to the plane. If the weight,  $W$ , of the barrel is 200 lbs., the length,  $l$ , of the incline 20 ft., and the height 10 ft., find, by the principle of work, the least force which must be exerted by the two men, and the work expended, *neglecting friction*, in drawing the barrel from the cellar.



RAISING BARRELS BY  
RETURN ROPE AND  
INCLINED PLANE.

Let the accompanying figure represent a vertical cross section through the middle of the barrel and the inclined plane. Then a statical force,  $P$ , applied at the c.g. of the barrel, would just balance its weight,  $W$ , and the reaction from the plane (not shown).

By the principle of work, *neglecting friction*—

*The work put in = The work got out.*

$P \times \text{its distance} = W \times \text{its distance}.$

$$P \times l = W \times h.$$

$$P = W \frac{h}{l} = 200 \frac{10'}{20'} = 100 \text{ lbs.}$$

But by passing the rope round the barrel, as explained in the question, this force  $P$  is halved on the ropes (see Lecture VI. on the pulley and snatch-block). Therefore the least force which the two men must exert in order *just* to move the barrel will be—

$$\frac{P}{2} = \frac{100}{2} = 50 \text{ bs.}$$

But this force acts through a distance  $2l = 40'$  ft.; therefore the work expended will be—

$$\frac{P}{2} \times 2l = 50 \text{ lbs.} \times 40' = 2000 \text{ ft.-lbs.}$$

Or, work got out =  $W \times h = 200 \text{ lbs.} \times 10' = 2000 \text{ ft.-lbs.}$

In this question we have a combination of the pulley and the inclined plane. The inner ends of the two ropes being fixed at the top of the inclined plane, the force with which the men act on the free ends is communicated throughout the ropes, so that the stress in the ropes on each side of the barrel balances the force  $P$ , that would be required to move the barrel up the incline if applied at its centre of gravity.

Or, the *theoretical advantage* due to the pulley part of the system is, . . . . .  $\frac{P}{\frac{2P}{2}} = \frac{2P}{P} = \frac{2}{1}$

Then for the inclined plane part we have by the "triangle of forces," or by the "principle of work," a *theoretical advantage* of—

$$\frac{W}{P} = \frac{l}{h} = \frac{20}{10} = \frac{2}{1}$$

Therefore, the *total* theoretical advantage is the *product* of the two separate advantages, viz.—

$$\frac{2P}{P} \times \frac{W}{P} = \frac{2}{1} \times \frac{2}{1} = \frac{4}{1}$$

Consequently, a force of 1 lb. applied at the free end of the rope would balance a weight of 4 lbs. on the incline. Or, as in the question, and, *neglecting friction*, a barrel weighing 200 lbs. requires a pull of 50 lbs. to move it up the inclined plane.

We have simply split up the total advantage in this way to show the student that the combined advantages of the several parts of a compound machine must equal the advantage of the whole. We might have said at once, as we have done before in other cases—

$$\text{The Theoretical Advantage} = \frac{W}{P} = \frac{200}{50} = \frac{4}{1}$$

NOTE.—I have this day (Sept. 9, 1892) witnessed the interesting operation of lowering four very large 25-ton steam boilers of the marine type, down an incline of about 100 feet in length by the method described in the foregoing question. *One man*, by aid of an ordinary block and tackle, supplied the requisite restraining force on the free end of the rope.

## LECTURE IX.—QUESTIONS.

1. Prove by the triangle of forces (drawn to scale) the relation between the weight  $W$  of a body resting on a smooth inclined plane, the reaction,  $R$ , from the plane, and the force,  $P$ , necessary to just balance the weight—(1) when the force,  $P$ , acts parallel to the plane; (2) when it acts parallel to the base; (3) when it acts at an angle,  $\theta$ , to the plane.

2. A ball, weighing 100 lbs., rests on an inclined plane, being held in position by a string which is fastened to a bracket so as to be parallel to the plane. The height of the plane being  $\frac{1}{3}$  of the length, find the tension of the string and the pressure perpendicular to the plane. Establish your results by reasoning on known principles, such as the principle of work or that of the parallelogram of forces. *Ans.*  $P = 33\frac{1}{3}$  lbs., and  $R = 94\frac{1}{3}$  lbs.

3. Prove the relation between  $W$ ,  $P$ , and  $R$ , acting on a body resting on a smooth inclined plane by the "*principle of work*" for cases 1, 2 and 3 in this Lecture. An incline is 1 ft. in vertical height for 15 in length. A weight of 100 lbs. rests on the plane and is held up by friction; make a diagram for estimating the pressure on the plane, and find its amount. *Ans.* 99·7 lbs.

4. Friction being neglected, find the force, acting parallel to the plane, which will support 1 ton on an incline of 1 ft. vertical and 10 ft. along the incline. Prove the formula which you employ. If the incline were 1 ft. vertical and 280 ft. along the incline, find the force in pounds which would support 1 ton. *Ans.* 224 lbs., and 8 lbs.

5. A smooth incline plane has a vertical side of 1 ft., and a length of 10 ft.; what work is done in pulling 10 lbs. up 8 ft. of the incline? *Ans.* 8 ft.-lbs.

6. When a body is raised through a given height, how is the work done estimated? A body weighing 8 cwt. is drawn along 100 ft. up an incline, which rises 2 ft. in height for every 5 ft. along the incline; the resistance of friction being neglected, find the work done. *Ans.* 35,840 ft.-lbs.

7. A smooth incline is 8 ft. long, and the total vertical rise from the bottom to the top thereof is 2 ft. What amount of work is performed in drawing a weight of 100 lbs. up 4 ft. of the incline, and what is the least force which will do this work? *Ans.* 100 ft.-lbs.; 25 lbs.

8. Friction is neglected, and it is found that a force acting horizontally will move 10 lbs. up 5 ft. of an incline rising 1 in 4. Find the work done, and find also the force parallel to the plane which will just support the weight of 10 lbs. *Ans.* 12·5 ft.-lbs.; 2·5 lbs.

9. A car laden with 20 passengers is drawn up an incline, one end of which is 160 ft. above the other; the car, when empty, weighs 3 tons, and the average weight of each passenger is 140 lbs. Find the number of ft.-lbs. of work done in ascending the incline, neglecting friction. *Ans.* 1,523,200 ft.-lbs., or 680 ft.-tons.

10. It will be observed that draymen sometimes lower heavy casks into cellars by means of an inclined plane and a rope. One end of the rope is secured to the upper end of the inclined plane, and is then passed under and over the cask, the men holding back by means of the loose end. Now, supposing the incline to be at an angle of 45 degrees, explain the mechanical principles that are here applied, and find the advantage. *Ans.*  $2\sqrt{2} : 1$ .

11. A barrel weighing 5 cwt. is lowered into a cellar down a smooth slide inclined at an angle of 45 degrees with the vertical. It is lowered by means of two ropes passing under the barrel, one end of each rope being fixed, while two men pay out the other ends of the ropes. What pull in lbs. must each man exert in order that the barrel may be supported at any point? *Ans.* 99 lbs. nearly.

## LECTURE X.

**CONTENTS.**—Friction—Heat is Developed when Force overcomes Friction—Laws of Friction—Apparatus for Demonstrating First and Second Laws of Friction—Experiment I.—Example I.—Angle of Repose or Angle of Friction—Experiment II.—Diagram of Angles of Repose—Limiting Angle of Resistance—Experiment III.—Apparatus for Demonstration of the Third Law of Friction—Experiment IV.—Lubrication—Anti-Friction Wheels—Ball Bearings—Work done on Inclines, including Friction—Example II.—Questions.

**Friction.**—Whenever a body is caused to slide over another body, an opposing resistance is at once experienced. This natural resistance is termed *friction*.\* The true cause of friction is the roughness of the surfaces in contact. The smoother the sliding surfaces are made the less will be the friction. Friction cannot, however, be entirely eliminated by any known means, for even the most microscopical protuberances on the smoothest of surfaces seem to fit into corresponding hollows on other equally smooth places, so that some force is required to make the one body slide over the other.

Friction has its advantages as well as its disadvantages. For example, if it were not for friction we could not walk, neither could a locomotive start from a railway station, nor could it be brought to rest in the usual speedy manner. Friction is also essential to the utility of nails, screws, wedges, driving belts, &c. On the other hand, power is often expended in overcoming friction with the result of much wear and tear in machinery. For example, in the case of working the slide valves of locomotive engines as much as twenty horse-power is required in moving these essential parts when running at full speed.†

It is the duty of the engineer to reduce friction to a minimum in the case of the bearings of engines, shafting, and machines generally, in order that a minimum of work may be expended in moving them. He has, however, also to devise means of producing a maximum of friction in the case of certain pulleys, grips, clutches, brakes, and such like appliances, where motion has to be transmitted by aid of friction, or bodies in motion (such as a

\* French writers call friction a *passive resistance*, because it is only apparent when one body tends to move or pass over another.

† See the Author's "Elementary Manual on Steam and the Steam Engine," page 182, for an arithmetical example.

moving train) have to be brought to rest quickly when nearing a station.

**Heat is Developed when Force overcomes Friction.**—When a body is kept moving by a force, part (or in certain cases it may be the whole) of the mechanical force is expended in overcoming frictional resistance. This *lost work* is directly transformed into *heat* in the act of overcoming the frictional resistance through a distance. *For example* :—A person slips down a vertical rope by holding it between his hands and his legs. The force of gravity impels him downwards, overcoming the frictional resistance between his hands and limbs and the rope, with the consequence that they become severely heated, especially if he happens to slip down quickly. A boy takes a run, and then slides along a level piece of ice. The foot-pounds of work stored up in him just before he begins to slide are expended partly in overcoming the frictional resistance between the soles of his boots and the ice, and partly in the frictional resistance between his clothes and the air. As a consequence, he will find that by the time he gets to the end of the slide his soles are considerably warmed. If the ice were perfectly level, infinitely long, and if there were absolutely no friction between it and his boots, and if there were no frictional resistance between him and the air, then he would slide on *for ever* ! If we could diminish the frictional resistance between the skin of a ship and the water, and between the exposed parts of the ship and the air, *to nothing*, then all that would be required to transport her across the Atlantic would be a strong force applied at the start until she attained the desired speed, when she would proceed forward, and arrive at her destination with undiminished velocity ! In reality, however, we find it necessary to employ steam engines of 10,000 horse-power continuously in order to propel an Atlantic “greyhound” of 5000 tons at twenty knots in the calmest of weather. About one-half of this power is absorbed in overcoming the frictional resistance of the ship through the water and air, and the other half in the frictional and other losses due to the working of the propelling machinery. Examples of the conversion of mechanical work into heat are so familiar to you all, being in fact brought prominently before your notice every day of your existence, that we need not further enlarge upon this question except to remind the student of Dr. Joule’s discovery of the rate of exchange between heat and work. He found by experiment that if work is transformed into heat, every 772 ft.-lbs. of work will produce 1 heat unit, or that quantity of heat which would raise 1 lb. of water 1° Fahr.\*

\* For further examples and an explanation of Dr. Joule’s experiments see the Author’s Treatise on Steam and the Steam Engine.

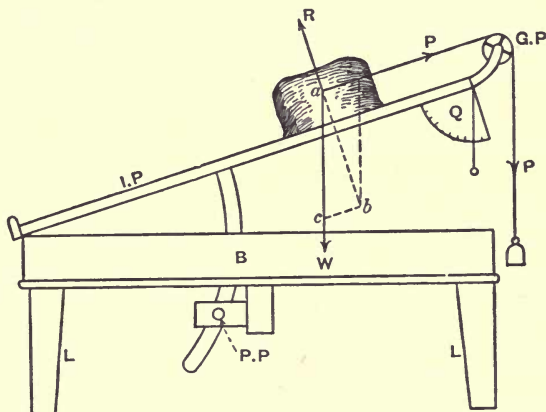
**Laws of Friction.**—From 1831 to 1834 General Morin carried out an extensive series of experiments at Metz on friction for plane surfaces, with different areas, pressures and velocities, from which he arrived at certain conclusions. These conclusions were for a long time regarded as constituting the fundamental laws of friction. They have been since proved to be only true within the limits of his experiments, for, they do not hold good for great pressures and high velocities, neither are they true for fluid, rolling, or axle friction. For the latest and most reliable experiments we must refer to the Proc. of the Inst. of Mechanical Engineers, 1883, 1885, 1888, and 1891.

1st Law. *Friction is directly proportional to the pressure between two surfaces, if they remain in the same condition.*

2nd Law. *Friction is independent of the areas in contact.*

3rd Law. *Friction is independent of velocity.*

**Apparatus for Demonstrating First and Second Laws of Friction.**—Nevertheless, it will be both interesting and instructive to students to have these three laws demonstrated by the following simple apparatus:—



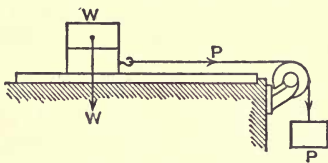
APPARATUS FOR DEMONSTRATING THE FIRST AND SECOND LAWS OF FRICTION.

INDEX TO PARTS.

IP represents Inclined plane.	B represents Box for planes.
GP    "    Guide pulley.	PP   "    Pinching pin.
Q     "    Quadrant.	LL   "    Legs.

The inclined plane IP is fitted with a joint at its left-hand end, and after slackening the pinching pin PP, it may be raised to any desired angle or fixed in a level position by tightening the pin. The desired position is found by reading off the angle opposite the plumb-ball line on the graduated degree scale of the quadrant Q. In the box B may be kept planes of glass, brass, iron, steel, &c., as well as the different kinds of wood to be experimented upon. These planes are fixed on IP, in a central position, by means of a catch, and the bodies to be laid upon them should be fitted with a small hook opposite their *c.g.*, to which a fine flexible silk cord can be attached and passed over the guide-pulley GP, which should turn very freely on its bearings.\* The pull P is best effected by attaching to this cord a small tin pail into which shot may be dropped one by one until the body moves freely on the plane. The pail and shot may then be unhooked and weighed in a balance.

**Demonstrations of the First Law of Friction.**—EXPERIMENT I.—Fix the inclined plane IP in a truly *horizontal* position.



**PROOF OF FIRST LAW OF FRICTION.**

Take from the box B, say, a long strip of planed yellow pine and a small block of the same kind of wood, and let its weight be W. Adjust the strip along the middle line of IP by means of the sneck or catch, and place the block therein. Attach the silk thread to the hook on the forward side of the block, and pass the same over the practically frictionless pulley. Hang a little tin pail from the free end of the silk thread, and drop small shot one by one into the pail until the block moves freely over the yellow pine strip when aided by a little tapping on the table. Unhook the pail containing the shot, and weigh it as carefully as you weighed the block of yellow pine. Let it equal P units.

Then P is the force which just overcomes the *directly opposing passive resistance*, called friction, between the surface of the yellow

\* The guide-pulley bracket should be fitted with a stiff joint and with a telescope arm, so that the pulley may be raised or lowered in order to bring the direction of the pull P on the cord parallel to the plane, or parallel to its base, or adjusted to any desired angle with respect to the plane, in order to demonstrate Cases 1, 2 and 3 of the inclined plane in Lecture IX. By having, say, a  $\frac{1}{2}$ " slot along the middle of the plane, and by lowering the pulley, Case 2, wherein the pull on the body is parallel to the base, may be readily demonstrated; and by pulling out the telescope arm of the bracket, and turning up the bracket, Case 3, wherein the pull makes an angle,  $\theta$ , with the plane, may be verified.

pine block and strip; and the ratio  $\frac{P}{W}$  is termed the *co-efficient of friction*. Now put another block of weight  $W$  on the top of the one just tested, so as to double the pressure on the sliding surface, and put in shot until the block moves when aided by a little vibration, so as to overcome the greater resistance to starting the body in motion than to keep it moving.\* You find on weighing the pail and shot that it is now  $2P$ . Consequently the co-efficient of friction has not altered, for  $\frac{2P}{2W}$  is the same fraction as  $\frac{P}{W}$ .

EXAMPLE I.—Suppose you take a very small block of wood (say  $1/10''$  thick,  $2''$  long and  $1''$  broad; in fact, so light that its weight is negligible), and place a 1-lb. weight on the top of it; you will find that 5.75 oz. are required to cause motion of this piece of wood over the surface of the yellow pine strip. You therefore conclude that the *co-efficient of friction* is

$$\mu = \frac{P}{W} = \frac{5.75 \text{ oz.}}{16 \text{ oz.}} = .353, \text{ or friction} = .353 W.$$

Now, place a 2-lb. weight on the upper piece of wood, and you find that it requires more shot in the pail to move it. Weigh the pail and the shot again just after you have obtained free movement of the one body over the other, and you will find that it amounts to 11.5 oz.

Consequently,  $\frac{P}{W} = \frac{11.5}{32} = .353$  as before.

If, however, you put a 10-lb. weight on the upper piece of wood, you will obtain a different result, thus proving the first law and the variation therefrom; because in this latter case the pressure is so great, compared with the first and second experiments, that the grains of the upper piece of wood enter those of the lower, and bring into play another condition of affairs—viz., the gripping action of the one set of grains on the other set. If you had taken a large plank of yellow pine, weighing, say, 100 lbs., and had placed it on another similar plank, the co-efficient of friction would have a certain value. If you had even put a 100-lb. weight on the upper plank, the co-efficient of friction might not have varied perceptibly. But if you placed a weight of 1000 lbs.

\* Statical friction, or the friction of repose, is that resistance which opposes the commencing of the motion. If a body be allowed to rest on another for some time, it requires more force to move it than if it had only been stationary for a few seconds.

on the upper plank, the co-efficient of friction would be considerably altered. Hence you observe that this first law only holds good between narrow limits.\*

**Angle of Repose, or Angle of Friction.**—**EXPERIMENT II.**—Another way of proving the first law of friction is to disconnect the silk thread and the shot-pail from the upper body, and tilt up the inclined plane to such an angle,  $\alpha$ , with the horizontal that (with the aid of a little tapping) the weighted block of yellow pine just slides slowly down the incline. Here we have simply the force of gravity acting on the body and overcoming friction. At the moment the body just begins to slide we have the weight,  $W$ , of the body acting vertically downwards,  $R$  the reaction from the plane at right angles to the surface, and  $F$ , the passive resistance of friction, acting parallel to the plane in the direction of  $aP$  in the first figure in this Lecture. Now, these three forces act from the *c.g.* of the body, and they are in equilibrium.  $R$  is equal to the resolved part of  $W$  at right angles to the plane (or  $R = W \cos \alpha$ ), and it represents the pressure between the surfaces.  $F$  is the resolved part of  $W$ , parallel to the plane (or  $F = W \sin \alpha$ ). and  $\frac{F}{R}$  is the co-efficient of friction.

$$\therefore \frac{F}{R} = \frac{W \sin \alpha}{W \cos \alpha} = \tan \alpha = \frac{h}{b} = \frac{\text{height of plane}}{\text{base of plane}}$$

The angle  $\alpha$ , to which the plane must be inclined before the free body will slip over the fixed one, has been termed the “*angle of repose*” or “*angle of friction*.”

*Therefore, the tangent of the angle of repose is equal to the co-efficient of friction.*

But  $\frac{P}{W}$  was proved by the previous experiment to be also equal to the co-efficient of friction,

$$\therefore \frac{P}{W} = \frac{F}{R} = \frac{h}{b} = \tan \alpha = \mu$$

Or,  $P = \mu W \quad \text{and} \quad F = \mu R$

\* Sir Robert Stawell Ball, when Professor of Mechanism at the Royal College of Science, Ireland, tried a careful experiment in the above way with a smooth horizontal surface of pine 72" x 11", and a slide, also of pine, 9" x 9" grain crosswise. He loaded and started the slide, and applied a force sufficient to maintain it in uniform motion, and he found that on increasing the load from 14 to 112 lbs., by increments of 14 lbs., the co-efficient of friction diminished from .336 to .262. From these experiments he constructed the empirical formula for this case that  $F = .9 + .266 R$ , where  $F$  is the frictional resistance and  $R$  the reaction from the surface or net load.

where  $\mu$  is the Greek letter universally adopted to represent coefficients of friction.

The accompanying figure is a diagram of the "*angles of repose*" for various common materials, together with the numerical values of  $\mu$ , or their co-efficients of friction.

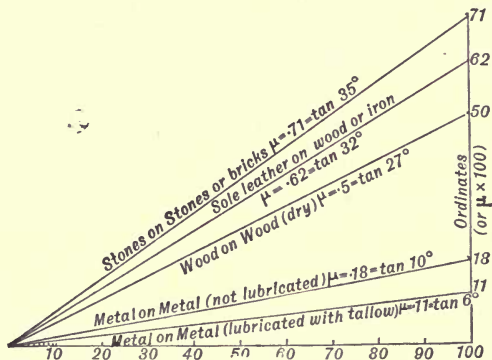


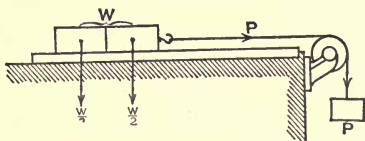
DIAGRAM OF ANGLES OF REPOSE.

**Limiting Angle of Resistance, or Sliding Angle.**—A third way of proving the first law of friction is to place the bodies so that the sliding surface is perfectly level. Then begin by pressing the upper body through the intervention of a compression spring-balance fitted with a sharp point, so that it will not slip off, and with a clinometer to indicate the angle through which it is tilted away from the perpendicular. Now gradually incline your pressure to the perpendicular, until you arrive at such an angle as will just cause the upper body to slide over the under one. This angle is termed the "*sliding angle*," or "*limiting angle of resistance*," because it is the limit, or maximum angle which the reaction from the surface can make with the perpendicular to the surfaces, for the reaction must act in the directly opposite direction to the pressing force.\* Again, apply the spring-balance, but with double the registered pressure, and you can just incline this force to the same angle as before. If, however, you press with ten times the former force, you would probably be able to act at a greater angle than before. It will be seen from this experiment that

*The Limiting Angle of Resistance = The Angle of Repose.*

\* Here the weight of the upper body is supposed to be negligible in comparison with the inclined pressure upon it.

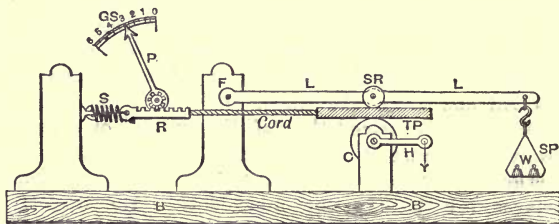
**Demonstration of the Second Law of Friction.**—**EXPERIMENT III.**—Take a block of planed yellow pine, and cut it into two equal pieces at right angles to the planed surface. Place one piece on the horizontal strip of yellow pine (used in previously demonstrating the first law), with the planed side next to it,



**PROOF OF SECOND LAW OF FRICTION.**

and put the other piece on the top of it, as shown by the second figure in this lecture. Now ascertain the horizontal pull,  $P$ , required to overcome friction. Then attach the top piece to the bottom one, as shown by the accompanying figure, so that the area of the surface in contact is doubled, and you will find that the same horizontal force,  $P$ , will cause it to move. If you take a long planed block and cut it into ten equal pieces, each of the same size as one of the above pieces, and try the experiment in a similar manner, you will be able to increase the area of contact tenfold, and you will then find that the

ratio  $\frac{P}{W}$  is not exactly the same with the surface of one block in contact with the strip, as when the surface of the whole ten came into action at once. The result of increasing the area in contact may also be tried by placing the blocks on the inclined plane, and observing the angle to which the plane is tilted when they begin to slide down the plane.



**APPARATUS FOR DEMONSTRATING THE FIRST AND THIRD LAWS OF FRICTION.**

**INDEX TO PARTS.**

L represents Lever.  
F " Fulcrum.  
SR " Small roller.  
SP " Scale-pan.  
TP " Test-piece.  
C " Cylinder.

H represents Handle.  
R " Rack.  
S " Spiral spring.  
P " Pointer.  
GS " Graduated scale.  
B " Base of apparatus.

**Demonstration of the Third Law of Friction.**—The preceding figure represents the apparatus belonging to the Applied Mechanics Department of the Royal College of Science, South Kensington (as described by Prof. Goodeve in his “Manual of Applied Mechanics”), for demonstrating the first and third laws of friction.

If the weights, *W*, be removed from the scale-pan *SP*, then there will be but a slight pressure between the lower surface of the test-piece *TP*, and the roller cylinder *C*. Consequently, on turning the handle *H* in the direction of the arrow, there will be a slight pull on the cord, causing the pointer *P* to move a degree or two over the graduated scale *GS*. The pointer should therefore be set back to zero.

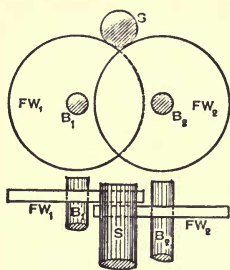
**EXPERIMENT IV.**—Put a weight, *W*, of say 5 lbs., into the scale-pan, and turn the cylinder slowly by the handle as before. The pointer deflects so many degrees. Increase the weight *W* to 10 lbs., and the pointer instantly indicates twice the amount of friction; put in 15 lbs., and it shows treble the friction; thus demonstrating the first law. Then turn the handle faster and faster, and the pointer remains fairly stationary, thus proving within certain limits that friction is independent of the velocity.\*

**Lubrication.**—Lubricants, such as tallow, grease, soft soap, and many kinds of oils, are used to reduce friction. Both skill and knowledge are required to decide upon the best kind of lubricant and the proper amount for different cases. Lubrication and lubricants should receive greater attention from the engineer, for the satisfactory working and length of life of most machines depend so largely upon effective lubrication. Where very heavy pressures and high speeds are experienced as in some cases of electrical machinery, it pays to use the very best kind of oil, and to distribute it to all the bearings from one common centre under pressure by means of a force-pump. It thereby flows in a continuous stream through the bearings to a filtering tank, from which it is again and again pumped on its soothing mission for months on end, without change or great loss in quantity. This is a very different state of matters from the “travelling oil-can” system, where the amount applied may vary, and the times of application may be erratic, according to the opinion of the attendant.

**Anti-Friction Wheels.**—In the case of delicate machinery, such as in Atwood’s machine for ascertaining by experiment the acceleration of gravity, and in Lord Kelvin’s mouse-mill for driving the paper rollers of his Syphon Recorder, when receiving

\* See Molesworth’s Pocketbook of Engineering Formulæ, and the Transactions of the Institution of Mechanical Engineers, for results of friction experiments with shafts run at different speeds.

telegraphic signals from long submarine cables, anti-friction wheels are used for the purpose of reducing the friction to a minimum. The accompanying figures illustrate one pair of anti-friction wheels.



ANTI-FRICTION  
WHEELS.

The spindle  $S$ , which carries the driving-wheel, instead of resting on two ordinary bearings, is supported by two wheels at *each* end, so that a rolling contact is produced between it and the wheels. This form of contact implies far less friction to begin with, than a sliding or scraping contact. Besides, the small amount of force required to overcome the friction between the spindle and the rims of these wheels, has a great advantage or leverage given to it, in as far as, it acts with an arm equal to the radius of the wheels  $FW_1$  and  $FW_2$ . This enables it to turn them with great ease at a slow rate in the very small bearings  $B_1$  and  $B_2$ .

In merely overcoming friction at a bearing, there is a considerable advantage in using large pulleys; for, the force necessary at the periphery of the pulley to overcome the friction at the bearing, is inversely proportional to the radius or diameter of the pulley. (See Lecture XI. fig. 1).

**Ball Bearings.**—Another example of the effect of rolling contact reducing friction is found in the use of ball bearings, which are now so common in all kinds of cycles and in high-class foot-driven lathes.\*

When it is necessary to move heavy beams, guns, &c., a common practice is to place them on rollers or on two channel iron girders  $\square$  with round cannon-shot between them, when a comparatively small force, properly applied, will have the desired effect.

We will have to return to this subject in the Advanced Course when dealing with the friction between shafts and their bearings, and the various means that have been adopted for minimising the same. In the meantime, we will complete this Lecture with an example of work done on an incline when friction is included.

**Work done on Inclines, including Friction.**—The method of calculating the work expended in moving a body along a *smooth* inclined plane was fully dealt with in Lecture IX.; consequently, the student is prepared, after what has been said about friction in

\* Refer to Lecture XVI., p. 183.

this Lecture, to consider the case of pulling a body up or down a plane when the co-efficient of friction between the body and the plane is known.

The *total work* expended is evidently divisible into two distinct portions—

(1) The work done *with* or *against* the action of gravity, according as the body is moved down or up the inclined plane =  $W \times h$  (where  $h$  is the height of the plane).

(2) The work done *against* friction =  $F \times l$  (where  $l$  is the length of plane passed over).

The work to be done against friction is the same whether the body is urged up or down the incline; for it is equal to the co-efficient of friction  $\times$  the reaction of the plane  $\times$  the distance through which it is moved.

Or,  $F \times l = \mu \times R \times l$

But by Lecture IX.  $R \times l = W \times b$ ;  $\therefore F \times l = \mu \times W \times b$

Or, the work done against friction in moving a body along the inclined distance  $l$ , is equal to the work done in moving the same body along a horizontal distance  $b$ , equal to the base of the incline.

If the work to be done in overcoming friction, is *equal* to the work capable of being done on the body by *gravity*, the body will be in equilibrium, and the inclination of the plane is equal to the angle of repose.

If the work to be done in overcoming friction is *less than* the work which gravity can do on the body, the body will slide down the incline, or, in technical language, the machine will overhaul.

EXAMPLE II.—What is the co-efficient of friction, and how is it ascertained? There is an inclined plane of 1 foot vertical to 10 feet horizontal; what work is done in moving 700 lbs. 5 feet along the plane, the co-efficient of friction being .08? (S. and A. Exam. 1892.)

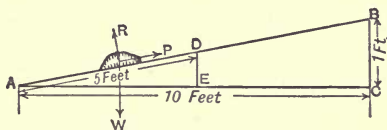


FIGURE FOR EXAMPLE II.

ANSWER.—The co-efficient of friction for two bodies in contact is the *passive resistance* (opposing the motion of the one over the other) divided by the reaction or normal pressure between the surfaces in contact—

$$\text{i.e.,} \quad \text{Co-efficient of friction} = \frac{\text{Friction}}{\text{Reaction}} = \frac{F}{R} = \mu$$

For methods of ascertaining co-efficients of friction, see the text in this Lecture.

*Total work done = work done against gravity + work done against friction.*

Referring to the accompanying figure, we see that—

$$(1) \text{ Work done against gravity} = W \times DE$$

$$(2) \text{ Work done against friction} = F \times AD$$

$$\therefore \text{ Total work done} = \underline{\underline{W \times DE + F \times AD}}$$

We have therefore only to substitute the numerical values corresponding to these letters in order to arrive at the result. From the question  $W = 700$  lbs. From the figure we see that  $DE$  is parallel to  $BC$ ; consequently by Euclid the  $\triangle^s$ ,  $ADE$ , and  $ABC$  are similar in every respect; and therefore

$$DE : BC :: AD : AB ; \text{ or, } DE = \frac{BC \times AD}{AB}$$

$$\text{But, also by Euclid, } AB = \sqrt{AC^2 + BC^2} = \sqrt{10^2 + 1^2} = 10.05 \text{ ft. (nearly)}$$

$$\text{Consequently, } DE = \frac{BC \times AD}{AB} = \frac{1 \times 5}{10.05} = .497 \text{ ft.}$$

$$\text{And, } F = \mu R$$

From the question we are told that  $\mu = .08$ , and we learn from Lecture IX. that

$$R : W :: AC : AB ; \text{ or, } R = \frac{W \times AC}{AB} = \frac{700 \times 10}{10.05} = 696.5 \text{ lbs.}$$

$$\therefore F = \mu R = .08 \times 696.5 = 55.72 \text{ lbs.}$$

$$\text{Hence Total Work} = W \times DE + F \times AD$$

$$\begin{aligned} \text{" " "} &= 700 \times .497' + 55.72 \times 5' \\ \text{" " "} &= 347.9 \text{ ft.-lbs.} + 278.6 \text{ ft.-lbs.} \\ \text{" " "} &= 626.5 \text{ ft.-lbs.} \end{aligned}$$

**NOTE.**—For the work done against friction quite a simple way would have been to have taken the formula deduced on the previous page—

$$\text{viz. : } F \times l = \mu \times W \times b = \mu \times W \times AE = .08 \times 700 \times 4.97 = 278.6 \text{ ft.-lbs.}$$

$$\text{For, } \frac{AE}{AC} = \frac{DE}{BC} \therefore AE = \frac{AC \times DE}{BC} = \frac{10 \times .497}{1} = 4.97.$$

**APPROXIMATE ANSWER.**—Since the inclination of the plane is so very small in this case, we might have assumed that

$$R = W ; AB = AC, \text{ and } DE = \frac{1}{2} BC$$

Then,

$$(1) \text{ Work done against gravity} = W \times DE = 700 \times \frac{1}{2} = 350 \text{ ft.-lbs.}$$

$$(2) \text{ Work done against friction} = F \times AD = .08 \times 700 \times 5 = 280 \text{ ft.-lbs.}$$

$$\therefore \text{ Total work} = W \times DE + F \times AD = 350 + 280 = \underline{\underline{630 \text{ ft.-lbs.}}}$$

## LECTURE X.—QUESTIONS.

1. What is friction, and how does it act? What is developed when force overcomes friction? How do you measure the result?

2. Explain by sketches and concise description how the laws of friction may be tested experimentally. What is meant by the "co-efficient of friction," "angle of repose," "angle of friction," and "sliding angle" or "limiting angle of resistance"?

3. How is the co-efficient of friction between two surfaces ascertained approximately by experiment? When two rough surfaces are pressed together, how much may the line of pressure be inclined to the common perpendicular to the surfaces in contact before motion ensues?

4. What is the co-efficient of friction when the angle of repose is—(a)  $5^{\circ} 42'$ ; (b)  $11^{\circ} 18'$ ; (c)  $16^{\circ} 42'$ ; (d)  $21^{\circ} 48'$ ; (e)  $26^{\circ} 36'$ ; (f)  $30^{\circ}$ ; (g)  $45^{\circ}$ ? Draw the angles to scale. *Ans.* (a) .1; (b) .2; (c) .3; (d) .4; (e) .5; (f) .5774; (g) 1.

5. An inclined plane is 100 feet long and 20 feet high. A body weighing 100 lbs. is pulled up from the bottom to the top, and then down again. If the co-efficient of friction between the body and the plane is .5, what work was expended in each case? What would require to be the co-efficient of friction in order that the body might just slide down of its own accord? *Ans.* 6,900 ft.-lbs.; 2,900 ft.-lbs.;  $\mu = \frac{h}{b} = \frac{\sqrt{6'}}{12} = .204$ .

6. What is the co-efficient of friction, and how is it ascertained? There is an inclined plane of 1 foot vertical to 5 feet horizontal; what work is done in moving 100 lbs. through 100 feet along the plane, the co-efficient of friction being .1? *Ans.* 2940 ft.-lbs.

7. An incline is 80 feet long, with a rise of 20 feet. A body weighing 100 lbs. is drawn 40 feet along the incline; what work is expended if the co-efficient of friction is .6? *Ans.* 3,323 ft.-lbs.

8. A weight of 5 cwts. resting on a horizontal plane requires a horizontal force of 100 lbs. to move it against friction. What is the co-efficient of friction? *Ans.* .18.

9. A plank of oak lies on a floor with a rope attached to it. When the rope is pulled horizontally with a force of 70 lbs. it just moves, but when pulled at an angle of  $30^{\circ}$  to the floor a force of 60 lbs. moves it. What is the weight of the plank and the co-efficient of friction between it and the floor? *Ans.* 116.6 lbs.; .6.

10. Suppose a locomotive weighs 30 tons, and that the share of this weight borne by the driving wheel is 10 tons. Then, if the co-efficient of friction between the wheels and the rails be .2, what load will the engine draw on the level if the required co-efficient of traction be 10 lbs. per ton of train load? What load will this engine draw at the same rate up an incline of 1 in 20? *Ans.* 448 tons (including engine); 36.72 tons (including engine).

11. State the laws of friction, and explain the contrivance known as *friction wheels*. What is the advantage of ball bearings for bicycles? Sketch in section such a bearing.

12. What are lubricants, and for what purposes are they used in machinery? What kind of lubricant would you use for the moving parts of a very high-speed engine and direct-driven dynamo, and how would you apply it so as to be able to use it over and over again?

13. What is friction? What is meant by limiting friction, by sliding friction, and by the co-efficient of friction? A weight of 5 cwts. resting on a horizontal plane, requires a horizontal force of 108 lbs. to move it against friction. What in that case is the value of the co-efficient of friction? *Ans.* .192.

14. How would you experimentally determine the nature of the friction between clean, smooth surfaces, say of oak, and what sort of law would you expect to find?

15. Describe any experiment which you have made or seen for finding the laws of solid friction. What are the laws so found? Are they quite true? How do they differ from the laws of fluid friction?

16. Sketch and describe an apparatus for determining the co-efficient of sliding friction between two planed surfaces of oak.

If you have made this or a similar experiment describe the behaviour of the sliding piece, and any troubles you may have had. State how you would conduct the experiment so as to establish the principal facts concerning such friction.

(B. of E., 1902.)

17. The tractive resistance of a train weighing 335 tons is 11 lbs. per ton. If the effective horse-power of the locomotive is 600, estimate the uniform speed obtainable when ascending an incline of 1 in 200. *Ans.* 2662 ft. per min. or 30 miles per hour.

(C. & G., 1904, O., Sec. A.)



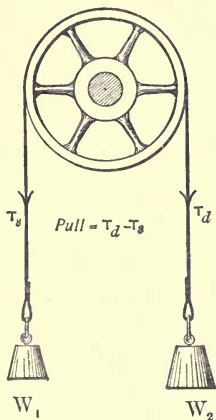
## LECTURE XI.

CONTENTS.—Difference of Tension in the Leading and Following Parts of a Driving Belt—Brake Horse-Power transmitted by Belts—Examples I. II.—Velocity Ratios in Belt Gearing—Examples III. IV.—Open and Crossed Belts—Fast and Loose Pulleys—Belt Gearing Reversing Motions—Stepped Speed Cones with Starting and Stopping Gear—Driving and Following Pulleys in Different Planes—Shape of Pulley Face—Questions.

WE shall devote this Lecture to the transmission of power by belting and to belt-gearing.

**Difference of Tension in the Leading and Following Parts of a Driving-Belt.**—In Lecture VI., when discussing the case of the simple pulley, we assumed that the belt or rope passing over the pulley was perfectly flexible, and that there was no friction

at the axle of the pulley. Consequently, we found that equal weights would balance each other, or that the tension of the two sides of the belt were equal. A little consideration of the subject will show that when one pulley is driven from another one by an endless belt or rope, the tension on the driving side must be greater than that on the following side.



DIFFERENCE OF TENSION  
DUE TO FRICTION.

1. Take the case of an ordinary vertical pulley with its axle or shaft resting in two bearings (one on each side of the pulley), with a belt or rope passed over it, and with weights attached to the free ends of the same. Here we must have a certain amount of friction between the axle and its bearings, which can only be overcome by a force applied to the circumference of the pulley.

Let .  $F_1 = \left\{ \begin{array}{l} \text{Force required to overcome friction at the circum-} \\ \text{ference of the axle or shaft.} \end{array} \right.$

Let . .  $r_1$  = Radius of the axle.

„ . .  $F_2$  = { Force required to overcome the friction of the axle  
when acting at the circumference of the pulley.

„ . .  $r_2$  = Radius of the pulley to centre of belt.

Then,  $F_1 \times r_1 = F_2 \times r_2$ .

Let . .  $W_1$  = { Weight attached to the left-hand side of the belt,  
and which therefore produces a tension on the  
slack side= $T_s$ .

„ . .  $W_2$  = { Least weight on the right-hand side of the belt  
that will produce motion, and which therefore  
produces a tension on the driving side= $T_d$ .

Then taking moments about the centre of the axle, we have—

$$W_1 \times r_2 + F_2 \times r_2 = W_2 \times r_2$$

$$\text{Or,} \quad T_s \times r_2 + F_2 \times r_2 = T_d \times r_2$$

Dividing both sides of the equation by  $r_2$  we get

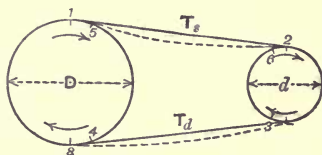
$$T_s + F_2 = T_d$$

$$\therefore F_2 = T_d - T_s$$

Or, expressed in words, the force  $F_2$ , acting at the circumference of the pulley (which is required to overcome the friction of the axle) is equal to the tension  $T_d$  on the driving or forward side of the pulley, *minus* the tension  $T_s$  on the slack or following side.

In order that the periphery of the pulley may move at the same rate as the under face of the belt, we must have sufficient tension on each part of the latter, and the co-efficient of friction between them must not have less than a certain value. Too great adhesion between them would result in a loss of work, for in that case an extra force would have to be applied solely for the purpose of pulling the belt from the pulley.

2. Take the case of one vertical pulley of diameter  $D$ , driving another vertical pulley of diameter  $d$  by means of an endless belt, rope, or chain in the direction of the arrows shown on the accompanying figure. Whenever the pulley  $D$  is moved, the tension on the driving side  $T_d$  tends to stretch the belt on that side, and this tension increases until the pulley  $d$  begins to move; whereas the tension on the following or slack side,  $T_s$ , is gradually diminished until the difference of the tensions ( $T_d - T_s$ ) produces a uniform velocity of the belt. Of course the tension on the slack side must be sufficient to pre-



BELT DRIVING.

vent the slipping of the belt on either of the pulleys if the periphery of the driven pulley is to keep pace with the periphery of the driving one. In order that there may be a minimum chance of the belt slipping, its *slack side* should always run *from the top side of the driving pulley*. By so arranging the drive, the sag of the belt on the slack side will cause it to encompass a greater length of the circumferences of both pulleys. The motion of the belt will be easier, and the wear and tear of the bearings will be less, because there will be less total stress ( $T_d + T_s$ ) tending to draw the pulleys together for the transmission of a certain horse-power, than if the slack side left the under side of the driving-pulley. Referring to the previous figure, if the slack side leaves the top side of the pulley D, it grips the same from position 4, round the back of the pulley to 5, and the pulley *d* from 6 round to 3; whereas, if D were rotated in the opposite direction, we should have the slack side entering on it at 1, and only gripping it as far as position 8; entering on *d* at 7, and only gripping it to position 2, thus having far less grip on the pulleys and thereby encouraging the natural tendency of the belt to slip on the pulleys.\*

#### Brake Horse-power transmitted by Belts.

Let . . .  $V$  = Velocity of belt in feet per minute.

„ . . .  $P = (T_d - T_s)$  the net pull causing motion in lbs.

$$\text{Then, B.H.P.} = \frac{VP}{33,000} = \frac{V(T_d - T_s)}{33,000}.$$

Let . . .  $D$  = Diameter of driving pulley in feet =  $2r$ .

Then . . .  $\pi D$  = Circumference of driving pulley in feet =  $2\pi r$ .

Let . . .  $n$  = Number of revolutions of pulley per minute.

Then . . .  $V = \pi Dn = 2\pi rn$  = velocity of belt (with no slip).

$$\text{And, the . . . B.H.P.} = \frac{\pi DnP}{33,000} = \frac{2\pi rnP}{33,000}$$

**EXAMPLE I.**—A pulley 6' in diameter is driven at 100 revolutions per minute and transmits motion to another pulley by means of a belt without slip. If the tension on the driving side of the belt is 120 lbs. and on the slack side 20 lbs., what is the brake horse-power being transmitted?

\* The previous figure should have been drawn with the full and dotted lines at  $T_s$ , reversed, but the student will easily follow the explanation.

ANSWER.—Here  $r = 3'$ ;  $n = 100$ ;  $P = (T_d - T_s) = (120 - 20) = 100$  lbs.

$$\therefore \text{B.H.P.} = \frac{2\pi r n P}{33,000} = \frac{2 \times \frac{22}{7} \times 3 \times 100 \times 100}{33,000} = 5.71$$

EXAMPLE II.—What must be the number of revolutions per minute of a driving pulley 6' in diameter, in order that it may transmit 5.71 B.H.P. by a belt to another pulley, if the net pull on the belt is 100 lbs.?

ANSWER.—Here we have the same data to go upon as in Example I., except that we are given the B.H.P. instead of the revolutions per minute. Then, transposing every quantity except  $n$  (the revolutions per minute) to one side of the above equation, we have

$$n = \frac{(\text{B.H.P.}) \times 33,000}{2\pi r P} = \frac{5.71 \times 33,000}{2 \times \frac{22}{7} \times 3 \times 100} = 100 \text{ r.p.m.}$$

In precisely the same way, if you were given the power to be transmitted, the revolutions per minute, the difference of tension on the two sides of the belt, and you were asked for the diameter of the pulley, the formula would appear thus—

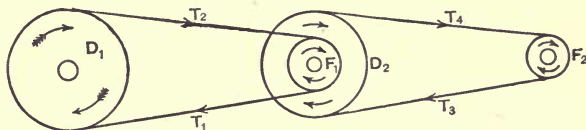
$$D = \frac{(\text{B.H.P.}) \times 33,000}{\pi n P}$$

If it was the difference of tension in the belt that was asked for, then—

$$(T_d - T_s) = P = \frac{(\text{B.H.P.}) \times 33,000}{\pi D n}$$

You would (after arranging the formula in this way, so as to keep the unknown quantity on one side of the equation) simply have to substitute the numerical values corresponding to the different symbols, and then cancel out the figures in numerator and denominator, in order to reduce the long multiplication and division to a minimum, and thereby arrive at the result as quickly as possible.

**Velocity Ratios in Belt Gearing.**—Let two or more pulleys be connected by belting in the manner shown by the accompanying figure. Then, if there is no slipping of the belts, the circum-



VELOCITY RATIOS IN BELT GEARING.

ferential speeds of the pulleys will be the same as the velocity of the belts passing round them.

Let  $D_1, D_2$  = Diameters of the drivers.

„  $F_1, F_2$  = Diameters of the followers.

„  $N_{D_1}, N_{D_2}$  = Number of revolutions per minute of the drivers.

„  $N_{F_1}, N_{F_2}$  = Number of revolutions per minute of followers.

Then, taking the first pair of pulleys,  $D_1$  and  $F_1$  we have—

*Circumferential speed of driver 1 = Circumferential speed of follower 1.*

$$\text{i.e.} \quad \pi D_1 N_{D_1} = \pi F_1 N_{F_1}$$

(Divide both sides by  $\pi$ )

$$D_1 N_{D_1} = F_1 N_{F_1} \quad \therefore N_{D_1} = \frac{F_1 N_{F_1}}{D_1}$$

Or, *The product of the diameter of the driver and its number of revolutions per minute.* } = *The product of the diameter of the follower and its number of revolutions per minute.*

$$\text{Or,} \quad \frac{D_1}{F_1} = \frac{N_{F_1}}{N_{D_1}} \quad (1)$$

*i.e., The ratio of the diameters of the pulleys is in the inverse ratio of their speeds or revolutions per minute.*

Treating the motion of the second set of pulleys in *exactly* the same way, we have—

Circumferential speed, of  $D_2$  = circumferential speed, of  $F_2$

$$\therefore \pi D_2 N_{D_2} = \pi F_2 N_{F_2}$$

(Divide both sides by  $\pi$ )

$$D_2 N_{D_2} = F_2 N_{F_2}$$

$$\text{Or,} \quad \frac{D_2}{F_2} = \frac{N_{F_2}}{N_{D_2}} \quad (2)$$

(But the revolutions of  $F_1$  and of  $D_2$  are the same)  $\therefore N_{F_1} = N_{D_2}$

$$\text{Or,} \quad D_2 N_{F_1} = F_2 N_{F_2} \quad \therefore N_{F_2} = \frac{D_2 N_{F_1}}{F_2}$$

$$\text{Consequently} \quad \frac{N_{D_1}}{N_{F_2}} = \frac{\frac{F_1 N_{F_1}}{D_1}}{\frac{D_2 N_{F_1}}{F_2}}$$

$$\left\{ \begin{array}{l} \text{(Dividing both numerator} \\ \text{and denominator by } N_{F_1}) \end{array} \right\} \frac{N_{D_1}}{N_{F_2}} = \frac{\frac{F_1}{D_1}}{\frac{D_2}{F_2}} = \frac{F_1 \times F_2}{D_1 \times D_2}$$

Or, we might have arrived at the same result by multiplying equations (1) and (2) together. Thus—

$$\frac{D_1}{F_1} \times \frac{D_2}{F_2} = \frac{N_{F_1}}{N_{D_2}} \times \frac{N_{F_2}}{N_{D_1}}; \text{ but } N_{F_1} = N_{D_2} \therefore N_{F_2} = N_{D_1} \left( \frac{D_1 \times D_2}{F_1 \times F_2} \right)$$

$$\text{Or, } \frac{\text{Speed of first driver}}{\text{Speed of last follower}} = \frac{\text{Product of diameters of followers}}{\text{Product of diameters of drivers}}$$

$$\text{Or, } N_{D_1} \times D_1 \times D_2 = N_{F_1} \times F_1 \times F_2$$

$$\text{i.e., } \left. \begin{array}{l} \text{Speed of first driver} \times \text{dia-} \\ \text{meters of the drivers} \end{array} \right\} = \left\{ \begin{array}{l} \text{Speed of last follower} \times \text{diale-} \\ \text{meters of the followers.} \end{array} \right.$$

In the same way we may treat any number of drivers and followers by this general formula—viz.,

$$\left. \begin{array}{l} \text{Speed or number of revolutions} \\ \text{per minute of the first driver} \\ \times \text{the successive diameters of} \\ \text{the drivers} \end{array} \right\} = \left\{ \begin{array}{l} \text{Speed of the last follower} \times \text{the} \\ \text{successive diameters of the} \\ \text{followers.} \end{array} \right.$$

Precisely the same rule holds good for discs driven by contact friction and for wheel gearing, as you will find from the next lecture; but in friction gearing and wheel gearing the driver and the follower move in different directions, whereas in belt gearing they move in the same or in the opposite direction, according as the driving belts are “open” or “crossed.”

**EXAMPLE III.**—Referring to the previous figure, suppose that a driving pulley,  $D_1$ , is connected by a belt to a follower,  $F_1$ , whilst it moves at 100 revolutions per minute. If the diameter of the driver is 6' and of the follower 3', what will be the number of revolutions *per minute* of the follower?

By the previous formula for two pulleys,

$$D_1 \times N_{D_1} = F_1 \times N_{F_1}$$

$$\therefore N_{F_1} = \frac{D_1 \times N_{D_1}}{F_1} = \frac{6' \times 100}{3'} = 200 \text{ r.p.m.}$$

**EXAMPLE IV.**—Referring to the previous figure, suppose that a driving pulley  $D_1$  (4' diameter), is geared to a follower,  $F_1$  (2' in diameter), and that a second driver  $D_2$  (4' diameter), fixed to the same shaft as  $F_1$ , is geared to a second follower  $F_2$  (1' diameter). If  $D_1$  makes 60 revolutions per minute, what is the speed of  $F_2$ ?

By the previous formula for four pulleys,

$$N_{D_1} \times D_1 \times D_2 = N_{F_1} \times F_1 \times F_2$$

$$\therefore N_{F_2} = \frac{N_{D_1} \times D_1 \times D_2}{F_1 \times F_2}$$

$$N_{F_2} = \frac{60 \times 4' \times 4'}{2' \times 1'} = 480 \text{ r.p.m.}$$

**Open and Crossed Belts.**—By referring to the next figure, the student will observe that the left-hand end view shows what

is termed an open belt, OB, and that the right hand end view shows a crossed belt, CB. In the case of open belts, the driver and the follower rotate in the *same* direction (as may be seen from the second and third figures in this Lecture); whereas, with crossed belt driving, the follower revolves in the *opposite* direction to that of the driver, just as it does when direct friction or wheel-gearing is used.

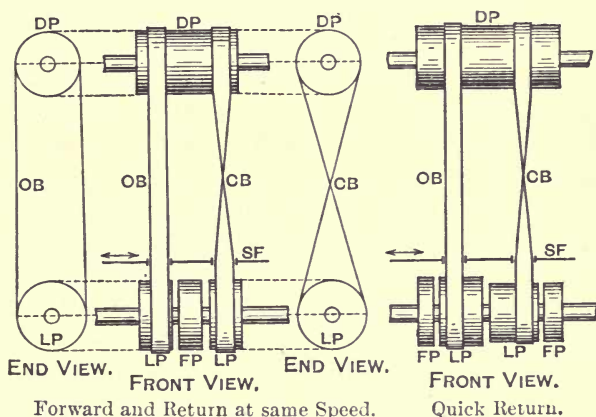
**Fast and Loose Pulleys.**—As will be seen from the two front views in the next set of illustrations, the open and the crossed belts are shown passing from the broad driving-pulleys DP, to the broad loose pulleys LP. Loose pulleys are generally bushed with gun-metal, and then bored out so as to fit their shafts easily. This permits them to rotate without turning the shaft upon which they bear. The pulleys, FP, are keyed hard on to the shafts, so that when the belt is forced over upon them by means of the shifting forks, SF, the machines connected with the same are set agoing. This simple combination of fast and loose pulleys therefore enables a machine to be stopped or started at pleasure, without interfering with the motion of the driving pulley and the belt. In ordinary cases where there is only one driving belt required, the loose pulley is of the same breadth as the fixed pulley.\*

**Belt-Gearing Reversing Motions.**—In many kinds of machine tools it is desirable to be able to drive the tool first in one direction and then in the opposite direction, as well as to start or stop it. This is frequently effected by a combination of open and crossed belts with fast and loose pulleys, as illustrated by the accompanying figure.

From what has just been said about open and crossed belts, as well as fast and loose pulleys, the student will have no difficulty in understanding this arrangement of reversing gear. If applied to a machine for planing metals, the shaft which is keyed to the fixed pulley FP would be connected either through wheel gearing and a rack, or through a central screw, to the travelling table of machine upon which the job to be acted upon is secured. Whenever the table had been moved backwards to the end of the required stroke by the crossed belt, the shifting fork SF would be pushed forward by an outstanding arm or kicker attached to

\* See the set of figures *after* the next, where B<sub>1</sub> is the driving belt engaging the fixed pulley, FP; and where LP is the loose pulley, to which the belt may be shifted by means of the shifting-fork, SF, whenever it is desirable to stop the speed cones, SC<sub>1</sub>, SC<sub>2</sub>, and the machine to which they are connected. In the first front view of the *next* set of figures, the driving pulley, DP, and both of the loose pulleys, LP, are drawn too narrow. They should have been represented half as wide again, in order to prevent the belts slipping over the outside edge, when the other belt is shifted on to the fixed pulley situated between them.

the side of the table at such a position as would cause the crossed belt CB to be shifted from the central fixed pulley to its loose one, and at the same time bring over the open belt from its loose pulley to the central fixed one. Whenever the planing tool had finished its cut on the metal, the shifting fork would be pulled backward by another similar outstanding arm or kicker (also attached to the travelling table of the planing machine, at a position just beyond the end of the required stroke for the particular job under operation), thereby shifting the open belt OB from FP, to its loose pulley, LP, and at the same time pulling over the crossed belt, CB,



## BELT GEARING REVERSING MOTIONS.

## INDEX TO PARTS.

DP represents Driving pulleys.  
 FP   "   Fixed pulleys.  
 LP   "   Loose pulleys.

OB represents Open belts.  
 CB   "   Crossed belts.  
 SF   "   Shifting forks.

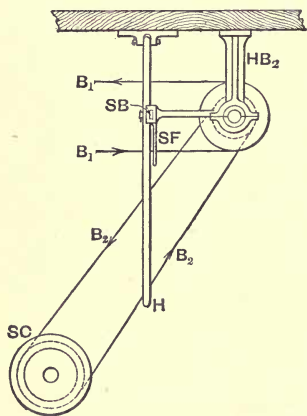
from its loose pulley to the central fixed pulley, thus causing the table to make the return stroke.

The left-hand front view, with its accompanying end views, show the necessary arrangements when the forward and backward velocities of the table are equal. The right-hand front view illustrates the case wherein the backward or non-planing motion is intentionally made quicker than the forward or cutting stroke, so as to save time, by having the back motion fixed pulley, FP, and its corresponding loose one, LP, made smaller than the forward set. The end views for this latter case would be similar to the former one, with the exception that the crossed belt would engage a smaller

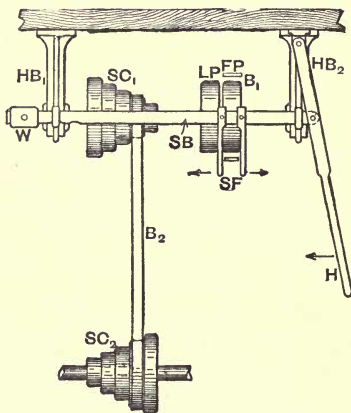
pulley of the same size as shown by the front view. This latter arrangement can evidently be employed to obtain a fast or a slow motion *in the same direction*, by simply having both belts open or both crossed.

### Stepped Speed Cones with Starting and Stopping Gear.

—In many machines, such as lathes, planers and other machine tools, it is very desirable not only to be able to start and stop them, but also to alter their speed so as to suit different classes of



END VIEW.



SIDE VIEW.

### STEPPED SPEED CONES WITH STARTING AND STOPPING GEAR.

#### INDEX TO PARTS.

HB <sub>1</sub> , HB <sub>2</sub> represent	Hanging brackets	B <sub>1</sub> , B <sub>2</sub> represent	Belts.
	for supporting	H	Handle.
	shaft, &c.	SB	Sliding bar.
SC <sub>1</sub> , SC <sub>2</sub> "	Speed cones.	SF	Shifting fork.
FP "	Fast pulley.	W	Weight to fix SB in
LP "	Loose pulley.		positions ← →.

work, without affecting the motion of the prime motor or that of the shop driving-shaft. These objects are generally attained by a combination of fast and loose pulleys with what are termed "stepped speed cones." The accompanying side and end views illustrate the arrangement as usually carried out in engineering works. When the starting-handle, H, is turned to the right hand, it pulls over the sliding-bar, SB, with its shifting-fork, SF, which moves the belt, B<sub>1</sub>, from the loose pulley, LP, to the fixed one, FP; thus setting the speed cones, SC<sub>1</sub>, SC<sub>2</sub>, and thereby the

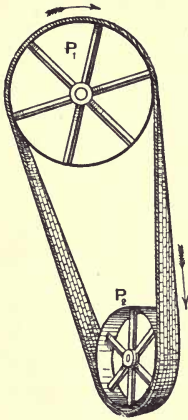
machine in motion. When the handle is turned to the left, it pushes the sliding-bar and shifting-fork also in that direction, thus moving the belt from the fixed to the loose pulley, which allow the cones and machine to come to rest. In each case the weight,  $W$ , causes a notch in the sliding-bar to engage with its left-hand supporting bracket, thereby preventing the shifting fork from pushing the driving belt too far, or off either pulley, and at the same time ensuring that it remains in the desired position. Both supporting brackets for the sliding-bar,  $SB$ , are merely right-angle extensions from the hanging brackets,  $HB_1$ ,  $HB_2$ , which carry the upper shaft with its cone and pulleys.

The upper and lower speed cones,  $SC_1$ ,  $SC_2$ , are generally made of the same size and shape, but they are always keyed to their respective shafts in opposite directions. Consequently, if it should be desirable to run the machine fast for light work, the belt,  $B$ , is shifted on to the largest pulley of the upper cone and the smallest one of the lower cone. If the machine is required to move slowly for heavy cuts, then the belt is placed on the smallest upper pulley and the largest lower one. Any desired intermediate speed between these extremes is obtained by adjusting the belt on one or other of the remaining sets of pulleys of the upper and lower cones.

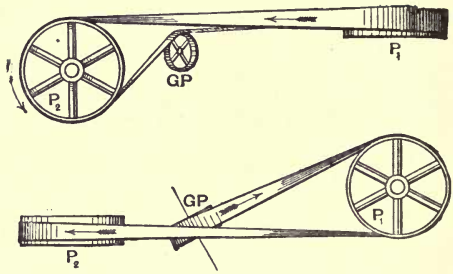
The student can easily prove to himself (by drawing down the arrangement to scale) that such stepped speed cones, if connected by a *crossed* belt on one pair of its pulleys, will produce the same tension in the belt with any other pair.\* With open belt-driving the tightness of the belt will not be the same when on one pair of the pulleys as when on another; but the difference is so small that it can generally be disregarded in practice without having recourse to tightening or slackening the same.

**Driving and Following Pulleys in Different Planes.**—It is often necessary to drive a follower placed in a different plane from the driver. The accompanying set of illustrations show very clearly how this is effected. The important precaution to be observed is, that the leading or on-going part of the belt *must* enter upon the follower in a *fair or direct line with its plane of rotation*. If this rule be attended to, then power may be transmitted between two non-parallel shafts, as shown by the first figure, even if their centre lines are in planes at right angles to each other—*i.e.*, when the belt is working with quarter-twist. When two shafts are in planes at right angles to each other, and

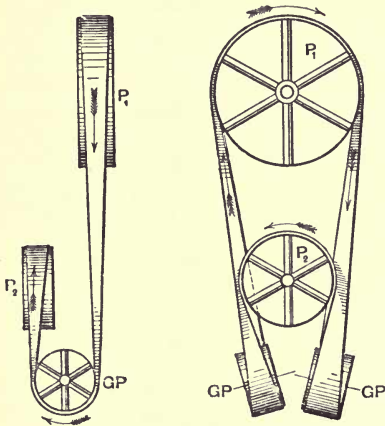
\* The algebraical proof of this will be considered in our "Advanced Book on Applied Mechanics." The student should refer to the general view and to the detail drawings of the stepped speed cones in the foot-driven screw-cutting lathe illustrated in Lecture XVI.



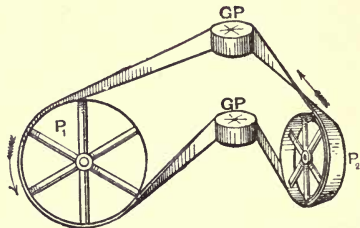
Tullis's Thick-sided Leather Chain Belt, Working Quarter-twist, and Transmitting Power between two shafts which are not parallel. No Guide Pulleys are required for this drive.



Flat Belt Working Quarter-twist and Transmitting Power between two right-angled shafts, with leading Guide Pulley (GP) to remove the twist from the Belt before it enters upon the Follower, and to give the belt more grip on the pulleys.



Flat Belt Transmitting Power between two parallel shafts not in the same plane by aid of guide pulleys (GP).



Flat Belt Transmitting Power over Guide Pulleys between two non-parallel shafts in the same plane.

it is found desirable to remove the twist from the belt before it enters upon the follower, then a guide-pulley, GP, must be used as shown by the second figure. When the shafts are parallel, but not in the same plane, then the power must be transmitted by aid of two guide-pulleys, as seen from an inspection of the third figure. Or, should the shafts not be parallel, but in the same plane, two guide-pulleys are necessary, as in the fourth figure. Guide-pulleys, if supported by spindles running in adjustable bearings or brackets, may be made serviceable as tightening-pulleys for the purpose of taking up the slack of the belt, and thus giving the necessary grip for transmitting more power with a steadier drive than can be obtained without them.

**Shape of Pulley Face.**—The student will have observed that the faces or rims of the fast and loose pulleys, as well as those of the stepped cones in the previous set of figures, are slightly curved. This convex curvature, or double coning, is purposely done in order to ensure that the belt may ride easily and fairly in the centre line of the pulley face without inclining to either side. A flat band, if placed on the smaller end of a revolving straight conical pulley, will naturally tend to rise to the larger end of the cone. Consequently, if each half of the face of a pulley is coned (or, which amounts to the same thing, if the rim of the pulley be curved so as to have its largest diameter in the middle of its face), each half of the breadth of the belt will have an equal tendency towards the middle of the pulley's rim. When very fast driving and sudden severe stresses are brought to bear upon a machine, as in the case of circular saws, morticing machines, and emery-wheel grinders, it is found necessary to fit the pulleys with side flanges, in addition to curving their rims, in order to prevent the belts from sliding off the pulley's face to one side or to the other.

N.B.—The student may be referred to the Author's Text-book on "Applied Mechanics and Mechanical Engineering," Chapters XVII. and XVIII., for further information on belt, rope, and chain gearing.

## LECTURE XI.—QUESTIONS.

1. In machinery, where one pulley drives another by means of an endless belt, there is a difference of tension in the two parts of the belt. Why is this? The pulley on an engine shaft is 5 feet in diameter, and it makes 100 revolutions per minute. The motion is transmitted from this pulley to the main shaft by a belt running on a pulley, and the difference in tension between the tight and slack sides of the belt is 115 lbs. What is the work done per minute in overcoming the resistance to motion of the main shaft?

*Ans.* 180,642 ft.-lbs.

2. Deduce from the "principle of work" a formula for the brake horse-power transmitted by a belt. The pull on the driving side of a belt is 200 lbs. and on the following side 100 lbs., whilst the belt has a velocity of 990 ft. per minute. Find the number of units of work performed in two minutes and the B.H.P. transmitted. *Ans.* 198,000 ft.-lbs., 3 B.H.P.

3. State and prove the rule for estimating the relative speeds of two pulleys connected by a belt. Also, the velocity ratio between the first driver and the last follower in belt gearing, where there are two or more drivers and a corresponding number of followers. [A main shaft carrying a pulley of 12 inches diameter and running at 60 revolutions per minute, communicates motion by a belt to a pulley of 12 inches diameter, fixed to a countershaft. A second pulley on the countershaft, of  $8\frac{1}{2}$  inches diameter, carries on the motion to a revolving spindle which is keyed to a pulley of  $4\frac{1}{4}$  inches diameter. Sketch the arrangement and find the number of revolutions per minute made by this last pulley. *Ans.* 123'5.

4. Two pulleys are connected by a driving belt, and the sum of their diameters is 30 inches; one pulley makes 2 revolutions while the other makes 3 revolutions; find their respective diameters. *Ans.* 18", 12".

5. An engine works normally at 106 revolutions per minute. At that speed it was found that it drove by belting a dynamo at 420 revolutions per minute, but to show off the electric lights at their normal candle power the dynamo had to be run at 460 revolutions per minute. At what speed was the engine being driven? *Ans.* 116 revolutions per minute.

6. A pulley of 3 feet radius rotates at 100 revolutions per minute and transmits motion to another pulley of 36 inches diameter. If there is 10 per cent. slip on the belt what will be the speed of the follower? What will be the net driving pull on the belt if 5 B.H.P. is transmitted by it? *Ans.* 180 revolutions per minute; 97'2 lbs.

7. Sketch an arrangement of pulleys and bands for obtaining a reversing motion from a shaft driven at a constant rate in one direction, and describe the action of the combination.

8. Sketch a combination of fast and loose pulleys as used for setting in motion, or stopping machinery. Explain the construction adopted for retaining a flat belt upon a pulley, pointing out where the fork is to be applied, and why.

9. Sketch and describe a good form of slow forward and quick return for a shaping machine.

10. Sketch and describe an arrangement for driving the table of a planing machine by means of a screw, so that the table may travel 50 per cent. faster in the return than in the forward or cutting stroke.

11. What is the object of using guide-pulleys in machinery? Mention instances of their use, and show how the directions of their axes are ascertained.

12. Describe, with a sketch, the mode of reversing the motion of the table in a planing machine, when a screw is employed to drive the table.

13. A rope transmits 20 horse-power to a rope pulley of 8 feet diameter; draw a section of the rope in its groove. If the pulley makes 100 revolutions per minute, what is the speed of the rope in feet per minute? What is the difference of the tensile forces in the rope on the two sides of the pulley? As it is the difference between the tensile forces in a belt or rope that is important for power, why is it necessary to have any pull on the slack side? *Ans.* 2513 ft. per min., and 262.5 lbs.

14. What are cone or speed pulleys? Describe the use of such pulleys in any machine with which you are acquainted. The spindle of a lathe can, by moving the belt on its cone pulleys, be driven at four hundred revolutions per minute when at its greatest and at 100 revolutions per minute when running at its lowest speed. If the revolutions of the driving shaft are kept constant throughout, and the largest diameter of the speed cones is 20", what must be the diameter of the smallest steps on the pulleys; the speed pulleys on the two shafts being of the same size? Sketch the pulleys in position. *Ans.* 10 inches.

15. Upon what does the limiting difference of tensions in the tight and slack sides of a moving belt depend? If the working stress in a belt of sectional area  $a$  square inches be  $f$  pounds per square inch, and the ratio of the tensions in the tight and slack sides be  $m$ , find the horse-power that can be safely transmitted when the speed of the belt is  $v$  feet per second. (C. & G., 1903, O., Sec. A.)

$$\text{Ans. H.P.} = \frac{f a v (1 - \frac{1}{m})}{550}$$

16. A belt transmits 60 H.P. to a pulley 16 inches in diameter running at 263 revolutions per minute. What is the difference of the tensions on the tight and slack sides? *Ans.*  $T_d - T_s = 1796$  lbs. (B. of E., 1904.)

17. State the condition that has to be satisfied in order that the same belt may work on two or more pairs of pulleys keyed to parallel shafts (1) when the belt is crossed, (2) when open. Explain also why a belt always climbs to the section of the pulley where the diameter is greatest. (C. & G., 1905, O., Sec. A.)

18. Explain any means with which you are acquainted for determining the brake horse-power of an engine provided with a fly-wheel.

A rope is wrapped once round a fly-wheel. One end of the rope carries a weight of 500 lbs., and the other end is led upwards, and is attached to a spring balance. When the revolutions are 105 per minute, the pull in the spring balance varied between 10 and 20 lbs. If the diameter of the wheel be 8 ft., and of the rope 1 in., find the average brake horse-power, *Ans.* 39.6 B.H.P. (C. & G., 1905, O., Sec. A.)

## LECTURE XII

CONTENTS.—Velocity Ratio of Two Friction Circular Discs—Pitch Surfaces and Pitch Circles—Pitch of Teeth in Wheel Gearing—Rack and Pinion Velocity Ratio in Wheel Gearing—Example I.—Principle of Work applied to Wheel Gearing—Examples II. III.—Questions.

**Velocity Ratio of Two Circular Friction Discs.**—If two truly centred circular discs or cylindrical rollers, having their shafts parallel to each other and free to turn in fixed bearings, be brought into firm contact; then, if one of them be driven round, and if there be no slipping, the other one will rotate in the *opposite* direction with the same circumferential speed or surface velocity (see the next figure).

Consequently, *their velocity ratio will be in the inverse ratio to their diameters.*

This may be proved in exactly the same way as we found the velocity ratio of two pulleys driven by a belt in Lecture XI.

Let  $D_1$  = Diameter of the driving disc.

„  $F_1$  = Diameter of the following disc.

„  $N_{D_1}$  = Number of revolutions per minute of  $D_1$ .

„  $N_{F_1}$  = Number of revolutions per minute of  $F_1$ .

Then, *The peripheral velocity of  $D_1$  = Peripheral velocity of  $F_1$*   
*i.e.*     .     .     .     .      $\pi D_1 N_{D_1} = \pi F_1 N_{F_1}$

Or,     .     .     .     .      $D_1 N_{D_1} = F_1 N_{F_1}$

*i.e. The Driver's diameter  $\times$  its speed = Follower's diam.<sup>r</sup>  $\times$  its speed.*

$$\therefore \frac{N_{D_1}}{N_{F_1}} = \frac{F_1}{D_1}$$

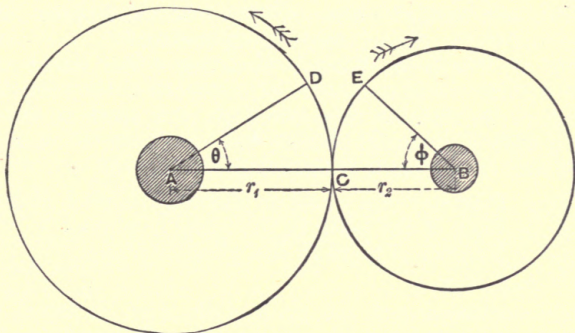
$$\text{i.e.} \quad \frac{\text{Speed of the Driver}}{\text{Speed of the Follower}} = \frac{\text{Diameter of Follower}}{\text{Diameter of Driver}}$$

This velocity ratio may also be proved in the following way :—

Let the two circles centred at A and B represent a cross section of the two friction discs in contact at C; and let them move by rolling contact through the angles  $\theta$  and  $\phi$  respectively in the same time.

Since the magnitude of an angle in circular measure is

always = the length of the arc subtended by the angle at the centre of the circle  $\div$  the radius of the circle.



VELOCITY RATIO OF TWO CIRCULAR DISCS.

Then,  $\angle DAC = \theta = \frac{\text{arc DC}}{r_1}$ ; and  $\angle EBC = \phi = \frac{\text{arc EC}}{r_2}$

But, the arc DC = the arc EC since there is no slipping. Consequently,

$$\frac{\text{The angular velocity of circle A}}{\text{The angular velocity of circle B}} = \frac{\theta}{\phi} = \frac{\frac{\text{DC}}{r_1}}{\frac{\text{EC}}{r_2}} = \frac{r_2}{r_1}$$

Or,\*

$$\frac{\text{The angular velocity or speed of driver, A}}{\text{The angular velocity or speed of follower B}} = \frac{\text{Radius of follower B}}{\text{Radius of driver A}}$$

**Pitch Surfaces and Pitch Circles.**—In the case of the two discs or rollers just considered, their cylindrical surfaces are termed the *pitch surfaces*; and the two circles in the previous figure (which is simply a representation of their cross section, or section in the plane of their rotation) are called the *pitch circles*.

\* The *angular velocity* of a rotating disc is the *angle* described by its radius in unit time.

The relation between angular velocity and linear velocity may be shown thus:—Let  $\omega$  = the angular velocity; whilst  $v$  = the linear velocity of a point at radius  $r$  from the centre of motion when the disc makes  $n$  revolutions in unit time;

$$\text{Then } \omega \times r = v; \text{ or, } \omega = \frac{v}{r}; \text{ but } v = 2\pi rn,$$

$$\therefore \omega = \frac{2\pi rn}{r} = 2\pi n.$$

When the resistance to motion of the follower is great, the discs have to be provided with teeth in order to prevent slipping.

Consequently, the *pitch surfaces* and the *pitch circles* of such toothed rollers, toothed wheels, or spur wheel and pinion, are the surfaces and the circles of their rolling contact.\*

**Pitch of Teeth in Wheel Gearing.**—The linear or the circular distance from the centre of one tooth to the centre of the next one, or the distance from one edge of a tooth to the corresponding edge of its neighbouring one, *as measured on the pitch circle*, is termed the pitch of the teeth of a wheel.

Let  $D$  = Diameter of a wheel or pinion at its pitch circle.

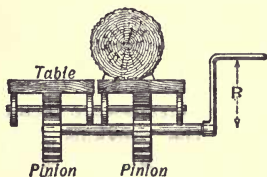
$p$  = Pitch of the teeth in the wheel or pinion.

$n$  = Number of teeth in the wheel or pinion.

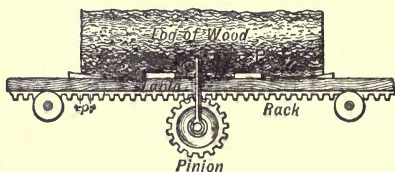
Then  $\pi D = p \times n$

For the circumference of the pitch circle must be equal to the pitch between any two neighbouring teeth  $\times$  the number of teeth in the wheel or pinion ; since the pitch between each pair of teeth must be the same all round the pitch circle, otherwise the wheel would not gear properly with any other wheel or pinion of the same pitch.

**Rack and Pinion.**—If a straight bar of iron be furnished with teeth on one side it is called a *rack*. It may therefore be considered as a wheel of infinite radius. When a rack has a pinion of the same pitch geared with it, the two form the useful combination termed the *rack and pinion*. It is employed for moving to and fro the tables of planing machines and large saw benches, as well as for elevating and lowering sluices in dams, &c.



END VIEW.



SIDE VIEW.

RACK AND PINION APPLIED TO A SAW-MILL TABLE.

The accompanying illustrations show the second of these applications, where two parallel racks are fitted to the under side of

\* When a large toothed wheel gears with a small one, the larger is termed a *spur-wheel* and the smaller a *pinion*. It is not possible in the space allotted to this elementary manual to enter into the best forms of the teeth of different kinds of wheel gearing. This subject is taken up in our "Advanced Text Book," Vol. I., Part II.

two movable tables or platforms. Upon the upper side of one of the tables is laid a log of wood adjusted in the desired position by wedges. The tables are each carried and guided by four rollers turning on fixed spindles. To the projecting end of the pinion shaft there is fitted a lever handle, so that by merely turning this handle in one direction, the racks, tables, and log of wood are pushed forward upon the projecting circular saw which revolves between the platforms, and if turned in the opposite direction they are drawn backwards. The pinions with their shaft and handle, have no linear motion, for the shaft is simply free to rotate in fixed bearings.

The *rack and pinion* with their handle constitute a modification of the wheel and axle, or lever and winch barrel, where the resistance offered by the rack and its load is overcome by a force applied to the handle. Every revolution of the handle turns the pinion, and consequently moves the rack through a linear distance equal to the circumference of the pinion's pitch circle. The principles of moments and of work can therefore be applied to this machine in exactly the same way as we applied them to the wheel and axle and the winch.

If  $P$  = Pull acting on the handles,

$R$  = Radius of handle,

$r$  = Radius of pinion's pitch circle,

$W$  = Weight or resistance overcome;

$$\begin{array}{lcl} \text{Then} & . & . \\ & & P \times 2\pi R = W \times 2\pi r \\ & & P \times R = W \times r \end{array}$$

$$\text{Theoretical advantage} \quad . \quad . \quad = \frac{W}{P} = \frac{R}{r} = \frac{\text{P's velocity}}{\text{W's velocity}}.$$

**Velocity Ratio in Wheel Gearing.**—From what has been said about belt gearing, pitch surfaces, pitch circles, and pitch of teeth, it must be at once apparent to the student that the same rule which was worked out in Lecture XI., in connection with belt gearing, will equally apply to the case of wheel gearing, where there are an equal number of drivers and followers. In the accompanying figure, where there are three drivers and three followers,

Let  $D_1, D_2, D_3$  = Diameters of the drivers.

„  $F_1, F_2, F_3$  = Diameters of the followers.

„  $N_{D_1}, N_{F_3}$  = Number of revolutions in the same time of the *first* driver and the *last* follower.

Then, following the same reasoning as was expounded in Lecture XI. for the velocity ratio of belt gearing, we have

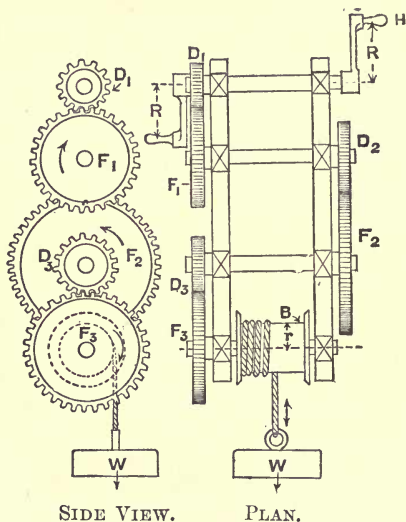
The speed of the *first driver*  $\times$   $\left\{ \begin{array}{l} \text{the successive diameters of} \\ \text{the drivers} \end{array} \right\} = \left\{ \begin{array}{l} \text{The speed of the last follower} \\ \times \text{ the successive diameters} \\ \text{of the followers.} \end{array} \right.$

$$\text{i.e., } N_{D_1} \times D_1 \times D_2 \times D_3 = N_{F_3} \times F_1 \times F_2 \times F_3$$

$$\text{Or, } \frac{N_{D_1}}{N_{F_3}} = \frac{F_1 \times F_2 \times F_3}{D_1 \times D_2 \times D_3}$$

The speed of *first driver* =  $\frac{\text{Product of the diameters of the followers.}}{\text{Product of the diameters of the drivers.}}$   
 The speed of *last follower*

In the above equation we may substitute the radii, or the cir-



WHEEL GEARING IN A TRIPLE PURCHASE WINCH

cumferences, or the number of teeth in the drivers and in the followers respectively, for their diameters; consequently,

Let  $r_{D_1}, r_{D_2}, r_{D_3}$  = Radii of the respective drivers.  
 „  $C_{D_1}, C_{D_2}, C_{D_3}$  = Circumferences „  
 „  $n_{D_1}, n_{D_2}, n_{D_3}$  = Number of teeth in „  
 „  $r_{F_1}, r_{F_2}, r_{F_3}$  = Radii of the respective followers.  
 „  $C_{F_1}, C_{F_2}, C_{F_3}$  = Circumferences „  
 „  $n_{F_1}, n_{F_2}, n_{F_3}$  = Number of teeth in „

$$\text{Then, } N_{D_1} \times r_{D_1} \times r_{D_2} \times r_{D_3} = N_{F_3} \times r_{F_1} \times r_{F_2} \times r_{F_3}$$

$$\text{Or, } N_{D_1} \times C_{D_1} \times C_{D_2} \times C_{D_3} = N_{F_3} \times C_{F_1} \times C_{F_2} \times C_{F_3}$$

$$\text{Or, } N_{D_1} \times n_{D_1} \times n_{D_2} \times n_{D_3} = N_{F_3} \times n_{F_1} \times n_{F_2} \times n_{F_3}$$

EXAMPLE I.—Three drivers of 10, 20, and 30 teeth each, gear respectively with three followers of 40, 80, and 120 teeth each. Ascertain the velocity ratio between the *first* driver and the *last* follower.

By the above formula—

$$N_{D_1} \times n_{D_1} \times n_{D_2} \times n_{D_3} = N_{F_3} \times n_{F_1} \times n_{F_2} \times n_{F_3};$$

$$\therefore \frac{N_{D_1}}{N_{F_3}} = \frac{n_{F_1} \times n_{F_2} \times n_{F_3}}{n_{D_1} \times n_{D_2} \times n_{D_3}}$$

Substituting the corresponding numerical values for the letters, we get

$$\left\{ \begin{array}{l} N_{D_1} = 10 \times 20 \times 30 \\ N_{F_3} = 40 \times 80 \times 120 \end{array} \right. = \frac{4 \times 4 \times 4}{1} = \frac{64}{1}$$

**Principle of Work applied to Wheel-gearing.**—Referring to the previous figure, it is perfectly evident from the former applications to other machines of the “principle of work,” that, *neglecting friction*, the force applied (to the handles of the machine)  $\times$  the distance through which it acts, will be equal to the weight raised  $\times$  the distance through which it is elevated.

- Let  $P$  = Push applied to the handles in lbs.  
 „  $R$  = Radius or leverage at which  $P$  acts.  
 „  $W$  = Weight raised by the rope on the barrel  $B$ .  
 „  $r$  = Radius or leverage with which  $W$  acts.  
 „  $D_1, D_2, D_3$  = Diameters of the driving wheels.  
 „  $F_1, F_2, F_3$  = Diameters of the following wheels.  
 „  $N_{D_1}$  = Number of revolutions of the *first* driver,  $D_1$ , or of the handles,  $H$ .  
 „  $N_{F_3}$  = Number of revolutions *in the same time* of the *last* follower,  $F_3$ , or of the barrel,  $B$ .

Then, by the principle of work and *neglecting friction*—

$$P \times \text{its distance} = W \times \text{its distance.}$$

$$\text{i.e., } P \times 2\pi R \times N_{D_1} = W \times 2\pi r \times N_{F_3}$$

(Divide both sides of the equation by  $2\pi$ )

$$\therefore P \times R \times N_{D_1} = W \times r \times N_{F_3}$$

$$\text{Or, } \frac{P \times R}{W \times r} = \frac{N_{F_3}}{N_{D_1}}; \text{ or } \frac{P}{W} = \frac{N_{F_3} \times r}{N_{D_1} \times R}$$

\* It is evident that in order to obtain the distance through which  $P$  acts, we must multiply the circumference of the circle described by the handles by the number of revolutions they make; and in the same way the circumference of the barrel must be multiplied by the revolutions which it makes in the same time, in order to get  $W$ 's distance.



But by the previous equation for velocity ratios,

$$\frac{N_F}{N_D^3} = \frac{D_1 \times D_2 \times D_3}{F_1 \times F_2 \times F_3}$$

i.e.,  $\frac{P \times R}{W \times r} = \frac{D_1 \times D_2 \times D_3}{F_1 \times F_2 \times F_3}$

$$\text{Or, } P \times R \times F_1 \times F_2 \times F_3 = W \times r \times D_1 \times D_2 \times D_3$$

Hence the general rule for work done in wheel-gearing  $P \times \text{its leverage} \times \text{the diameters (or radii, or circumferences, or number of teeth) of all the followers} = W \times \text{its leverage} \times \text{the diameters (or radii, or circumferences, or number of teeth) of all the drivers}$ .

EXAMPLE II.—If four men exert a constant force of 15 lbs. each on the handles of a compound crab or winch (such as that illustrated by the previous figure), and if the leverage of the handles is 15", whilst the weight to be raised acts on the barrel or drum at a leverage of 5", what load will they lift if the respective diameters of the drivers are 12", 20", and 20"; and of the followers, 36", 80" and 100", neglecting friction?

ANSWER.—In this case,  $P = 4 \times 15 = 60$  lbs.;  $R = 15$ ";  $r = 5$ ";  $D_1 = 12$ ";  $D_2 = 20$ ";  $D_3 = 20$ ";  $F_1 = 36$ ";  $F_2 = 80$ ", and  $F_3 = 100$ ".

By the above formula and by substituting the corresponding numerical values we have—

$$P \times R \times F_1 \times F_2 \times F_3 = W \times r \times D_1 \times D_2 \times D_3$$

$$60 \times 15'' \times 36'' \times 80'' \times 100'' = W \times 5'' \times 12'' \times 20'' \times 20''$$

$$\therefore W = \frac{60 \times 15 \times 36 \times 80 \times 100}{5 \times 12 \times 20 \times 20}$$

$$\text{Or, } W = 60 \times 3 \times 3 \times 4 \times 5 = 10,800 \text{ lbs.}$$

EXAMPLE III.—If 40 % of the force applied to the handles be absorbed in overcoming internal friction in the above example of a winch, what weight can then be raised by the four men, each acting, as before, with a constant force of 15 lbs.?

ANSWER.—If 40 % of the applied force be lost in overcoming friction, then only 60 % is left for effective work, or the efficiency or *modulus* of the machine is said to be 0.6.\*

Consequently,  $100 : 60 :: 10,800 \text{ lbs.} : x \text{ lbs.}$

$$\therefore x = \frac{60 \times 10,800}{100} = 6480 \text{ lbs.}$$

\* The term *modulus of a machine* is only another expression for the more appropriate phrase, *efficiency of a machine*.

## LECTURE XII.—QUESTIONS.

1. When two circular discs with fixed centres are in firm contact and roll uniformly together, state and prove the rule for estimating their relative speeds of rotation.

2. Define the pitch circle of a toothed wheel. When two pitch circles, A and B, of diameters 2 and 3 respectively, roll together, prove that the angular velocity of A is to that of B as 3 to 2. Three spur wheels, A, B, C, with parallel axes, are in gear. A has 8 teeth, B has 32 teeth, and C has 42 teeth. How many turns will A make upon its axis while C goes round 8 times? Why is B termed an *idle wheel*? *Ans.* 42 turns. (*See Note to Question 10 re Idle Wheel.*)

3. What is the *pitch* of a tooth in a spur wheel? Two parallel shafts, whose axes are to be as nearly as possible 2 feet 6 inches apart, are to be connected by a pair of spur wheels, so that while the driver runs at 100 revolutions per minute, the follower is required to run at only 25 revolutions per minute. Sketch the arrangement, and mark on each wheel its diameter and the number of teeth, supposing the pitch of a tooth to be  $1\frac{1}{2}$  inch.

*Ans.* The follower is 48 inches diameter with 120 teeth.

The driver is 12

4. Define the "pitch surface" and the "pitch circle" of a toothed wheel. Two parallel axes are at a distance of 10 inches, and they are to rotate with velocities as the numbers 2 and 3 respectively. What should be the diameters of the pitch circles of a pair of wheels which would give this motion. Find pitch of teeth on the smaller wheel if the larger has 24 teeth? *Ans.* 12 ins. and 8 ins. ;  $1\cdot57$  inch.

5. Sketch and describe the "*rack and pinion*," and give instances from personal observation of its application. A pinion of  $3\cdot2$ " diameter has teeth of 1" pitch, and gears with a straight rack applied to a sluice gate. If the weight of the sluice and rack be 100 lbs. and the lever handle describes a circle of  $40\cdot2$ " in each turn, what force must be applied to the handle to lift the gate? How many feet will the sluice be lifted by six turns of the handle? *Ans.* 25 lbs. ; 5 ft.

6. Sketch the arrangement known as the rack and pinion. Apply the "principle of moments" and the "principle of work" to find the relation between the force applied and the weight raised by aid of this machine. A pinion has sixteen teeth of  $\frac{7}{8}$ -inch pitch in gear with a rack. If the pinion makes  $3\frac{1}{2}$  turns, through what distance has the rack been moved? If the pinion is turned by a handle 14 inches long, and with a force of 35 lbs. applied to the handle, find the force with which the rack is urged forward. *Ans.* 49 inches ; 223 lbs.

7. Deduce the formula for the velocity ratio in wheel gearing where there are three drivers and three followers, and state the rule derived therefrom in general terms. Three drivers of 20, 30, and 40 teeth respectively gear with three followers of 40, 60, and 80 teeth. If the first driver makes 160 revolutions, how many revolutions will the last follower make? *Ans.* 20.

8. In the previous question, if the handles attached to the first driver have each a radius of 15", and the drum connected to the last follower be 15" diameter, what force must be applied to the handles in order that they may lift 1120 lbs. supposing that the efficiency of the machine is 70 per cent.? *Ans.* 100 lbs.

9. The hour and minute hands of a clock are on the same arbor or axis, and the hour hand takes its motion from the minute hand. Devise some train of wheels for connecting the two hands.

10. How would you determine the "pitch circles," and the proper "pitch of the teeth" for a pair of spur-wheels? What would be the diameter of the pitch circle of a spur-wheel having 80 teeth of  $\frac{3}{4}$ -inch pitch?  
*Ans.* 19 inches.

Three spur-wheels A, B, C are on parallel axes, and are in gear. A has 10 teeth, B has 35 teeth, and C has 55 teeth. How many revolutions upon its axis will be made by A for every 4 revolutions of C? Why is B called an idle wheel and what is its use? *Ans.* 22 revs.

*Note re "Idle or Intermediate Wheel."*—When a wheel is carried on a separate axle and is interposed between two other wheels (or is introduced into a train of wheels), merely for the purpose of *changing the relative directions of rotation* of the first and last wheel, then such intermediate wheel is called an *idle* wheel, because it does not affect the numerical value of the train, but only its sign. For examples, see Vol. I. of my "Text Book of Applied Mechanics and Mechanical Engineering," Lecture  
 XI

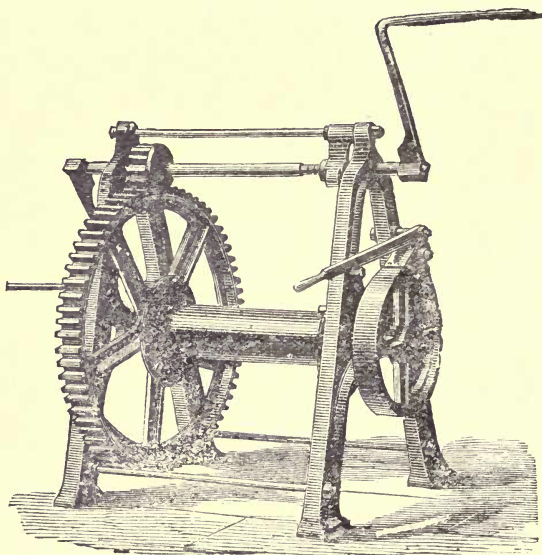


## LECTURE XIII.

**CONTENTS.** — Single-purchase Winch or Crab—Example I.—Double-purchase Winch or Crab—Example II.—Wheel Gearing in Jib-Cranes—Questions.

IN this Lecture we will apply the principles and formulæ discussed in the previous one to a few practical applications of gearing in machines for lifting weights.

**Single-purchase Winch or Crab.**—The comparatively small working advantage of the simple hand-driven wheel and axle or



**SINGLE-PURCHASE WINCH OR CRAB.**

By Messrs. Loudon Bros., Glasgow. .

handle and winch barrel (illustrated in Lecture V.) renders it unfit for lifting greater weights than one or two hundredweight. Consequently, whenever heavier loads have to be raised by manual

labour, one of the most useful machines that can be employed is the single-purchase crab. As will be seen from the accompanying perspective view, this machine consists of a pair of lever handles fitted to the squared ends of a round shaft carrying a pinion. This pinion gears with a spur-wheel keyed to a lower shaft, upon which is also fixed a drum or barrel. To a hook or eye on the inside neck of the left-hand flange of this barrel the rope or chain (to be connected to the load) is attached. Therefore, the turning of the handles causes the barrel to rotate and wind the rope upon it, thereby elevating the load. Both shafts turn in bearings bored in the cast-iron end standards or **A** frames. These frames are bound tightly together and kept at a fixed distance apart by three wrought-iron collared stays, secured on the outside by screw nuts. To the outside right-hand end of the barrel shaft there is keyed a friction pulley acted on by a steel brake-strap, for the purpose of enabling the labourers to lower a load gently or quickly without enduring the stress and danger of pulling back on the handles. In fact, after applying the brake-strap by its outstanding handle, they can lift the claw pawl which is hinged on the top stay (and which keeps the pinion in gear with the spur-wheel when in the position shown on the figure) and by pulling the upper shaft to the right, disengage the pinion from its wheel. Then, by adjusting the pawl into the other groove of this shaft, they are free to lower the load by the brake without having the handles flying round. Between the right hand flange of the barrel and its neighbouring **A** frame there is a ratchet-wheel (not seen on the figure). This ratchet-wheel is generally cast along with the barrel. Its pawl, which is hinged to the inner side of the standard, can therefore be dropped down so as to engage with a tooth of the stop-wheel, whenever it is necessary to cease heaving up a heavy weight; thereby preventing the machine overhauling, and giving the labourers freedom to leave the handles and attend to other duties.

**EXAMPLE I.**—In a single-purchase crab the lever handles are each 16" long, the diameter of the barrel is 8"; the pinion or driver has 12 teeth, and the wheel or follower 60 teeth. If two men apply a constant force of 20 lbs. each to the handles, and are just able to raise a weight of 600 lbs. to a height of 20 feet in two minutes, find—(1) the theoretical advantage; (2) the working advantage; (3) the work put in for every foot the weight is lifted; (4) the work got out for every foot the weight is lifted; (5) the efficiency; (6) the percentage efficiency of the machine; (7) the H.P. developed by the two men.

**ANSWER.**—Referring to the notation in last Lecture, we have  $P = 2 \times 20 \text{ lbs.} = 40 \text{ lbs.}$ ;  $R = 16''$ ;  $r = 4''$ ;  $n_D = 12 \text{ teeth}$ ;  $n_F = 60$

teeth ;  $W_T$  = the theoretical weight that would be raised if there were no friction ;  $W_A = 600$  lbs. (the actual weight raised) ;  $h = 20$  feet.

$$(1) \text{ Theoretical advantage} = \frac{W_T}{P}$$

By the principle of work (*neglecting friction.*)

$$P \times \text{by its distance}^* = W_T \times \text{its distance}^*$$

$$P \times 2\pi R \times n_F = W_T \times 2\pi r \times n_D$$

$$P \times R \times n_F = W_T \times r \times n_D$$

$$\therefore W_T = \frac{P \times R \times n_F}{r \times n_D}$$

(Substituting the above numerical values we get)

$$W_T = \frac{40 \times 16 \times 60}{4 \times 12} = 40 \times 4 \times 5 = 800 \text{ lbs.}$$

$$\text{Consequently, } \frac{W_T}{P} = \frac{800}{40} = \frac{20}{1}$$

$$(2) \text{ Working advantage} = \frac{W_A}{P} = \frac{600 \text{ lbs.}}{40 \text{ lbs.}} = \frac{15}{1}$$

(3) *Work put in for every foot  $W_A$  is raised.* From equation (1) we see that for every foot  $W_A$  is raised  $P$  must have gone through 20 feet, since the velocity ratio is  $\frac{20}{1}$

$$\therefore P \times 20 = 40 \text{ lbs.} \times 20 = 800 \text{ ft.-lbs.}$$

$$(4) \text{ Work got out for every foot } W_A \text{ is raised} \\ = W_A \times 1' = 600 \text{ lbs.} \times 1' = 600 \text{ ft.-lbs.}$$

$$(5) \text{ The efficiency} = \frac{\text{Work got out}}{\text{Work put in}} = \frac{600 \text{ ft.-lbs.}}{800 \text{ ft.-lbs.}} = .75$$

$$(6) \text{ The percentage efficiency} = .75 \times 100 = 75 \%$$

$$(7) \text{ The H.P. developed by the two men} = \frac{\text{Work put in per minute}}{33,000}$$

$$\therefore \text{H.P.} = \frac{8000 \text{ ft.-lbs.}}{33000 \text{ ft.-lbs.}} = \frac{1}{4} \text{ bare, or } \frac{1}{8} \text{ of a horse-power per man.}$$

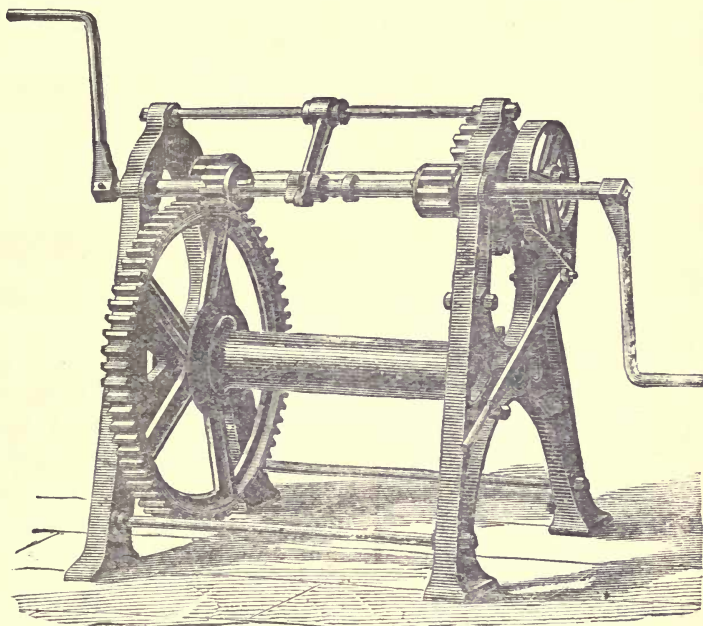
\* It is evident that—

$$\frac{P \times 1 \text{ turn of handles}}{W_T \times 1 \text{ turn of barrel}} = \frac{\text{Number of teeth in the driver}}{\text{Number of teeth in the follower.}}$$

$$\text{Or, } P \times 2\pi R : W_T \times 2\pi r :: n_D : n_F$$

$$\therefore P \times 2\pi R \times n_F = W_T \times 2\pi r \times n_D$$

**Double-Purchase Winch or Crab.**—It will be observed, from an inspection of the accompanying photographic view of a “Double-purchase Crab,” that the chief difference between it and the single-purchase one is, that it has another pinion and wheel, with a view of increasing the actual or the working advantage, and thus enabling the same manual force to lift a greater load, although by taking a longer time. It is also larger, heavier, and stronger.



DOUBLE-PURCHASE WINCH OR CRAB.

By Messrs. Loudon Bros., Glasgow.

As will be seen from the figure, it may be used as a single-purchase winch by simply lifting the claw-pawl hinged on the top stay, and pushing the handle shaft forward until its left-hand pinion gears with the large spur wheel, and then letting the pawl drop on to bearing to the right hand of the two collars on this shaft. By so doing, the right-hand pinion or first driver (when in double-purchase gear) is freed from the first follower, and both are inactive during the time it is used in single purchase, but the second

driver is still in gear and is turned round by the spur wheel. The brake strap pulley is keyed to the second shaft (carrying the first follower and second driver), and can be used for lowering the load without the handles coming into action (as described in the previous case) by placing the claw-pawl *between* the two collars in the first motion shaft. When the pawl is in this position, both of the pinions on this shaft are out of gear. The machine may be locked and the load left suspended by dropping the ratchet into the ratchet-wheel cast on the right-hand end of the barrel in the same way as with the single-purchase crab. A triple-purchase winch was illustrated in Lecture XII., and the student should again refer to the plan and the side elevation of its gearing.

**EXAMPLE II.**—Four men exert a force of 20 lbs. each, on the handles of a double-purchase crab, which are 15" long. The driving pinions have 12 teeth each, the followers 24 and 48 teeth respectively, and the diameter of the barrel is 10". Find the weight that can be raised if 25 per cent. of the work put in be absorbed in overcoming friction.

**ANSWER.**—Here  $P = 4 \times 20 = 80$  lbs. ;  $R = 15''$  ;  $n_{D1} = 12$  ;  $n_{D2} = 12$  ;  $n_{F1} = 24$  ;  $n_{F2} = 48$  ;  $r = 5''$ .

By the formula deduced in the previous lecture from the principle of work (neglecting friction),

$$P \times R \times n_{F1} \times n_{F2} = W_T \times r \times n_{D1} \times n_{D2}$$

$$80 \times 15 \times 24 \times 48 = W_T \times 5 \times 12 \times 12$$

$\begin{matrix} & 3 & 2 & 4 \\ & \times & \times & \times \end{matrix}$

After cancelling, we get—

$$80 \times 3 \times 2 \times 4 = W_T = 1920 \text{ lbs.}$$

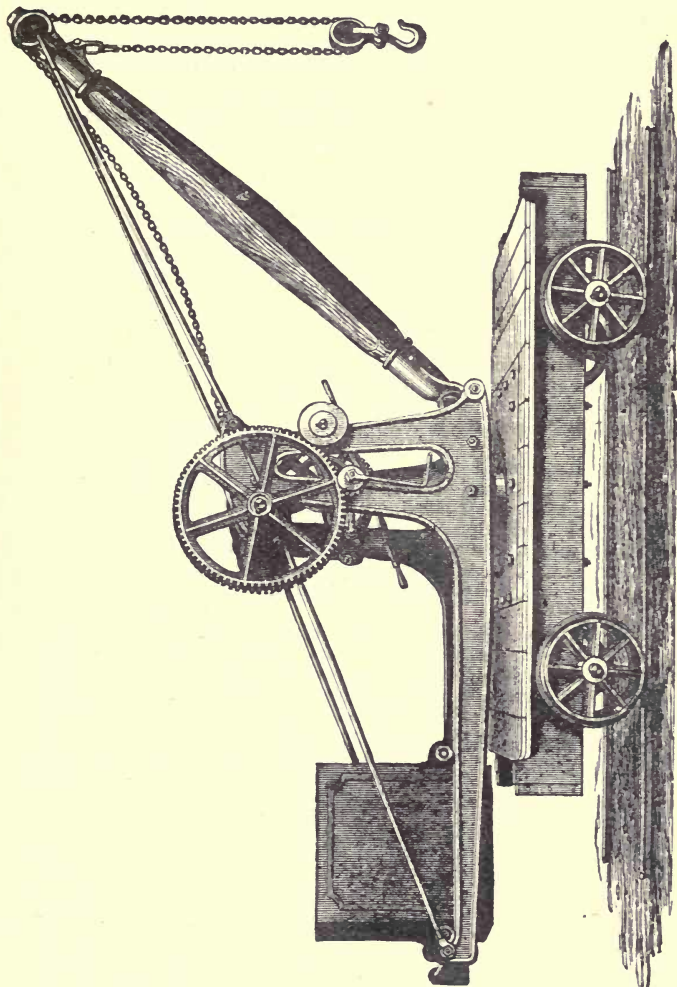
If 1920 lbs. of work be expended by the men and 25 per cent. of this be lost work, there remains 75 per cent. as useful work.

Or, . . . 100 : 75 :: 1920 lbs. :  $W_A$ .

Weight actually raised =  $W_A = 1440$  lbs.

**Wheel Gearing in Jib Cranes.**—In Lecture VIII. the side view of a jib crane was given for the purpose of exemplifying the stresses on the jib, tie-rods, and central pillar. We now illustrate a swing jib crane on a bogie and rails, to show that the framework and lifting gear are simply those of an inverted double-purchase crab with the toothed wheels placed outside the standards instead of inside as in the ordinary winch. The snatch block pulley (previously referred to in Lecture VII.), to the hook of which the load is attached, doubles the theoretical purchase or advantage of the winch gearing, and therefore one, two or more men can lift nearly double the weight by aid of this simple addition to the machine. Large cranes of this description are

fitted with slewing or horizontal turning gear, to enable the load when lifted to be swung round before depositing it in a truck,



SLEWING JIB CRANE WITH BACK BALANCE WEIGHT, SUPPORTED BY BOGIE AND RAILS.  
(By Messrs. P. & W. MacLellan, Glasgow.)

hold of a ship, or on a machine tool. This latter gear consists of a horizontal wheel on the top of the vertical central cast-iron

supporting boss, with which is geared a bevel pinion, actuated by aid of a lever handle.

In order to prevent the whole machine being capsized by a heavy load, there is a back balance weight, and further the bogie wheels can be clamped to the rails. The back balance weight also tends to cancel the severe right angle stress on the central pillar which was specially taken notice of in Lecture VIII. We will defer the description of heavy steam power cranes, tripods and shear legs to our Advanced Course.

### LECTURE XIII.—QUESTIONS—(continued)

10. What do you understand by the efficiency of a machine, and how is it measured? In a single purchase crab, the pinion has 12 teeth and the wheel has 78 teeth, the diameter of the barrel being 7 inches, and the length of the lever handle 14 inches. It is found that the application of a force of 15 lbs. at the end of the handle suffices to raise a weight of 280 lbs. Find the efficiency of the machine. *Ans.* 0.72; or 72 per cent.

10. In a crane an effort of 122 lbs. just raises a load of 3265 lbs. What is the mechanical advantage? If the efficiency be 60 per cent., what is the velocity ratio? *Ans.* Mech. Adv. 26.76 : 1; Vel. Ratio 44.6 : 1.

(B. of E., 1903.)

11. In a crane, a force of 3 lbs. applied at the handle is found to raise a weight of 42 lbs., and a force of 8 lbs. a load of 120 lbs. If the relation between the force applied and the weight raised is represented by the straight line law, obtain the equation expressing the relationship between them; and if the velocity ratio between the force applied and the weight raised is 18, estimate the efficiency of the crane when lifting a load of 200 lbs. *Ans.*  $F = 5W/78 + 4/13$ ; 84.6%.

(C. & G., 1904, O., Sec. A.)

12. In an electrically driven overhead crane a weight of 5 tons is raised at the rate of 90 feet per minute. What is the horse-power? Convert this into watts. The motor drives through gearing whose efficiency is 70 per cent. How many amperes of current must be supplied to the motor at a voltage of 220 if the efficiency of the motor is 87 per cent.? *Ans.* 30.5 H.P. 22,787 watts; 170 amperes.

[Note that 1 horse power = 746 watts; and 1 ampere multiplied by 1 volt is 1 watt.]

(B. of E. 1905.)

## LECTURE XIII.—QUESTIONS.

1. Where wheelwork is employed to modify motion, as in a crane, or in the double-gearred headstock of a lathe, how is the change of motion calculated? Write down the formula employed.

2. Sketch a side elevation and end view of a single purchase crab, and describe the same by aid of an "index to parts." Apply the principle of work in solving the following question:—The lever handle of a crab is three times the diameter of the drum, and the wheelwork consists of a pinion of 16 teeth driving a wheel of 80 teeth; what weight will be lifted by a force of 30 lbs. acting at the end of the lever handle? *Ans.* 900 lbs.

3. Describe, with a freehand sketch, a single purchase lifting crab. The leverage of the handle of the crab is 16 in., and there is a pinion of 20 teeth driving a wheel of 100 teeth, the diameter of the barrel being 8 in. Assign the relative proportions of the working parts, and estimate the theoretical advantage. What weight would be raised by a man exerting a force of 15 lbs. on the lever handle, neglecting friction? *Ans.* 300 lbs.

4. A weight of 4 cwt. is raised by a rope which passes round a drum 3 feet in diameter, having on its shaft a toothed wheel also 3 feet in diameter. A pinion, 8 inches in diameter, and driven by a winch-handle 16 inches long, gears with the wheel. Find the force to be applied to the winch-handle in order to raise the weight. *Ans.* 112 lbs.

5. In a lifting crab the lever handle is 14 inches long, the diameter of the drum is 6 inches, and the wheel and pinion have 57 and 11 teeth respectively. Find the weight in pounds which could be raised by a force of 50 lbs. applied to the lever handle, friction being neglected. *Ans.* 1209 lbs.

6. In a crane there is a train of wheelwork, the first pinion being driven by a lever handle; and the last wheel being on the same axis as the chain barrel of the crane. The wheelwork consists of a pinion of 11 gearing with a wheel of 92, and of a pinion of 12 gearing with a wheel of 72, the diameter of the barrel being 18 inches and that of the circle described by the end of the lever handle being 36 inches; find the ratio of the pull to the weight raised, friction being neglected. *Ans.* 11 : 1104.

7. In a 30-ton crane the tension of the chain as it runs on the winding barrel is  $7\frac{1}{2}$  tons, the barrel is 2 feet in effective diameter, and the spur wheel connected with it is 4 feet in diameter on the pitch line; what pressure will come upon the teeth of the spur wheel, supposing such pressure to act on the pitch line (friction is neglected)? *Ans.* 3.75 tons.

8. The crank of an engine is 2' long, and the diameter of the fly-wheel is 10'; also the fly-wheel has teeth on its rim, and drives a pinion 3' in diameter. If the mean pressure on the crank pin be  $7\frac{1}{2}$  tons, what is the mean driving pressure on the teeth of the pinion? *Ans.* 3 tons.

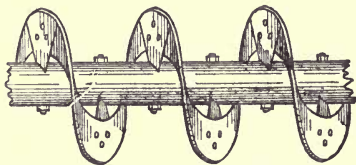
9. Draw to scale a side elevation, end view and plan of a double purchase crab, and describe the same by aid of an "index to parts." If four men each exert a constant force of 15 lbs. on the handles of such a crab; if the handles have a leverage of 16 inches whilst the barrel is 16 inches diameter, and if the drivers have 12 teeth each while the followers have 24 and 60 teeth respectively; find the weight which they could balance neglecting friction. If 30 per cent. of the work put in, be taken up in overcoming friction, what load can they lift? State (1) theoretical advantage; (2) working advantage; (3) work put in when lifting the load 1 foot; (4) the work got out; (5) the percentage efficiency; (6) the height through which they would lift the load in 1 minute if each man developed  $\frac{1}{2}$  H.P. *Ans.* 1200 lbs.; 840 lbs.; (1) 20 : 1; (2) 14 : 1 (3) 1200 ft.-lbs.; (4) 840 ft.-lbs., (5) 70 per cent.; (6) 13.75 ft.

## LECTURE XIV.

**CONTENTS.**—Screws—The Spiral, Helix, or Ideal Line of a Screw Thread—The Screw viewed as an Inclined Plane—Characteristics and Conditions to be Fulfilled by Screw Threads—Different Forms of Screw Threads—Whitworth's V-Threads—Whitworth's Tables of Standard V-Threads, Nuts and Bolt Heads—Seller's V-Thread—The Square Thread—The Rounded Thread—The Buttress Thread—Right and Left-hand Screws—The Screw Coupling for Railway Carriages—Single, Double and Treble Threaded Screws—Backlash in Wheel and Screw Gearings—Questions.

**Screws.**—Every one is more or less familiar with the form and uses of the screw nail for securing pieces of wood together, and of the bolt with its nut for fixing metal plates in position; but every one is not so familiar with the principle upon which screws are generated and act, or with the best shape to be given to a screw under different circumstances. We shall therefore endeavour in this Lecture to explain these points in an elementary manner, instancing a few examples of the practical applications of screws, but reserving for the following Lecture questions on the work done by screws and their efficiency.

**The Spiral, Helix, or Ideal Line of a Screw Thread.**—A very good idea of the form of a screw is obtained from the accompanying figure, which represents one means of elevating or trans-



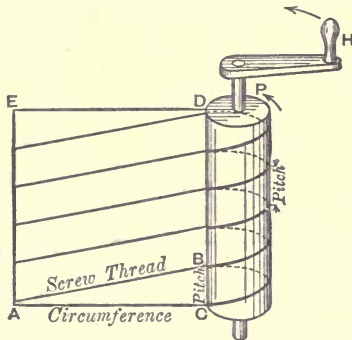
SPIRAL OR SCREW FOR MOVING GRAIN.

ferring grain, flour or other powdered substances from one part of a milling works to another. It consists of a steel band twisted around a cylindrical shaft in a continuous and uniformly pitched spiral. This shaft and screw are placed in a trough, tube or pipe. The grain or powdered substance is fed in at one end of the pipe, and by rotating the screw with a wheel or lever fixed

to one end of the shaft, the loose material is gradually pressed forward until it reaches the other end, from which it may be dropped into sacks or put through another process. It is evident from an inspection of the figure that as the screw is turned round by the lever, the particles of matter are forced *along the face of the continuous inclined plane* formed by the spiral steel band.

*The principle upon which the screw acts is, therefore, a combination of the inclined plane and the lever.*

To bring this view of the case still more forcibly before the student, take a cylinder and fix along the side thereof parallel to



FORMING A SCREW THREAD ON A CYLINDER.

its axis (by gum or drawing pins) a rectangle, ACDE, of paper or white cloth, having its sides, AC and DE exactly equal to the circumference of the cylinder. Then, when the envelope is wound round the cylinder by the turning of the handle, H (in the direction shown by the arrow at P), it exactly covers its cylindrical surface. On the outside of this rectangle when unfolded, draw any convenient number of parallel inclined black lines, AB, &c., equidistant from each other as shown by the figure, and again wrap it round the cylinder. These lines will be found to form a continuous spiral, helix, or screw-thread line from one end of the cylinder to the other. And the side AC of the right-angled triangle ACB forms the *circumference*, BC the *pitch*, AB the length of the *thread* (for one complete turn of the cylinder), and the angle BAC is the *inclination* or angle of the screw.

**The Screw Viewed as an Inclined Plane.**—Take another cylinder having an evenly pitched screw-thread line drawn upon it. Cut a sheet of flexible cardboard into the form of a right-angled triangle with its height BC or *h* equal to the *pitch* (or dis-

tance between two consecutive threads when measured parallel to the axis of the cylinder); AC or  $b$  equal to the *circumference* of the screw and wrap it round the cylinder, taking care to keep BC parallel to the axis. Then the hypotenuse AB or length  $l$  of the inclined plane will coincide with the contour of the screw-thread for one complete turn, and BAC or,  $\alpha$ , is the *angle* of the thread to the plane at right angles to the axis of the cylinder.

Now conceive this screw-thread instead of being a mere line to be an inclined plane of known breadth, as in the case of the grain elevator.\* Let the total weight of material being urged

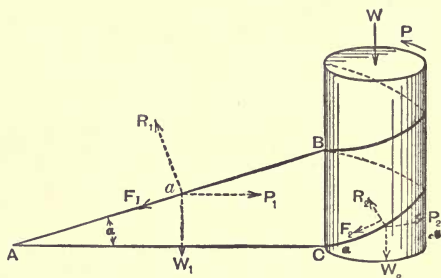


FIGURE TO PROVE THAT A SCREW THREAD IS AN INCLINED PLANE.

forward or upwards by the turning of the screw be  $W$  lbs., and let the resistance due to this load be uniformly distributed along the screw thread or inclined plane. Then, comparing the first and the third figures, it is evident that any small portion of the load having a weight  $W_2$  lbs. will have a corresponding reaction  $R_2$  lbs., and will require a part  $P_2$  lbs. (of the total force,  $P$ , applied to turn the screw at the radius at which this portion is situated) to move it along the screw-plane against the frictional resistance  $F_2$ .

Imagine the work done to be transferred to the inclined plane, AB, then any portion of the load having a weight  $W_1$  lbs. will have a corresponding reaction  $R_1$  lbs., and will require a part  $P_1$  lbs. (of the total force,  $P$ , applied parallel to the base to pull the whole load up the inclined plane) to move it along the plane against the frictional resistance  $F_1$ . Now, these forces act in identically the same way as the second case of the inclined plane, which was discussed in Lecture IX., consequently—

$$\begin{aligned} W_1 : P_1 : R_1 &:: AC : CB : AB \\ W_2 : P_2 : R_2 &:: AC : CB : AB \\ \therefore W : P : R &:: b : h : l \end{aligned}$$

\* Or, that the screw-thread has a certain *depth* as measured radially from the axis of cylinder.

$$\text{Or, } \frac{P}{W} = \frac{C B}{A C} = \frac{\text{height}}{\text{base}} = \frac{h}{b} = \frac{\text{pitch of thread}}{\text{circumference of screw.}}$$

We therefore see that a screw may be treated as an inclined plane where the force turning the screw—*i.e.*, overcoming the resistance to motion—acts parallel to the base of the incline. The same reasoning may be applied to any screw turning in a nut or to a nut turning on a screw.

**Characteristics of and Conditions to be Fulfilled by Screw Threads.**—The essential characteristics of a screw-thread are its *pitch*, *depth*, and *form*.

The principal conditions to be fulfilled by a screw-thread are: (1) *efficiency*; (2) *strength*; (3) *durability*.

(1) The *efficiency* depends on the pitch and the friction, and hence on the pitch and form of thread.

(2) The *strength* depends upon the form or the shearing thickness and depth, or area of the cross section parallel to the axis.

(3) The *durability* depends chiefly on the depth—that is, upon the extent of bearing surface.

**Different Forms of Screw Threads.**—Sir Joseph Whitworth, the famous tool and gun manufacturer, was so impressed with the great inconvenience and loss of money which arose from the use of different pitches and forms of threads for screws and nuts, that he published the following tables giving the dimensions of what has now become known as the Whitworth standard. Prior to 1841, the year in which Whitworth proposed the adoption of standard sizes for screws, and for several years afterwards, different engineering works in this country not only used different pitches for screws of the same diameter, but it was no uncommon thing to find a want of uniformity in the same shop. Now, every one in Great Britain and her colonies uses the Whitworth standard sizes for V-threaded bolts and nuts of  $\frac{1}{4}$ -inch and upwards, and the British Association standard for smaller screws in electrical and philosophical instruments.

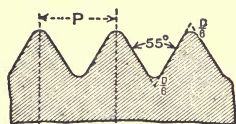
**Whitworth's V Thread.\***—The following figures of a Whitworth thread and nut, together with the tables, will serve to give full information regarding the number of threads per inch for different diameters of screw-bolts, nuts and bolt-heads, &c.

The angle between opposite sides of the threads and of the intervening spaces is  $55^\circ$ . One-sixth of the *depth* of the thread is rounded off at both the top and the bottom for the purpose of preventing a sharp nick at the bottom (which would weaken a

\* For a description of Whitworth's screw-taps, plates, stocks, dies and combs, see "Workshop Appliances" by Professor Shelley. And for a table of the B. A. Standard for Small Screws, see Munro and Jamieson's *Electrical Rules and Tables*, 16th ed., p. 67.

## WHITWORTH'S STANDARD FOR SCREWS WITH ANGULAR THREADS.

No. of Threads per Inch.	Old Sizes, Inches.	New Standard, Decimals of an Inch.	No. of Threads per Inch.	Old Sizes, Inches.	New Standard, Decimals of an Inch.	No. of Threads per Inch.	Old Sizes, Inches.	New Standard, Decimals of an Inch.
48	$\frac{1}{8}$	0.100	12	$\frac{5}{8}$	0.600	4	$2\frac{1}{2}$	2.375
40		0.125	11		0.625	4	$2\frac{1}{2}$	2.500
32		0.150	11		0.650	4	$2\frac{5}{8}$	2.625
24		0.175	11		0.675	$3\frac{1}{2}$	$2\frac{3}{4}$	2.750
24		0.200	11		0.700	$3\frac{1}{2}$	$2\frac{7}{8}$	2.875
24		0.225	10		0.750	$3\frac{1}{2}$	3	3.000
20	$\frac{1}{4}$	0.250	10	$\frac{3}{4}$	0.800	$3\frac{1}{2}$	$3\frac{1}{4}$	3.25
20		0.275	9	$\frac{7}{8}$	0.875	$3\frac{1}{4}$	$3\frac{1}{2}$	3.50
18		0.300	9	1	0.900	3	$3\frac{3}{4}$	3.75
18		0.325	8		1.000	3	4	4.00
18		0.350	7		1.125	$2\frac{1}{4}$	$4\frac{1}{4}$	4.25
16		0.375	7		1.250	$2\frac{3}{4}$	$4\frac{3}{4}$	4.50
16	$\frac{5}{16}$	0.400	6		1.375	$2\frac{3}{4}$	$4\frac{3}{4}$	4.75
14		0.425	6		1.500	$2\frac{3}{4}$	5	5.00
14		0.450	5		1.625	$2\frac{3}{4}$	$5\frac{1}{4}$	5.25
14		0.475	5		1.750	$2\frac{5}{8}$	$5\frac{1}{2}$	5.50
12		0.500	$4\frac{1}{2}$		1.875	$2\frac{1}{2}$	$5\frac{3}{4}$	5.75
12	$\frac{1}{2}$	0.525	$4\frac{1}{2}$	2	2.000	$2\frac{1}{2}$	6	6.00
12		0.550	$4\frac{1}{2}$	$2\frac{1}{8}$	2.125			
12		0.575	4	$2\frac{1}{4}$	2.250			



WHITWORTH VEE THREAD.

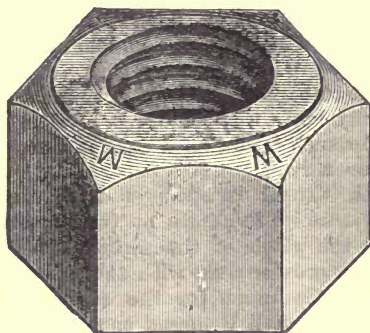
Angle of thread =  $55^\circ$ . One-sixth of depth is rounded off at top and bottom.

Number of threads to the inch in square threads =  $\frac{1}{2}$  number of those in angular threads.

Depth of threads = 0.64 pitch for angular = 0.475 pitch for square threads.

## WHITWORTH'S GAS THREADS.

Diameter in Inches.	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2
No. of threads per inch	28	19	19	14	14	11	11	11	11	11



WHITWORTH SCREW NUT.

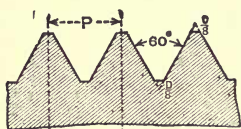
## WHITWORTH'S STANDARD NUTS AND BOLT-HEADS.

Diameter of Bolt, Inches.	Width across Flats.	Thickness of Nuts.	Thickness of Bolt- heads.	Diameter of Bolt at Bottom of Thread.	Diameter of Bolt, Inches.	Width across Flats.	Thickness of Nuts.	Thickness of Bolt- heads.	Diameter of Bolt at Bottom of Thread.
$\frac{1}{8}$	0.338	$\frac{1}{8}$	0.1093	0.0929	$1\frac{1}{8}$	1.8605	$1\frac{1}{8}$	0.9843	0.942
$\frac{3}{16}$	0.448	$\frac{3}{16}$	0.1640	0.1341	$1\frac{1}{4}$	2.0483	$1\frac{1}{4}$	1.0937	1.067
$\frac{1}{4}$	0.525	$\frac{1}{4}$	0.2187	0.1859	$1\frac{3}{8}$	2.2146	$1\frac{3}{8}$	1.2031	1.1615
$\frac{5}{16}$	0.6014	$\frac{5}{16}$	0.2734	0.2413	$1\frac{1}{2}$	2.4134	$1\frac{1}{2}$	1.3125	1.2865
$\frac{3}{8}$	0.7094	$\frac{3}{8}$	0.3281	0.2949	$1\frac{5}{8}$	2.5763	$1\frac{5}{8}$	1.4218	1.3688
$\frac{7}{8}$	0.8204	$\frac{7}{8}$	0.3828	0.346	$1\frac{3}{4}$	2.7578	$1\frac{3}{4}$	1.5312	1.4938
$\frac{1}{2}$	0.9191	$\frac{1}{2}$	0.4375	0.3932	$1\frac{7}{8}$	3.0183	$1\frac{7}{8}$	1.6406	1.5904
$\frac{9}{16}$	1.011	$\frac{9}{16}$	0.4921	0.4557	2	3.1491	2	1.75	1.7154
$\frac{5}{8}$	1.101	$\frac{5}{8}$	0.5468	0.5085	$2\frac{1}{8}$	3.337	$2\frac{1}{8}$	1.8593	1.8404
$1\frac{1}{16}$	1.201	$1\frac{1}{16}$	0.6015	0.571	$2\frac{1}{4}$	3.546	$2\frac{1}{4}$	1.9687	1.9298
$\frac{3}{4}$	1.3012	$\frac{3}{4}$	0.6562	0.6219	$2\frac{3}{8}$	3.75	$2\frac{3}{8}$	2.0781	2.0548
$1\frac{3}{16}$	1.39	$1\frac{3}{16}$	0.7109	0.6844	$2\frac{1}{2}$	3.894	$2\frac{1}{2}$	2.1875	2.1798
$\frac{7}{8}$	1.4788	$\frac{7}{8}$	0.7656	0.7327	$2\frac{5}{8}$	4.049	$2\frac{5}{8}$	2.2968	2.3048
$1\frac{5}{8}$	1.5745	$1\frac{5}{8}$	0.8203	0.7952	$2\frac{3}{4}$	4.181	$2\frac{3}{4}$	2.4062	2.384
1	1.6701	1	0.875	0.8399	3	4.531	3	2.625	2.634

bolt or a nut), as well as for ease in manufacturing them, since it would be practically impossible to maintain such perfectly sharp edges in the stocks and dies or in the combing tools with which such bolts and nuts are generally screwed. Besides, it would be most inconvenient to handle such sharp-pointed screws if they had edges tapering right off to  $55^\circ$ , and, moreover, it would serve no useful purpose, for such a thin edge cannot materially add to the strength of a screw-thread.

The Whitworth thread is stronger than any other, except the buttress one, since its thickness at the bottom of the thread is nearly equal to the pitch of the screw. The compression or grip is considerably greater than with the square thread, because the pitch is only half as much for the same size of bolt. The efficiency of the Whitworth V-thread as a means of transmitting motion is, however, small, since the reaction being at right angles to the face of the thread, a large part of the force employed in turning the screw is expended in tending to burst the enveloping nut. This very inefficiency, however, adds to its utility as a binder for all kinds of machinery, since a properly fitted nut when once screwed down, will not run back or overhaul, unless the pitch be very great and the threads be well oiled.

**Seller's V-Thread.**—In the United States of America, Seller's

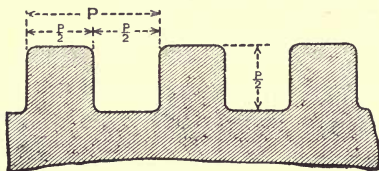


SELLER'S V THREAD.

V-thread is used. It differs from the Whitworth V-thread in that the angle between the opposite sides of the thread and between the spaces is  $60^\circ$  instead of  $55^\circ$ , also the depth is reduced by a sharp flat top and bottom, equal to one-eighth of the total depth, instead of being rounded. This is rather a curious

divergence from the usual American practice, where almost all other parts of their excellent machine tools are beautifully rounded off by symmetrical curves.

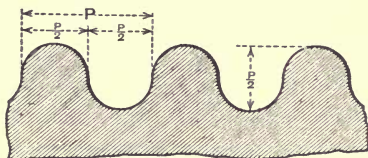
**The Square Thread.**—Since the bearing surface in this thread is very nearly at right angles to the direction of pressure and resistance it is much used for transmitting motion. Of the



SQUARE THREAD.

force applied to turn this screw there is only a small percentage dissipated in tending to burst the nut ; consequently, its efficiency is greater than that of the V-thread. As will be seen from the accompanying figure, the thickness of the thread and the width of the space are made equal, in single-threaded screws, therefore the shearing thickness is greatly reduced, and consequently its strength is less than the V-thread. The durability is, however, greater than in any other form of screw, for there is a larger bearing surface presented in the best manner to resist pressure.

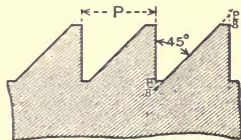
**The Rounded Thread**—This form is simply a modification of the square thread, in order to facilitate the quick engaging and



ROUNDED THREAD

disengaging of a leading motion screw by its nut in machine tools, or where a screw has to be subjected to rough usage. Its efficiency and durability are less than the square thread, but its strength is much greater, since the shearing thickness is greatly increased by the fillets at the bottoms of the thread.

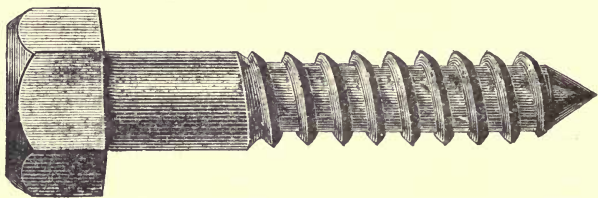
**The Buttress Thread.**—In such cases as the raising and lowering of heavy guns for the purposes of sighting and loading them, where the pressures are always in one direction, then this form of thread is adopted, because its strength is a maximum, the loss due to friction is a minimum, and there is very little tendency to burst the nut. The efficiency is quite equal to that of the square thread, although the durability is lessened by the fact that a certain amount of wear would diminish the depth of the thread. The strength is, however, nearly double, since the shearing thickness is double. It therefore possesses the advantages of the V and the square thread where pressures have to be applied in one direction.



BUTTRESS THREAD.

A slight modification of the buttress thread is used for wood screws. These bolts take a very firm hold of any material into which they can be screwed. Consequently, they are used for screwing thick planks of wood together, and binding down plates or

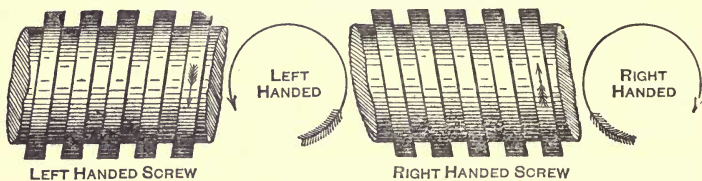
other planks where vibration and stresses would start and lessen the grip of the ordinary V-thread. They are much used by ship



COACH OR WOOD SCREW WITH SEMI-BUTTRESS THREAD.

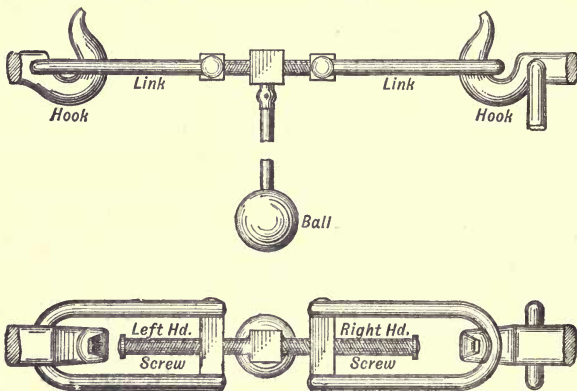
carpenters and erectors of light scaffolding, and are sometimes called holding-down bolts.

**Right- and Left-hand Screws.**—A right-hand screw, when being turned forward or into a nut, rotates in a right-handed way or in the direction of motion of the hands of a watch, whereas



a left-hand screw moves in the opposite or left-handed direction, as shown by the direction of the circular arrows in the above figure.

The Screw-coupling for Railway Carriages is a very good



SCREW COUPLING FOR RAILWAY CARRIAGES.

example of the use of right- and left-hand screws. When two carriages are brought together, the free link hanging from the hook of one of them is placed on the hook of the other one. The porter then turns the central lever by rotating the ball in a circle, thereby screwing both the right- and the left-hand screws into their respective nuts, which consequently draws the hooks toward each other, and couples the carriages tightly together.

EXAMPLE.—If the pitch of each screw is  $\frac{1}{2}$ ", the length of the lever arm or distance from the axis of the screw to the centre of the ball is 14"; and if the railway porter pulls the ball with a force of 40 lbs. when the carriages are brought tightly together, what will be the tension on the screw threads?

ANSWER.—Here  $p = \frac{1}{2}$ ";  $b = 2\pi R = 2 \times \frac{22}{7} \times 14" = 88"$ ;  $P = 40$  lbs.

The formula for the ratio of  $P$  to  $W$  in the case of a single screw given in this Lecture is

$$\frac{P}{W} = \frac{p}{b}, \text{ or } W = \frac{P \times b}{p}$$

$$\therefore W = \frac{40 \times 88}{\frac{1}{2}} = 7040 \text{ lbs.}$$

But there are two screws, and for every complete turn made by  $P$ , the stress  $W$  would be moved through twice the pitch of one screw or through  $2 \times \frac{1}{2} = 1$ ".

$$\therefore W = \frac{P \times b}{2p} = \frac{40 \times 88}{1} = 3520 \text{ lbs.}$$

NOTE.—We may answer this question directly from the "*Principle of Work*." Students should be trained to work out each question from *first principles* rather than from formulæ; for, by a too free use of formulæ they are apt to lose sight of principles.

Let the lever make *one complete turn*, then *each* nut will advance along its own screw a distance *equal to the pitch*. Therefore the two nuts, and consequently the two carriages, will be brought nearer by a distance equal to *twice the pitch*, or,  $= 2 \times p$ .

*By the principle of work, and neglecting friction—*

Work got out = Work put in

$$\text{Or, } \quad \quad \quad W \times 2p = P \times 2\pi R$$

$$\therefore \quad \quad \quad W = \frac{P \times 2\pi R}{2p}$$

$$\text{Or, } \quad \quad \quad W = \frac{40 \times 2 \times \frac{22}{7} \times 14"}{2 \times \frac{1}{2}} = 3520 \text{ lbs.}$$

Single, Double, and Treble-threaded Screws.—As has been previously stated, both the efficiency and the forward distance traversed in a single turn of a screw are directly as the pitch of

the thread, but the strength is proportional to the area of its cross section. Now, if for any purpose requiring a rapid movement of the nut or of a screw, the pitch must be increased; and if the screw consisted of a single-threaded square one, where the depth, thickness of the thread, and the width of the groove are each equal to half the pitch, the strength of the shaft upon which the screw is cut would be unnecessarily reduced. If the groove be made shallower and narrower, then two threads with two spaces having the same pitch as the single one, can be cut upon it so as to present about the same area of bearing surface to the pressure and at the same time afford quite as great a shearing thickness without interfering with the velocity ratio.\* If a very great velocity ratio should be required, then three or more threads with corresponding grooves may be cut in the shaft and nut.

**Backlash in Wheel and Screw-Gearings.**—Backlash is the slackness between the teeth of wheels in gear or between a screw and its nut. Suppose that two wheels are in gear, and that you move one of them in a certain direction until it turns the other, and then reverse the motion; if you can now move the pitch circle through, say,  $\frac{1}{8}$  inch, before the second wheel responds, this distance is the amount of backlash. In the same way, suppose you turn a screw in one direction until its nut moves, and then reverse the motion, the angle or proportion of a turn which you can now make before the nut responds, is the backlash of the screw and its nut. If a great amount of backlash be present in wheel-gearing, it causes vibration and a disagreeable rattling noise; and where severe stresses and sudden stoppages are common, the teeth are liable to be stripped. It can only be thoroughly prevented by cutting the teeth most accurately of the best rolling contact form by a tooth-cutting machine. All screws and nuts that are much worked are liable to backlash as they become worn, although when new they may have been very free from it, so that the best way of taking up the slack is to form the nut in two parts with flanges connected by screw-bolts, which may be tightened from time to time so as to take up the wear, and thus keep one side of the threads in one half of the nut, bearing hard against one side of the threads of the screw, and those in the other half against the other side.

\* The screw of the fly-press, figured on p. 249, is a double-threaded one.

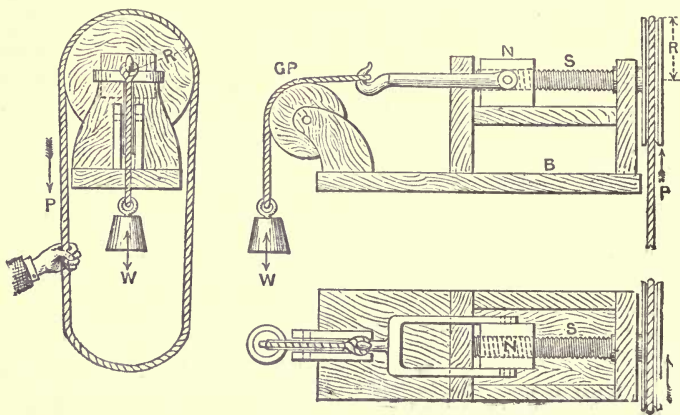
## LECTURE XIV.—QUESTIONS.

1. Explain how a screw is a combination of the lever and inclined plane, and illustrate your remarks. Find the theoretical advantage or ratio of  $W$  to  $P$  in the case of a screw of 1 inch pitch and 3.2 inches diameter; if the lever or spanner key be 7 feet long. *Ans.* 528:1.
2. Given a cylinder and a sheet of paper of sufficient size to cover the cylindrical surface, show how you would trace an evenly pitched spiral or screw line on the cylinder. Mark on your sketch the pitch, circumference, and angle of the screw-thread.
3. Trace a screw-thread line on a cylinder. Draw a triangle to represent the pitch, circumference and angle of the thread, and show the direction of all the forces on the supposition that there is a total pressure,  $W$  lbs., on the end of the cylinder acting parallel to its axis and balanced by a force,  $P$  lbs., acting at its circumference in a plane at right angles to the axis, with a total friction of  $F$  lbs. on the screw-thread.
4. What are the essential characteristics of a screw-thread? Upon which of these do (1) the efficiency, (2) the strength, (3) the durability of a screw depend?
5. Sketch and describe all the forms of screw-threads which you have seen in practice. State their representative advantages and disadvantages, and for which kind of work each kind is most suitable.
6. Define the pitch of a screw. In the Whitworth angular screw-thread, what is the angle made by opposite sides of the thread? To what extent is the thread rounded off at the top and bottom? Distinguish between a *single* and a *double-threaded* screw; in what cases should the latter be used? Why are holding down bolts made with angular threads?
7. Distinguish between a right-handed and a left-handed screw. Sketch the screw-coupling which is commonly used to connect two railway carriages, and explain the action of the combined screws. If the pitch of each screw is  $\frac{3}{8}$  inch and the lever-arm from the axis of the screw to the centre of the ball is 12 inches, with what force will the carriages be pulled together by a force of 50 lbs. applied to the ball on the end of the arm? *Ans.* 5028 lbs.
8. Draw a single, double, and treble square-threaded screw to a  $\frac{1}{16}$ th scale, where the outside diameter of the screw-thread is 10 inches and the pitch 6 inches. Explain the advantages of using a double or treble thread instead of a single one for transmitting rapid motion against a considerable resistance.
9. Why is the angular-threaded Whitworth or Seller's screw better adapted than the square, rounded, or buttress thread for the bolts which are used to bind pieces of machines, &c., together?
10. What is meant by *backlash*? How may backlash be prevented in a crew, and in wheel gearing?

## LECTURE XV.

**CONTENTS.**—Efficiency, &c., of a Combined Lever, Screw, and Pulley Gear—  
 Example I. — Bottle Screw-Jack — Example II. — Traversing Screw-Jack—Screw Press for Bales—Screw Bench Vice—Example III.—  
 Endless Screw and Worm-Wheel—Combined Pulley, Worm, Worm-Wheel and Winch Drum—Worm-Wheel Lifting Gear—Example IV.—  
 Questions.

**Efficiency, &c., of a Combined Lever, Screw, and Pulley Gear.**—Construct an apparatus of the following description, having a horizontal Whitworth V-screw of, say,  $p''$  pitch, with cylindrical ends and flanges supported by bearings, so that the screw cannot move longitudinally, but with a nut free to travel from one end of the screw to the other, along a slide or guide



APPARATUS FOR DEMONSTRATING THE ACTION AND  
 EFFICIENCY OF SCREW GEAR.

## INDEX TO PARTS.

W represents	Weight to be lifted.	P represents	Pull on pulley rope
GP	Guide Pulley.	R	Radius of pulley.
N	Nut.	B	Base or support
S	Screw.		

which prevents it from turning round. Apply a force,  $P$ , to a rope passed over the V-grooved pulley of radius,  $R$ , keyed to the end of the screw shaft, until it moves the nut with the hook, rope, and weight,  $W$ , attached thereto, as shown by the accompanying side elevation, plan and end view of the apparatus.\*

EXAMPLE I.—If the radius,  $R$ , of the turning-pulley be 12", the pitch,  $p$ , of the screw 1", and the gross pull,  $P$ , required to lift a weight of 100 lbs. be 4 lbs.: find (1) the velocity ratio; (2) the theoretical advantage; (3) the working advantage; (4) the work put in to lift  $W$  1 foot; (5) the work got out; (6) the percentage efficiency.

ANSWER.—We have got in this question all the necessary data required to find the various answers except  $n$ , the number of turns which the screw will have to make in order to lift  $W$  1 foot. Since the pitch of the screw is 1", each turn thereof will elevate or lower the weight 1", according as it is turned the one way or the other; consequently, if the screw makes 12 turns, the nut and the weight will move through 12", therefore  $n=12$  turns.

---

\* It is evident that, in addition to the friction between the screw and the nut, there is friction at the several bearings, at the nut slide, and in the bending of the ropes. Consequently, if the student were to place in succession weights at  $W$  of, say, 10, 20, 30, 40 lbs., &c., and ascertain by aid of a Salter's spring balance (hooked into the rope which passes round the turning-pulley), the corresponding pulls required to lift these several weights, and to plot down the results on squared paper with the weights as abscissæ and the pulls as ordinates, and then to draw a line through the intersections of the vertical and horizontal lines drawn through the corresponding values, he would obtain a characteristic curve for the friction of the machine as a whole. If he took the precaution to balance the initial friction of the machine (when there was no weight attached at  $W$ ) by hanging such a small weight at  $P$  as would just move the nut towards the turning-pulley, he would find upon repeating the above experiments (keeping the small additional weight on all the time) and replotting the results as now recorded by the spring balance, that the second frictional curve would approach much nearer to a straight line than the former one. In fact, its deviation therefrom would simply prove that the friction of the movable bearing surfaces *was not directly proportional to the load*. To arrive at the characteristic friction curve for the screw alone, he would have to find out by trial the proportion of the several pulls applied, which were spent in overcoming friction at all other points except between the screw and the nut. To those students who have the time and opportunity for carrying out experiments in applied mechanics, the apparatus illustrated above will prove interesting and instructive. The figures are drawn from the machine constructed in the author's engineering workshop for the purpose of enabling his students to make similar tests to those suggested above. A square, or a rounded, or a buttress-thread may be substituted for the V-Whitworth one, and sound information may thus be obtained about different forms of screws, which will make a stronger and more lasting impression on some students than merely studying books.

By the principle of work :—

$$(1) \text{ The Velocity Ratio } = \frac{\text{P's distance in 1 turn of driving pulley}}{\text{W's distance in the same time}}$$

$$\text{Or, } \cdot \cdot \cdot = \frac{\odot^{\text{co}} \text{ of pulley}}{\text{pitch of screw}} = \frac{2\pi R}{p} = \frac{75.4}{1}$$

$$(2) \text{ The Theoretical Advantage } \cdot \cdot \cdot \} = \frac{\text{Weight lifted if there were no friction}}{\text{Pull applied}}$$

$$\cdot \cdot \cdot = \frac{W_T}{P} = \frac{2\pi R}{p} = \frac{75.4}{1}$$

$$(3) \text{ The Working Advantage } \cdot \cdot \cdot \} = \frac{W}{P} = \frac{100 \text{ lbs.}}{4 \text{ lbs.}} = \frac{25}{1}$$

$$(4) \text{ The Work Put in to lift } W \text{ 1 foot } \cdot \cdot \cdot \} = 2\pi R n P$$

$$\cdot \cdot \cdot = \frac{2 \times 22 \times 12'' \times 12 \times 4}{7 \times 12''} = 301.56 \text{ ft.-lbs.}$$

$$(5) \text{ The Work Got out in raising } W \text{ 1 foot } \cdot \cdot \cdot \} = W \times 1' = 100 \text{ lbs.} \times 1' = 100 \text{ ft.-lbs.}$$

$$(6) \text{ The Percentage Efficiency } \cdot \cdot \cdot \} = \text{Efficiency} \times 100$$

$$\cdot \cdot \cdot = \frac{\text{Work got out}}{\text{Work put in}} \times 100$$

$$\cdot \cdot \cdot = \frac{100 \text{ ft.-lbs.}}{301.56 \text{ ft.-lbs.}} = 33.7 \% *$$

**Bottle Screw-Jack.**—The importance of the screw as a simple machine for exerting great pressures, is very well exemplified by the screw-jack. This tool is used for replacing locomotives and railway carriages upon their rails, for elevating heavy girders into position, or for overcoming any great resistance through a small space which cannot be effected by a labourer and a lever. As will be seen from the accompanying figure it consists of a strong hollow bottle-shaped casting, with a projecting handle for facilitating the carrying of the tool from one place to another. In the upper end of the casting a square-threaded screw is cut

\* It is evident that with such a low percentage efficiency the weight when hanging from the rope will not be able to overhaul the machine. The student can calculate what pitch of screw would be required with the same co-efficient of friction before overhauling could take place.

parallel with the axis, and into this nut there is fitted a steel screw terminating in a spherical head, having two holes bored through it at right angles to each other. Into one or other of these holes an iron lever bar is fixed, so that by pulling or pushing on the outer end of the bar the screw is turned, and thus the head is gradually raised from the base. To avoid the tearing, grinding action that would ensue between the head and the object acted upon, the former is provided with a loose crown fitted on a central pin projecting from the round head.

Let  $L$  = Length of the lever arm in inches  
from centre of jack to where the  
force is applied.

„  $p$  = Pitch of screw in inches.

„  $P$  = Pull or push applied at radius  $L$ .

„  $W$  = Weight lifted or resistance overcome.

Then, by the *Principle of Work*, and neglecting friction, we have in one turn of lever—

$$P \times \text{its distance} = W \times \text{its distance}$$

$$\text{Or,} \quad P \times 2\pi L = W \times p$$

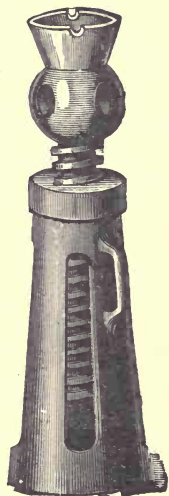
$$\therefore P = \frac{W \times p}{2\pi L}$$

EXAMPLE II.—A weight of 10 tons has to be lifted by a screw-jack, in which the pitch of the screw is  $\frac{1}{2}$ ". What length of lever will be required if a force of 70 lbs. be applied at the end of it? (1) Neglecting friction; (2) if the modulus or efficiency of the tool is only .4.

ANSWER.—(1) By the previous formula (neglecting friction)

$$L_1 = \frac{W \times p}{P \times 2\pi} = \frac{22400 \times .5'' \times 7}{70 \times 2 \times 22} = \frac{560}{22} = 25.45''$$

(2) Taking friction into account we see from the question that the efficiency is = .4, therefore the percentage efficiency is 40, or 60 per cent. of the work put in is lost work required to overcome friction between the screw and its nut. But as the length of the lever is directly proportional to the work put in, the theoretical length of the lever found above is only 40 per cent. of the actual or working length required.

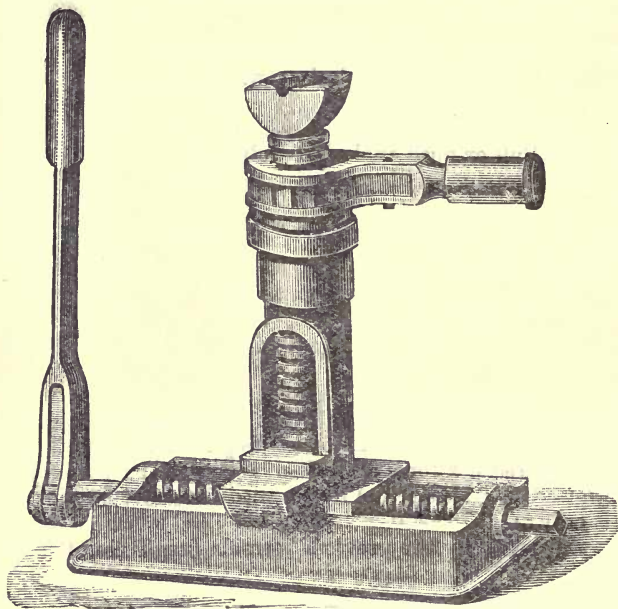


BOTTLE SCREW-JACK.

$$\therefore 40 : 100 :: 25.45 : L_2$$

$$L_2 = \frac{100 \times 25.45}{40} = 63.6''$$

**Traversing Screw-Jack.**—It is very often convenient, when using a strong heavy screw-jack, to be able to move the head a short distance to one side or the other, when near the object to which it is to be applied; or, after having raised a load with one or more jacks, to be able to traverse the jacks forward or backward through a short distance until the load is brought into



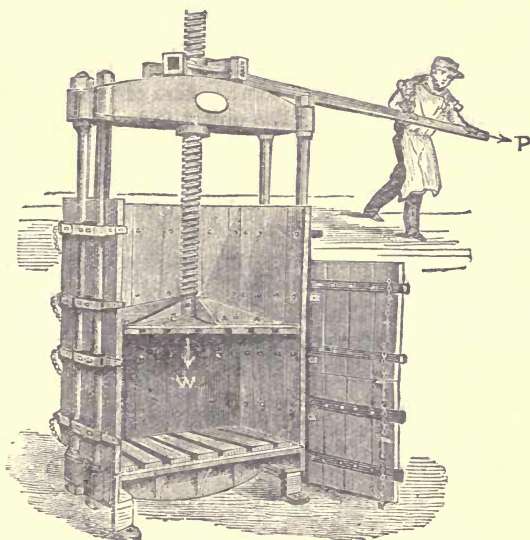
TRAVERSING SCREW-JACK WITH RATCHET-LEVERS.

(By P. & W. MacLellan, Glasgow.)

the desired position. These movements may be effected with a jack of the form shown by the accompanying figure. Further, this jack is provided with a side foot-step attached to and projecting from the lower end of the vertical screw. This foot-step can be placed under the flange of a low beam or rail, where it would be inconvenient or perhaps impossible to get the top head underneath the same. The nut of the horizontal traversing screw is

formed in, or fitted to the bottom of the vertical casting, and this screw is turned by a ratchet-lever which may be slipped on to one or other of the squared ends of its shaft. The upward and downward movement of the vertical screw is also affected by a ratchet-lever, and in this case without turning the screw, for the ratchet-wheel is fixed to the nut of its screw. The pawl of the ratchet-wheel and the vertical screw-nut to be turned round in either direction for elevating or lowering the load.

**Screw Press for Bales.**—When soft goods or hay have to be transported they may be squeezed into small bulk by means of a



SCREW PRESS FOR BALES.

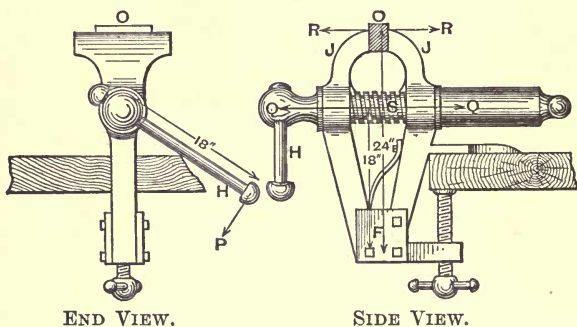
(By Loudon Bros., Glasgow.)

screw press, and bound firmly when under the press, by strips of hoop-iron passed round them and then riveted before the pressure is relieved. The bound bundle is then termed a bale. The operation will be understood by an inspection of the accompanying figure. The loose material is placed in the space between the rigid base and the movable plate of the press, the doors are closed and locked, the pressman applies himself to the end of the lever with a force, *P*, thereby turning the nut of the screw and forcing the movable plate downwards with a pressure, *W*, until the

desired compression of the goods has been attained. The doors are then opened and the strips of hoop iron (which were previously or are now placed in the grooves of the base and upper plate) are brought together and riveted. The lever is then turned in the opposite direction to relieve the pressure, and the bale is removed to the store or ship to make room for another quantity of goods being subjected to a similar action.

The same formula as we used for the screw-jack and for Example I. in connection with the combined lever, screw and pulley gear, naturally applies to this press, and to any similar appliance, such as a letter-copying press.\*

**Screw Bench Vice.**—A bench vice is essentially an instrument for seizing and holding firmly any small object whilst it is being acted upon by a chisel, file, drill, saw, or emery cloth, &c. Looking at the figure which illustrates the following example, it will be seen that the vice is a combination of two levers, a square-threaded screw, and a nut. The object O to be gripped is placed



SCREW BENCH VICE.

between the serrated jaws JJ. The lever handle H, on being turned, forces the screw S into its long nut, and thereby presses forward the outer jaw upon the object, by aid of the flange on the screw-head. This jaw is a lever, having a fulcrum at F, and therefore the pressure on the object is less than that on the screw-collar in the proportion of SF to OF. The bent flat spring between the limbs of the fixed and movable jaws serves to force the movable jaw away from the fixed one when the screw is turned backwards, and thus relieves the object without having to pull

\* Refer to index for page where the illustration of the Fly-press occurs. The statical pressures produced by this machine when used for punching holes, &c., may be treated in the same way.

this jaw back by the hand. It will be observed that the fixed jaw should have been continued to the floor level by a vertical supporting leg, in the case of such a big vice intended for rough heavy engineering work.

EXAMPLE III.—Sketch an ordinary bench vice. Apply the principle of work to find the gripping force obtained when a man exerts a pressure of 20 lbs. at the end of a lever 18 inches long, the screw having four threads per inch, the length from the hinge to the screw being 18 inches, and the length from the hinge to the jaws being 24 inches. (S. & A. Exam. 1892.)

ANSWER.—Let P represent Pull on end of handle  $H = 20$  lbs.

„ Q	„	Resistance offered by screw at S.
„ R	„	Reaction, or gripping force, exerted on object at O.
„ L	„	Length of handle $H = 18$ inches.
„ p	„	Pitch of screw $S = \frac{1}{4}$ inch.

Suppose the handle, H, to make one complete turn under the action of a constant force, P, at the extremity thereof, against a constant resistance, Q, acting along the axis of the screw.

[The student will observe that we suppose the forces P and Q to be constant, which is not correct for such a large movement as a complete turn of the handle, but which may be assumed here for the sake of simplicity. The reason for this is, that the resistance, R, will vary with the compression produced on the object at O. However, the ratio between P and R will remain a constant quantity.]

The work done by P during one turn of handle  $= P \times 2\pi L$

And „ on Q during the same time  $= Q \times p$ .

But, by the Principle of Work—

Work done by P = Work done on Q

$$\therefore P \times 2\pi L = Q \times p$$

Substituting the numerical values—

$$20 \times 2 \times \frac{22}{7} \times 18'' = Q \times \frac{1}{4}''$$

$$\therefore Q = \frac{20 \times 2 \times 22 \times 18 \times 4}{7} = 9051.43 \text{ lbs.}$$

But by the Principle of Moments—

$$R \times FO = Q \times FS$$

$$\therefore R = \frac{18}{24} \times Q = \frac{3}{4} Q$$

$$\text{i.e., } R = \frac{3}{4} \times 9051.43 = 6788.57 \text{ lbs.}$$

**Endless Screw and Worm-Wheel.\*** — When a screw is rotated between fixed bearings so that it cannot move longitudinally, it is called an *endless screw*, because the threads of the screw seem to travel onwards without ending.† When such a screw gears with a toothed wheel, having its teeth set obliquely at the same angle as the threads of the screw so as to bear evenly thereon, the wheel is termed a *worm-wheel*. The endless screw is sometimes called the *worm*, no doubt from its resemblance to that well-known humble animal which, when coiled up for rest, would not turn upon any one unless trod upon.

By this arrangement, motion may be transmitted from one shaft to another at right angles to each other, without any possibility of the machine overhauling; for although the velocity ratio is very great, the efficiency is comparatively small—considerably under 50 per cent. with single-threaded screws—owing to the friction between the worm and the wheel.‡

It is most important for the student to comprehend that if *the screw be a single-threaded one, it must make as many turns as there are teeth on the wheel, for every revolution of the latter*. If the screw is a *double-threaded one*, then for each revolution thereof it drives the wheel through a distance equal to the distance *between two teeth* on the pitch circle, and if *treble-threaded* through the pitches of *three teeth*. Thus, if  $N$  equal the number of teeth in the worm-wheel, then, with a single-threaded screw, for every turn of the same, the wheel will move a distance of  $\frac{1}{N}$ ; with a double-threaded worm  $\frac{2}{N}$ , and with a treble-threaded one  $\frac{3}{N}$  and so on.

The endless screw and worm-wheel is used in a very great variety of circumstances, from the turning of a big marine engine when in port, to the delicate movements in a telescope or a microscope.

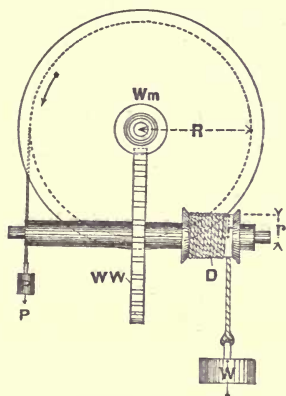
**Combined Pulley, Worm, Worm-wheel and Winch Drum.**—This combination is shown by the accompanying end and side views drawn from an experimental piece of apparatus in

\* Refer to the next figure.

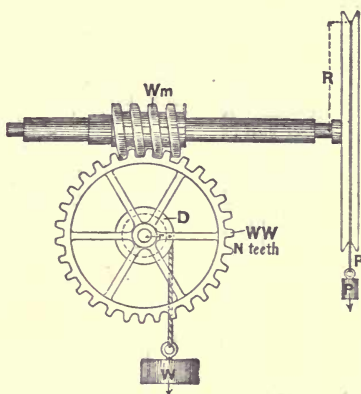
† The term *perpetual screw* would express more exactly its action, for when in motion, it continually screws the worm-wheel round.

‡ The greater the diameter of the screw and the smaller its pitch is, the better will be its bearing on the teeth of the wheel, but then the efficiency will be so small that there will be no chance of overhauling. This is the condition to be observed when the screw is intended to drive the wheel. If, however, it should be required to drive the screw by the wheel, or necessary that overhauling should take place, then the screw must be small in diameter, its pitch very great, and either double or treble threaded.

the Author's Laboratory, which is used by the students for ascertaining the efficiency of the machine, and for finding the co-efficient of friction between the endless screw and worm-wheel.



END VIEW.



SIDE VIEW.

PULLEY, WORM, WORM-WHEEL AND WINCH DRUM.\*

## INDEX TO PARTS.

P	represents Pull applied to pulley.	N	represents Number of teeth in WW.
R	„ Radius of pulley.	D	„ Drum, or diameter of winch barrel.
Wm	„ Worm or endless screw.	r	„ Radius of drum, D.
WW	„ Worm wheel.	W	„ Weight to be lifted.

By the *Principle of Work* (neglecting friction), if the drum, D, makes one turn, and if the worm be a single-threaded screw,

$$P \times \text{its distance} = W \times \text{its distance}$$

$$\text{Or,} \quad P \times 2\pi RN = W \times 2\pi r$$

(divide both sides by  $2\pi$ )

$$P \times RN = W \times r$$

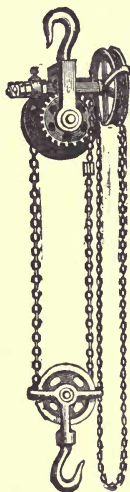
$$\therefore \frac{P}{W} = \frac{r}{RN} \dagger$$

\* Find out what is wrong with the above drawing.

† It will be evident to the student that, given any four of these five values, he can change this formula so as to find the fifth one; and, that he can experiment with this machine in precisely the same way as has been already explained in the case of the wheel and axle, block and tackle, Weston's pulley block and screw, &c., to ascertain its working advantage, co-efficient of friction and efficiency.

**Worm-wheel Lifting Gear.**—The accompanying figure shows a practical application of the endless screw and worm-wheel for the same purpose as the Weston's differential block is used—viz., the lifting of weights without fear of the tackle overhauling. A light-driving endless chain passes over a V-grooved pulley having ridges or teeth on the inner sides of the grooves, so as to fit the pitch of the links of the chain. This pulley is keyed to the outer end of a worm spindle, whose screw gears with a worm-wheel fixed to or cast along with a second V-grooved ridged pulley or drum, over which is passed the movable end of a heavier lifting chain after it has been reeved under a snatch-block pulley. In fact, it is simply the previous experimental apparatus in a handy and compact form.

WORM-WHEEL LIFTING GEAR. (By Holt &amp; Willets.)



**EXAMPLE IV.**—If in lifting tackle of the above description the driving pulley has a radius  $R = 5''$ , the number of teeth in the worm-wheel  $N = 20$ , and the driven pulley a radius  $r = 5''$ ; what weight suspended from the snatch-block hook could be lifted by a force of 10 lbs. applied to the forward side of the light chain—(1) Neglecting friction, (2) if the modulus or efficiency of the whole apparatus were only .25?

ratus were only .25?

**ANSWER.**—(1) Applying the previous formula, and taking account of the fact that the lifting chain is combined with a snatch-block, we have—

$$W = 2 \frac{P \times R \times N}{r} = \frac{2 \times 10 \times 5 \times 20}{5''} = 400 \text{ lbs.}$$

(2) Owing to friction, weight of chain and snatch-block, the actual result obtainable is only .25, or 25 per cent. of this theoretical value; consequently

$$\begin{aligned} 100 : 25 : 400 : x \\ x = \frac{25 \times 400}{100} = 100 \text{ lbs.} \end{aligned}$$

## LECTURE XV.—QUESTIONS.

1. A horizontal screw, of 1 inch pitch, is fitted to a sliding nut which is pulled horizontally by a cord passing over a fixed pulley, and having a weight,  $W$ , attached to it. To the free end of the screw there is fixed a pulley of 20 inches diameter, from the circumference of which a weight,  $P$ , hangs by a cord. Find the ratio of  $P$  to  $W$ . *Ans.*  $1 : 62.8$ .

2. In a set of combined lever, screw, and pulley gear, like that illustrated before Example I. in this Lecture,  $R = 6"$ ,  $P = 2$  lbs.,  $W = 50$  lbs., and the pitch of the screw is such that there are 2 threads to the inch; find (1) velocity ratio, (2) theoretical advantage, (3) working advantage, (4) work put in to lift  $W$  1 ft., (5) work got out, (6) percentage efficiency. *Ans.* (1)  $75.4 : 1$ ; (2)  $75.4 : 1$ ; (3)  $25 : 1$ ; (4)  $150.8$  ft.-lbs.; (5) 50 ft.-lbs.; (6) 33.1 per cent.

3. Describe, with sketches, the construction of an ordinary lifting jack in which the weight is lifted by means of a screw and nut. If the screw be 1 inch pitch, the lever 20 inches long, and the pressure applied at the end of the lever be 30 lbs.; what weight can be lifted (neglecting friction)? (Take  $\pi = 3.1416$ .) *Ans.* 3770 lbs.

4. In a screw-jack, where a worm-wheel is used, the pitch of the screw is  $\frac{1}{8}$  inch, the number of teeth on the worm-wheel is 16, and the length of the lever is 10 inches; find the gain in pressure. *Ans.*  $P : W :: 1 : 1609$ .

5. What practical objection is there to the use of screw gear of any description for obtaining great pressure? Take for example the case of the screw-lifting jack. Sketch in vertical section and plan, and describe, a traversing one to lift say 20 tons. Explain how the screw of the jack is raised and lowered without being turned round.

6. Sketch and describe the construction and action of a screw press for pressing goods so as to make them into bales for transport. What force must be applied at the end of a screw press lever 8' 4" in length, in order to exert on the goods a total pressure of 22,000 lbs. when the pitch of the screw is 1"? If 60 per cent. be lost in friction, what pressure would result from the application of this force on the lever? *Ans.* 35 lbs.; 8800 lbs.

7. Sketch an ordinary bench vice. Apply the principle of work to find the gripping force obtained when a man exerts a pressure of 15 lbs. at the end of a lever 15 inches long, the screw having 5 threads per inch, the length from the hinge to the screw being 12 inches, and the length from the hinge to the jaws being 16 inches. *Ans.* 5303.6 lbs.

8. Explain, with a sketch, the manner in which the principle of work is applied in determining the relation of  $P$  to  $W$  in the case of the endless screw and worm-wheel. The lever handle which works the screw being 14" long, the number of teeth in the worm-wheel 20, and the load being a weight of 1000 lbs. hanging upon a drum 12" diameter on the worm-wheel shaft, find the force to be applied at the end of the lever handle in order to support the weight. *Ans.* 21.43 lbs.

9. Explain the mechanical advantage resulting from the employment of an endless screw and worm-wheel. The lever handle which turns an endless screw is 14" long, the worm has 32 teeth, and a weight  $W$ , hangs by a rope from a drum 6" diameter, whose axis coincides with that of the worm-wheel. If a pressure  $P$  be applied to the lever handle, find the ratio of  $P$  to  $W$ . *Ans.*  $P : W :: 3 : 448$ . If in this question the worm be changed to (1) a double, and (2) a treble-threaded screw, what will be the respective ratios of  $P$  to  $W$ ? *Ans.* (1)  $1 : 74.7$ ; (2)  $1 : 49.8$ .

10. Describe, with the aid of a sketch, how the pressure upon the book is obtained in an ordinary copying-press. What should be the length of the double-ended lever, supposing that the force be always applied simultaneously to both ends of the lever, in order that with a screw having 6 threads to the inch, the combination may have a mechanical advantage of 216? *Ans.* Length of lever 6 inches.

11. Sketch in vertical section the common screw or bottle lifting jack. The lever in such a jack is single ended, and measures 24 inches in length, the pitch of the screw is  $\frac{3}{8}$  inch. What force applied at the end of the lever would be required to raise a load of 22 cwt., the effect of friction being neglected? *Ans.* 6·12 lbs.

12. Describe, with a sketch, the construction of an ordinary screw-jack with a lever handle and screw. If the pitch of the screw be  $\frac{3}{8}$  inch, the length of the lever handle 29 inches; what load could be lifted, neglecting friction, by a force of 19 lbs. applied to the end of the lever handle? *Ans.* 2 tons.

13. Describe either a screw-jack (pitch of screw  $\frac{1}{2}$ ", handle 19" long) or a simple winch for lifting weights up to 1 ton by one man. What is the mechanical advantage neglecting friction? Describe what sort of trial you would make to find its real mechanical advantage under various loads, and what sort of result would you expect to find?

14. The diameter of the safety valve of a steam boiler is 3 inches. The weight on the end of the lever is 55 lbs., and the distance from the centre of the valve to the fulcrum is 4·5 inches. What must be the length of the lever from the centre of the valve to the point of suspension of the weight, in order that the valve will just lift when the pressure of steam in the boiler is 80 lbs. per square inch? Neglect the weight of lever and valve. *Ans.* 41·3 inches.

15. Make a correct draughtsmanlike drawing of the next to the last figure in this lecture. Point out distinctly what is wrong with the end view if the side view be taken as correct.

16. Describe how you would proceed to determine experimentally for a screw-jack (1) the velocity ratio, (2) the actual effort required to lift a given load. (B. of E., 1905.)



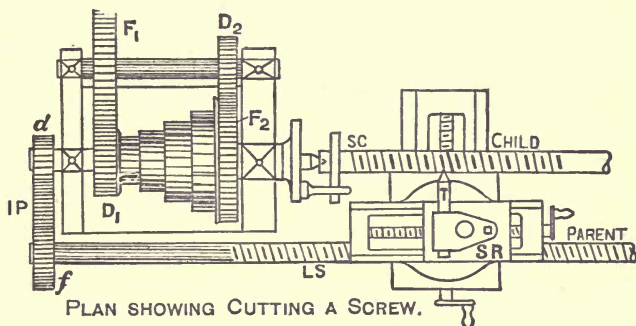
## LECTURE XVI.

**CONTENTS.**—General Idea of the Mechanism in a Screw-cutting Lathe—Motions of the Saddle and Slide Rest—Velocity Ratio of the Change Wheels—Rules for Calculating the Required Number of Teeth in Change Wheels—Examples I. II.—Movable Headstock for a Common Lathe—Descriptions of a Screw-cutting Lathe and of an Electrically Driven Hexagon Turret Lathe, with Frontis-Plates and complete sets of Detail Drawings—Questions.

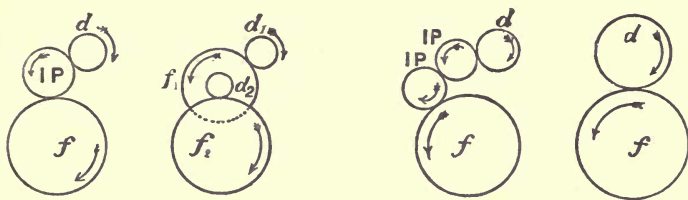
**General Idea of the Mechanism in a Screw-cutting Lathe.**—We will devote this Lecture to giving a general idea of the mechanism by which screws are cut in lathes, and the velocity ratio of the screw to be cut to the leading screw, together with a description of a complete set of illustrations prepared from working drawings of a new self-acting screw-cutting lathe.

Referring to the following figure, and to the general view of the 6-inch centre screw-cutting lathe (further on), it will be seen that the round metal bar on which the screw is to be cut is placed between the steel centres of the fixed and movable headstocks of the lathe. This bar has an eye-catch on its end next to the fixed headstock, which engages with a driving-stud connected to the face-plate. In order to obtain the necessary force to cut the screw, and to reduce the speed of the workshop motion shafts (in the case of a power lathe, or of the treadle shaft in the case of a foot lathe) to the required velocity, the fixed headstock is supplied with back motion gearing. The back wheels may be put into or out of gear with the lathe spindle wheels at pleasure, by a simple eccentric motion (in the case of the lathes herein illustrated), or, as is sometimes effected, by sliding the back shaft forward, so that its wheels clear those on the lathe spindle, and then fixing it there, by a tapered pin fitting through a hole in the framing and a groove cut in the shaft. But, when the back motion is required for the purpose of making a slow heavy cut, the follower  $F_1$  is thrown out of gear with the stepped cone pulley, so that the driver  $D_1$  (which is keyed to the cone) may turn the follower  $F_1$ ; and the driver  $D_2$  (which is keyed to the same spindle as  $F_1$ ) rotate the follower  $F_2$  (which is keyed to the lathe spindle), and hence revolve the face-plate and the bar, out of which the screw is to be formed.

On the back extension of the lathe spindle there is fixed a change wheel or small driver,  $d$ , which gears with a follower,  $f$  (keyed to the left-hand end of the leading or parent screw), either direct in the case of the cutting of a very finely pitched left-handed screw; or, through the intervention of a transmitting, or



PLAN SHOWING CUTTING A SCREW.



END VIEWS SHOWING CHANGE WHEELS.

FOR RIGHT-HANDED SCREWS.

FOR LEFT-HANDED SCREWS.

GENERAL IDEA OF MECHANISM IN A SCREW-CUTTING LATHE.

#### INDEX TO PARTS.

SC represents	Screw to be cut, or child.	$F_1, F_2$ represents	Followers of fixed headstock.
LS	Leading screw, or parent.	$d, f$	Driver and follower of change wheels.
SR	Slide rest.	IP	Idle pinion of change wheels.
$D_1, D_2$	Drivers of fixed headstock.		

what is technically termed an idle pinion,  $IP$ , in the case of a medium-pitched right-hand screw. (See also the end views of the change wheels above.)

It will therefore be seen that there are two independent motions to be considered—(1) the reducing gear from the speed of the

driving cone to that of the lathe-spindle or bar to be operated upon; and (2) the multiplying or reducing gear between the lathe-spindle and the leading screw. The former of these will be at once understood from the figures, and from what was said in regard to wheel-gearing in Lecture XII.

We shall now consider the second motion. Remembering that the pitch of the *parent* or leading screw is fixed and unalterable, and that on its truth depends to a large extent the accuracy with which the *child*, or screw to be cut, can be formed, it will be clear that we have only to connect these two parallel shafts with suitable gearing in order to transmit, by aid of the "*copying principle*" the characteristics of the parent to the child.\* This may be done in an equal or magnified or diminished degree, according as the pitch of the screw to be cut is equal or greater or less than that of the leading screw.

**Motions of the Saddle and Slide Rest.**—The base of the slide rest, or the saddle as it is technically termed, bears upon and is guided by the truly-planed shears (or upper framing of the lathe) parallel to the line joining the centres of the fixed and movable heads. In turning a *right-handed screw* the saddle is moved *from the movable headstock towards the fixed one, or from right to left*, by clamping it to the leading or guiding screw with a split nut attached to the under side of the saddle. In cutting a *left-handed screw* the saddle is moved by the same means, but in the opposite direction,—*i.e., from left to right*. In other words, it travels in the direction towards which the threads of the screw to be cut are inclined forward.

To the upper side of the saddle is bolted the slide-rest surmounted by the tool-holder. The rest is provided with two independent sliding motions, each actuated by a hand-turned screw, and guided by a true plane surface with dovetailed sides. These motions (for the purposes of turning parallel work and screws) are fixed at right angles to each other, the lower one being parallel to the centre line of the lathe, and the upper one at right angles thereto. Both motions are therefore independent of each other and of the sliding motion of the saddle. The turner is thereby enabled to adjust the cutting tool with great delicacy and accuracy with reference to the job to be operated upon, irrespective of the automatic travel of the supporting saddle.

**Velocity Ratio of the Change Wheels.**—As has been mentioned already, the change wheels are interposed between the

\* It is reported that Sir Joseph Whitworth, feeling the importance of a *thoroughly true leading screw*, spent an immense deal of money upon the scraping and finishing of a parent screw for a first-class lathe, from which many of the best screws in this country have been copied.

back end of the lathe spindle and the leading screw, for the purpose of transferring motion to the saddle, and determining, that the cutting tool shall be moved through a definite pitch for each rotation of the cylinder to be turned or screwed. Every turn of the leading screw moves the saddle and cutting tool through a distance equal to its pitch, and consequently if the bar to be screwed, turns at the same rate as the leading screw, the pitch of the screw cut upon it, will be the same as that of the leading screw. If it moves faster than the leading screw, the pitch will be less; and if slower, the pitch will be correspondingly greater. It therefore follows as a matter of course, that if we fit wheels on the lathe spindle and on the leading screw of the same diameter, or having the same number of teeth, the screw being cut will have the same pitch as the leading screw. If we fix a small pinion, or one with few teeth, on the lathe spindle and a wheel of large diameter, or many teeth on the leading screw, the pitch of the screw to be cut will be small, compared with that of the leading screw. Or, if the number of turns per minute of the leading screw be greater than that of the screw being cut, the pitch of the latter will be greater than that of the former, and *vice versa*.\*

**Rules for Calculating the Required Number of Teeth in Change Wheels.**—The following rules simply express the previous reasoning in the form of proportion. In applying them, the student should again refer to the end views of the change wheels in the first figure of this Lecture.

$$\frac{\text{Pitch of screw to be cut}}{\text{Pitch of guiding screw}} = \frac{\text{No. of teeth in 1st driver} \times \text{No. in 2nd driver.}}{\text{No. of teeth in 1st follower} \times \text{No. in 2nd follower.}}$$

Let  $p_c$  = Pitch of screw to be cut in inches, or fraction of inch, between two threads.

„  $p_g$  = Pitch of guiding screw „ „ „ „

„  $d_1, d_2$  = Diameters or number of teeth in drivers.

„  $f_1, f_2$  = Diameters or number of teeth in followers.

Then,

$$\frac{p_c}{p_g} = \frac{d_1 \times d_2}{f_1 \times f_2}.$$

Or,

$$p_c \times f_1 \times f_2 = p_g \times d_1 \times d_2,$$

\* What was said in Lectures XII. XIII. and XIV. enables the student to see clearly the velocity ratio between the cut screw and the leading screw. We need scarcely remind the student that the above statements refer to the pitch of a screw as the distance between two consecutive threads, and not to the number of threads per inch. If the number of threads per inch of its length are taken as the pitch, instead of the distance between two threads, the reverse ratio will hold good. Since a pitch of  $\frac{1}{4}$ " means 4 threads to the inch, a pitch of  $\frac{1}{3}$ " means 3 threads to the inch, and a pitch of  $\frac{1}{2}$ " means 2 threads to the inch. Or, the number of threads per inch is *inversely* proportional to the distance between two consecutive threads of the screw.

When the train of wheels is a compound one, as in this case, the two intermediate multiplying or reducing wheels,  $f_1$  and  $d_2$ , are fixed to any outstanding movable arm or quadrant at the left-hand end of the lathe, so as to bring them into gear with  $d_1$  and  $f_2$ . (See second view of the previous figure.)

If the train of wheels is a simple one, as in the first, third, and fourth views referred to above, where there is only one driver,  $d$ , and one follower,  $f$ , with, when necessary, one or more idle pulleys, IP, simply for the purpose of connecting  $d$  and  $f$  and of giving  $f$  the desired direction of rotation, then—

$$\frac{p_c}{p_g} = \frac{d}{f}, \text{ or } p_c \times f = p_g \times d.$$

Should the pitch of a screw be expressed by the *number of threads per inch of its length*—as is usually the case in tables of screws and change wheels—then you can either convert this number into the pitch proper, by taking its reciprocal—(i.e., by making the number of threads per inch the denominator of a fraction, with 1 for the numerator) or you may say—

Let  $t_c$  = Threads per inch of screw to be cut.

„  $t_g$  = Threads per inch of guiding screw.

Then, since the number of threads per inch are inversely proportional to the distance between any two consecutive threads,

$$\frac{t_c}{t_g} = \frac{p_g}{p_c} = \frac{f_1 \times f_2}{d_1 \times d_2}$$

Or, . . .  $t_c \times d_1 \times d_2 = t_g \times f_1 \times f_2$

If the train is a simple one, then

$$\frac{t_c}{t_g} = \frac{f}{d}; \text{ or, } t_c \times d = t_g \times f$$

**EXAMPLE I.**—The lathe illustration further on, has a guiding screw of  $\frac{1}{4}$ " pitch, or 4 threads to the inch. Calculate the number of teeth in the change wheel to be fixed to the end of the guiding or leading screw in order to cut a screw of 8 threads to the inch when the driver on the lathe-spindle has 40 teeth.

Compare the answer with the change-wheel table printed above the general view of the screw-cutting lathe, further on in this Lecture.

**ANSWER.**—Here  $t_c = 8$ ;  $t_g = 4$ ;  $d = 40$ ; and you are required to find  $f$ .

By above formula,

$$\frac{t_c}{t_g} = \frac{f}{d}; \quad \text{or, } \frac{8}{4} = \frac{f}{40} \therefore f = \frac{8 \times 40}{4} = 80 \text{ teeth.}$$

By using the previous formula, we have  $p_c = \frac{1}{8}"$  and  $p_g = \frac{1}{4}"$

$$\therefore \frac{p_c}{p_g} = \frac{d}{f}; \quad \text{or, } \frac{\frac{1}{8}}{\frac{1}{4}} = \frac{40}{f} \therefore f = \frac{\frac{1}{4} \times 40}{\frac{1}{8}} = \frac{8 \times 40}{4} = 80 \text{ teeth.}$$

It is at once evident from this example that you avoid having to multiply and divide by sometimes awkward fractions if you consider the number of threads per inch as the measure of the pitch of the screw, instead of the distance between two threads.

**EXAMPLE II.**—The guiding screw of a lathe is  $\frac{1}{2}"$  pitch, and you are required to cut screws of  $\frac{1}{10}"$  and  $\frac{1}{20}"$  pitch respectively. Determine the number of teeth in the follower, given the use of a driver having 20 teeth.

**ANSWER.**—For a screw of  $\frac{1}{10}"$  pitch, or 10 threads per inch, and using a driver of 20 teeth, we get by the above formula for a simple train,

$$\frac{t_c}{t_g} = \frac{f}{d}; \quad \text{or, } \frac{10}{2} = \frac{10}{2} \times \frac{10}{10} = \frac{100}{20} = \frac{f}{d}$$

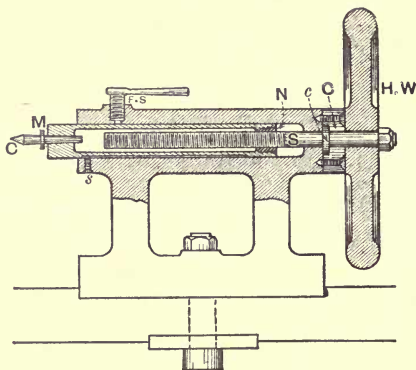
For a screw of  $\frac{1}{20}"$  pitch the number of threads per inch will be 20, and using a driver of 20 teeth, we find from the formula for a compound train—

$$\frac{t_c}{t_g} = \frac{f_1 \times f_2}{d_1 \times d_2} = \frac{80 \times 100}{20 \times 40} = \frac{f_1 \times f_2}{d_1 \times d_2}$$

Here we multiplied numerator and denominator by 20, in order to obtain suitable wheels, of which  $d_1$  will have 20 teeth. (See in the previous figure the second of the end views showing change wheels.)

**Movable Headstock for a Common Lathe.** — Before describing a complete screw-cutting lathe we will explain the use and construction of this part of a common small lathe for ordinary work. As will be seen from the accompanying rough sketch, it consists of a cast-iron poppet-head planed on its under side, so as to engage the breadth of the top of the shears. It may be bolted thereto in any desired position (along the length of the bed) by an underneath iron plate placed across the shears, and a single vertical bolt. The upper portion of the head is cylindrical, and is bored for about seven-eighths of its length to receive a round

hollow steel mandril, *M*, and for the remaining one-eighth to receive the spindle *S*. The mandril is fitted in front with a tapered centre, *C*, and behind with a screw nut, *N*. The centre is for carrying one end of the job to be operated upon by the turning tool, and the nut is for engaging the screwed part of the spindle *S*. On the back end of the spindle there is a collar, *c* (kept in position by a larger collar or guard, *G*, with small screws), and a hand-wheel, *HW*.\* Consequently, by turning this wheel in



MOVABLE HEADSTOCK FOR A COMMON LATHE.

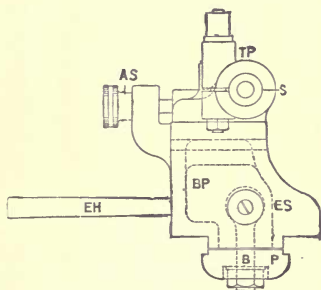
one direction the mandril and its centre are forced forward, and when moved in the opposite direction they are screwed backwards. To prevent the mandril turning round, it is fitted with a longitudinal slot on its underside, into which fits the flattened or rounded end of a small screw, *s*. A fixing stud, *FS*, with a handle, enables the mandril to be clamped to the head when it has been adjusted by the hand wheel and screwed spindle.

**Description of a Screw-cutting Lathe.**—By the favour of Messrs. John Lang & Sons we are enabled to give a general view, with a complete set of reduced working drawings, carefully indexed to every detail, of the very strong and superior 6-inch centre screw-cutting lathe, lately presented to the Author's Electrical Engineering Laboratory and Engineering Workshop by Mr. Andrew Stewart, of Messrs. A. & J. Stewart, and Clydesdale. This lathe weighs, with all its chucks and supernumerary parts, over 15 cwt. It has a bed 6 feet long, and admits a bar 3 feet

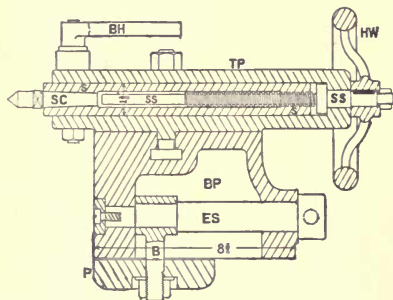
\* This arrangement of collar and guard is neither good nor strong, although frequently adopted in the case of small foot-lathes. The collar should be inside the bored head, behind the nut *N*.

2 inches between its centres. The bed is  $9\frac{1}{2}$  inches broad and  $6\frac{1}{2}$  inches deep. The *gap* is 9 inches wide and 6 inches deep; consequently the lathe can swing a job of 20 inches diameter clear of the leading screw, and one of 24 inches diameter when this screw is withdrawn from its bearings. The *speed-cone* has three pulleys, each  $2\frac{1}{2}$  inches broad, the diameter of the largest being 8 inches and that of the smallest 4 inches.

The makers have planed and scraped the *bed* to a true bearing surface, and have so fixed the gap piece that it cannot wear loose or spring the bed.



END VIEW.



LONGITUDINAL SECTION.

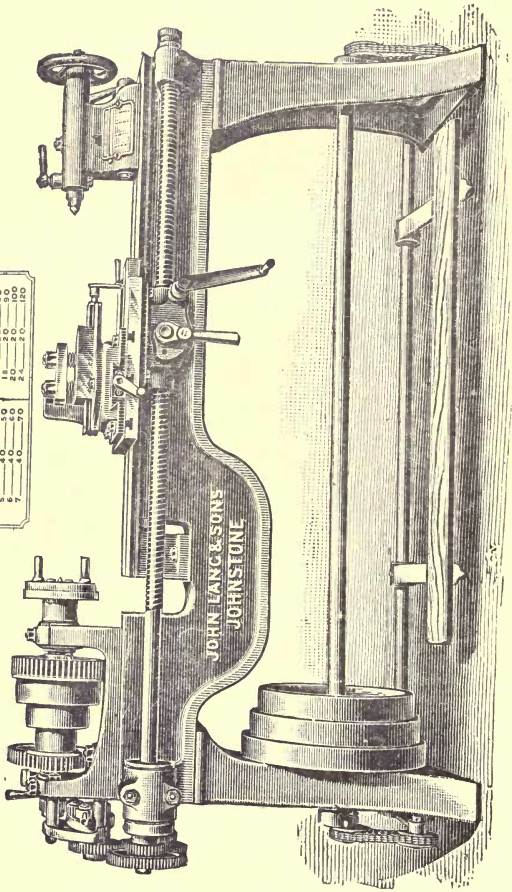
## MOVABLE HEADSTOCK OF SCREW-CUTTING LATHE.

## INDEX TO PARTS.

BP represents	Bottom part.	BH represents	Binding handle.
TP	Top part.	AS	Adjusting screw.
S	Spindle, or mandril.	ES	Eccentric spindle.
SC	Steel centre.	EH	Eccentric handle.
SS	Steel screw.	B	Bolt for clamping.
HW	Hand wheel.	P	Plate under B.

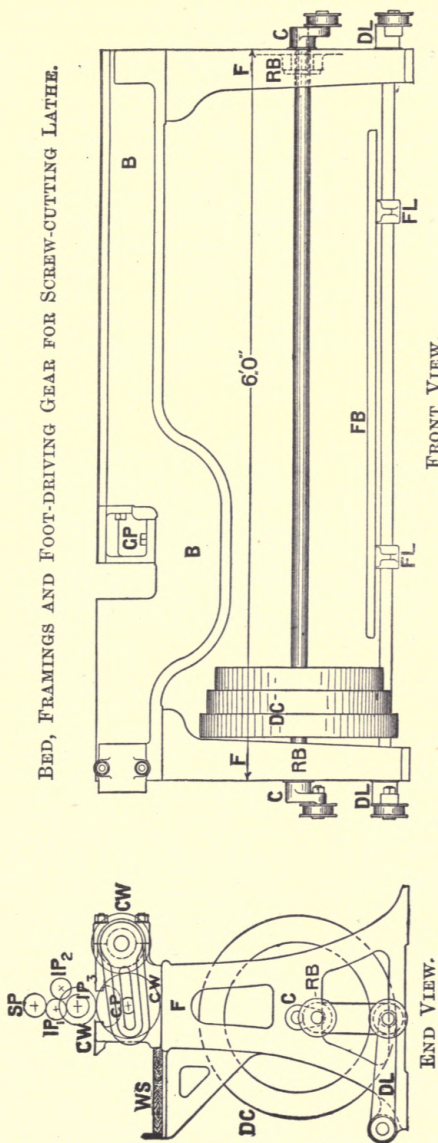
The *movable or loose headstock* is gripped to the bed by an eccentric motion worked by a handle, so that it may be instantly clamped in position without the trouble of finding a key to fit the usual nut, and then screwing it gradually home. The upper part of this head, which carries the mandril or spindle, has a side adjustment by means of a side screw, whereby the steel centre may be truly aligned with the corresponding centre of the fast headstock, or it may be moved to the one side or to the other in the case of taper turning. A small oil-holder is cast on the back side of the head to facilitate the oiling of the steel centre without having to look for an oil-can.

JOHN LANG & SONS			
LATHE MANUFACTURERS JOHNSTONE			
NAMES OF DRIVERS			
PER INCH			
1	80	20	80
2	40	20	40
3	40	30	10
4	40	35	12
5	40	40	16
6	40	50	20
7	40	70	24
			20
			120



SELF-ACTING SCREW-CUTTING TREADLE LATHE WITH HAND SURFACING MOTION.

BED, FRAMINGS AND FOOT-DRIVING GEAR FOR SCREW-CUTTING LATHE.

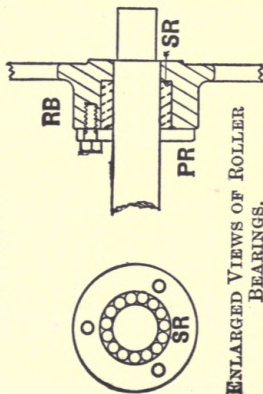


FRONT VIEW.

END VIEW.

INDEX TO PARTS.

B	represents Bed, or shears.	SR	represents Steel rollers.
GP	Gap piece.	PR	Plate for keeping rollers in position.
FF	Frames.	CW	Change-wheels.
DC	Driving cone.	CP	Change-plate or quadrant for attaching CW.
CC	Cranks.	IP <sub>1</sub> IP <sub>2</sub>	Idle pinions connecting CW and SP.
FB	Foot board.	SP	Spindle pinion.
FL	Foot levers.		
DL	Driving levers.		
WS	Wooden shelf.		
RB	Roller bearings.		



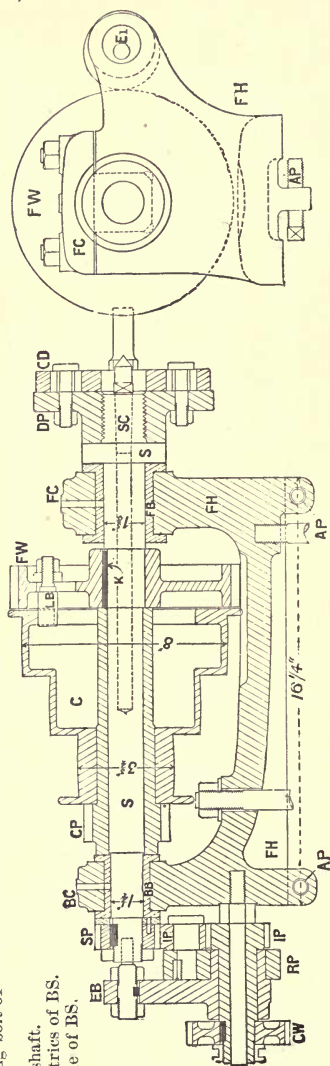
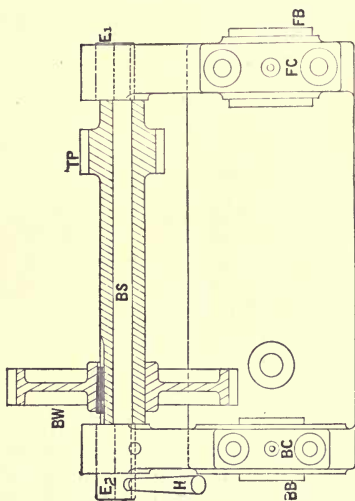
ENLARGED VIEWS OF ROLLER BEARINGS.

## INDEX TO PARTS.

FH for Fast headstock.  
 AP " Adjusting pins.  
 FB " Front bush.  
 FC " Front cover.  
 BB " Back bush.  
 BC " Back cover.  
 S " Spindle.  
 C " Cone for speeds.  
 CP " Cone pinion (or  $D_1$  in formula).  
 BW " Back wheel (or  $F_1$  in formula).  
 TP " Tube and pinion (or  $D_2$  in formula).  
 FW " Front wheel (or  $F_2$  in formula).  
 K " Key.  
 LB " Locking bolt of FW.  
 BS " Back shaft.  
 $E_1, E_2$  " Eccentrics of BS.  
 H " Handle of BS.

## INDEX TO PARTS.

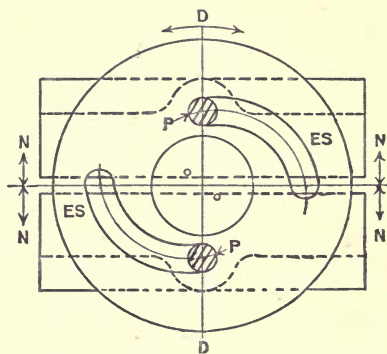
DP for Driving plate.  
 CD " Clements driver.  
 SC " Steel centre.  
 SP " Spindle pinion (or  $d$  in formula).  
 IP " Idle pinions.  
 EB " End brackets.  
 RP " Reversing plate.  
 CW " Change-wheels (or  $f$  in formula).



The *spindle* of the *fast headstock* is made of hard crucible steel ground accurately cylindrical, where it fits into parallel gun-metal bearings. These bearings are of extra diameter and length. This spindle is bored hollow for 12 inches of its length, in order to admit small rods for making terminals and screws in electrical engineering work. The *speed-cone* is turned inside and outside, and properly balanced. A specially strong and simple *reversing gear* has been fitted to the back end of this headstock, whereby the machine-cut steel pinions for turning right and left hand screws may be put into or out of gear by simply depressing or elevating a reversing handle. The *back-motion gear* is actuated by means of a handle and eccentrics on each end of the back-motion shaft; whilst the *front wheel* (or last follower,  $F_2$ , as we have symbolised it in the formula) is locked to the cone or thrown out of gear therewith in the usual way—viz., by a bolt fitting into a sliding slot in the cone and a projecting nut on the side of the toothed wheel.

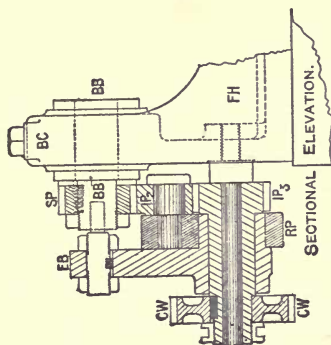
The *saddle* has T slots on its upper side for the purpose of bolting work to it that requires boring out, and which necessitates the removal of the slide rest. A quick hand traverse motion is provided for the saddle by means of a rack and pinion motion, quite independent of the sliding motion of the leading screw. The leading screw is turned to the standard pitch of  $\frac{1}{4}$  inch, or four threads to the inch. The engaging nut is made in halves, so that it may grip the leading screw fairly at the top and bottom of the threads.\*

\* In order to make the construction and action of the split nut which engages the leading screw clearer, we show here an enlarged view with the halves of the nut,  $N \leftarrow \rightarrow N$ , slightly apart, and the disc handle removed, so as to bring into full view the two eccentric slots, ES, which guide the two steel pins, P and P, fixed on N and N. By comparing this view with the others under heading "Saddle and Slide," the student will see how, by merely turning the disc handle DH the disc D is moved round through nearly a quarter of a circle, and the eccentric slots ES cause the pins, P, P, to move closer to or further away from the centre of the disc D, and consequently move the two parts of the nut, N, N, in or out of gear with the leading screw.

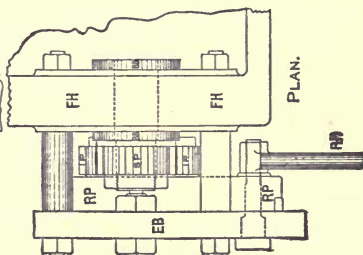


ENLARGED VIEW OF SPLIT NUT FOR LEADING SCREW, &c.

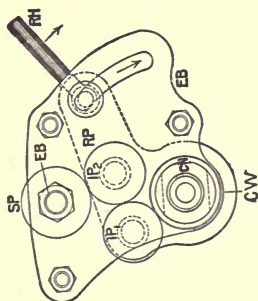




SECTIONAL ELEVATION.

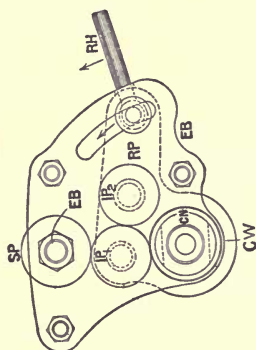


PLAN.



END VIEW.

*Showing IP<sub>2</sub> in gear for cutting a Left-handed Screw.*



END VIEW.

*Showing IP<sub>1</sub> in gear with SP for cutting a Right-handed Screw.*

## DIFFERENT VIEWS OF THE CHANGE-WHEEL GEAR.

### INDEX TO PARTS.

FH represents Fast headstock.	EB represents End bracket for supporting RP, &c.
BB " Back bush.	RP " Reversing plate to carry IP <sub>1</sub> , IP <sub>2</sub> .
BC " Back cover.	RH " Reversing handle.
SP " Spindle pinion.	CW " Change-wheels, with change-wheel nut, CN.
IP <sub>1</sub> " Idle pinion for right-hand screws.	
IP <sub>2</sub> " Idle pinion for left-hand screws.	

A *compound slide rest* is fitted to the top of the saddle, having large bearing surfaces with adjustments for taking up the wear, and a swivel arrangement for conical boring.

All the *toothed wheels*, including the change-wheels, have had their teeth cut directly from the solid casting, by the makers' special tool for that purpose, so that back-lash, and consequently noise and vibration arising from fast-speed driving may be minimised as far as possible.

The *driving shaft* has anti-friction steel roller-bearings. It is connected to the foot-treadle at each end by a pulley, chain, and crank. The driving-cone is so stepped that the belt has equal tension on any corresponding pair of driving and driven pulleys. It is sufficiently heavy to act as a fly-wheel. It is balanced along with the treadle to secure an easy, steady drive. A power-drive may be applied if desired, but the author believes that, as students should work in pairs or in sets of three in a laboratory, they will take a deeper interest in their experiments if they have turned out everything by their own skill and labour, than if motive power were freely supplied to them.

Of *heavy chucks* there are a very complete set, including a four-jaw expanding chuck, clement driver, drill chucks for both the fast and loose head spindles, &c.

The student should now go over each drawing *most carefully* by aid of the corresponding index to parts, and compare the drawings with an actual screw-cutting lathe.

**Hexagon Turret Lathe.**—Before completing this section it will be necessary to illustrate one of the latest and most important labour-saving appliances. The hexagon turret lathe is principally used for repetition work, and it is generally found in large engineering shops (where this class of work is carried on to a large extent) to pay the firm to spend some time in designing suitable cutting tools. The lathe with hexagon turret shown in frontis-plate, and the several detail views of turret head and saddle as designed by Messrs. Alfred Herbert, Ltd., Coventry, who have kindly given me assistance in supplying the necessary drawings and photos from which these figures were reproduced. The special features of this lathe are—(1) the ease with which the speeds and feeds can be changed, also the feeds reversed; (2) the greatly improved set of tool holders on the turret head; (3) the improved lever operated chuck for gripping the bar; and (4) the new stop motion, by which each tool has its own longitudinal and transverse stops.

*Description of Lathe.*—Fig. 1 on page 189 shows a direct-gearred motor-driven headstock for this lathe. The motor is

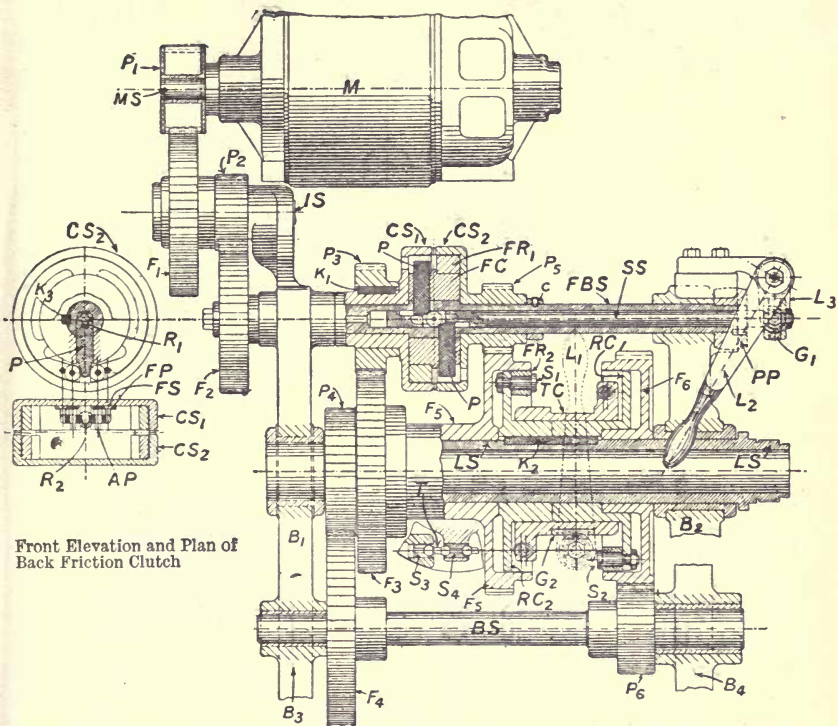


FIG. 1.—SECTIONAL PLAN OF AN ELECTRICALLY-DRIVEN HEXAGON TURRET LATHE, SHOWING ARRANGEMENT OF GEARS AND FRICTION CLUTCHES.

## INDEX TO PARTS.

M	represents	Electric motor.	PP	represents	Pulley for working force
MS	"	Motor spindle.			pump.
P <sub>1</sub> -P <sub>6</sub>	"	Pinions.	LS	"	Lathe spindle.
F <sub>1</sub> -F <sub>6</sub>	"	Followers or wheels.	RC <sub>1</sub> , RC <sub>2</sub>	"	Friction ring carriers.
IS	"	Intermediate gear stud.	G <sub>1</sub> , G <sub>2</sub>	"	Gluts.
K <sub>1</sub> -K <sub>3</sub>	"	Keys fixed in shafts.	TC	"	Toggle carrier.
CS <sub>1</sub> , CS <sub>2</sub>	"	Friction clutch sleeves.	T	"	Toggle.
R <sub>1</sub> , R <sub>2</sub>	"	Rollers.	S <sub>1</sub> , S <sub>2</sub>	"	Studs.
P	"	Plunger or wedge.	FR <sub>1</sub> , FR <sub>2</sub>	"	Friction rings.
FS	"	Friction screw.	S <sub>3</sub> , S <sub>4</sub>	"	Shoes for friction ring
FP	"	Friction plate.			and toggle carrier.
AP	"	Adjusting pin.	L <sub>1</sub>	"	Lever for actuating RC <sub>1</sub> .
FC	"	Friction clutch centre.			and RC <sub>2</sub>
FBS	"	Friction back shaft.	BS	"	Back gear shaft.
C	"	Collar or ruff on shaft FBS.	B <sub>1</sub> , B <sub>2</sub>	"	Bearings for LS.
SS	"	Sliding shaft.	B <sub>3</sub> , B <sub>4</sub>	"	Bearings for BS.
L <sub>2</sub> , L <sub>3</sub>	"	Levers for working SS.			

MS for Mitre Shaft.  
 GB Guide Bracket.  
 TMG Turret Mitre Gear.  
 TGB Turret Gear Bracket.  
 GC Gear Case.  
 GCC Gear Case Cover.  
 Sp<sup>2,3</sup> Studs.  
 PP Plunger Pinion.  
 PB<sub>1,2</sub> Plunger Bracket and Bush.  
 SRB Stop Rod Bracket.  
 SR Stop Rod.  
 HT Hexagon Turret.  
 B Cast Steel Bush.  
 SP Steel Plug.  
 C Collar.  
 CP Clamp Plate.  
 LS Locking Screw.  
 TMG Turret Mitre Gear.  
 G Gib.  
 H Handle.  
 SW<sub>2,6</sub> Spur Wheels.

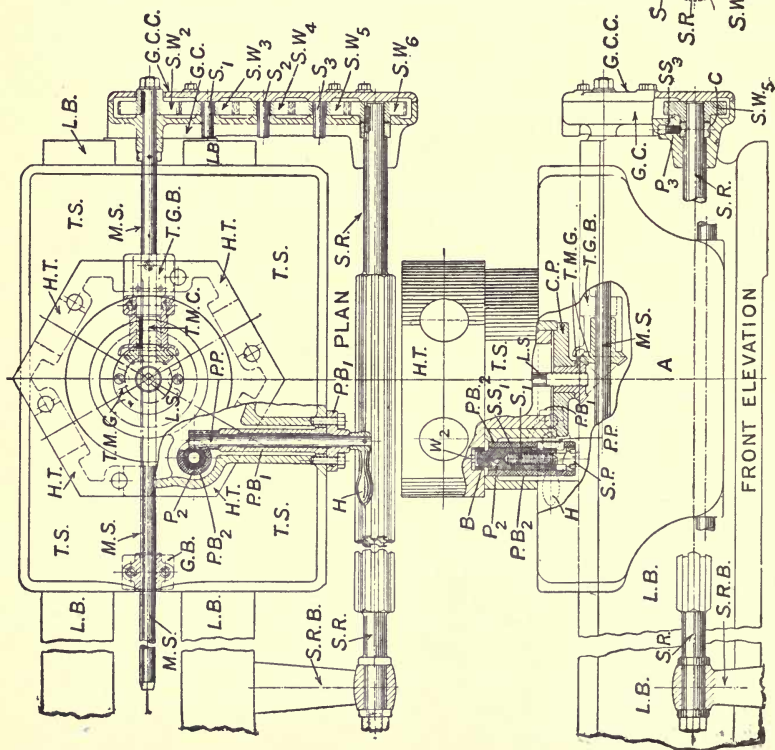


FIG. 2.—GENERAL ARRANGEMENT OF SADDLE AND TURRET FOR HEXAGON TURRET LATHE.

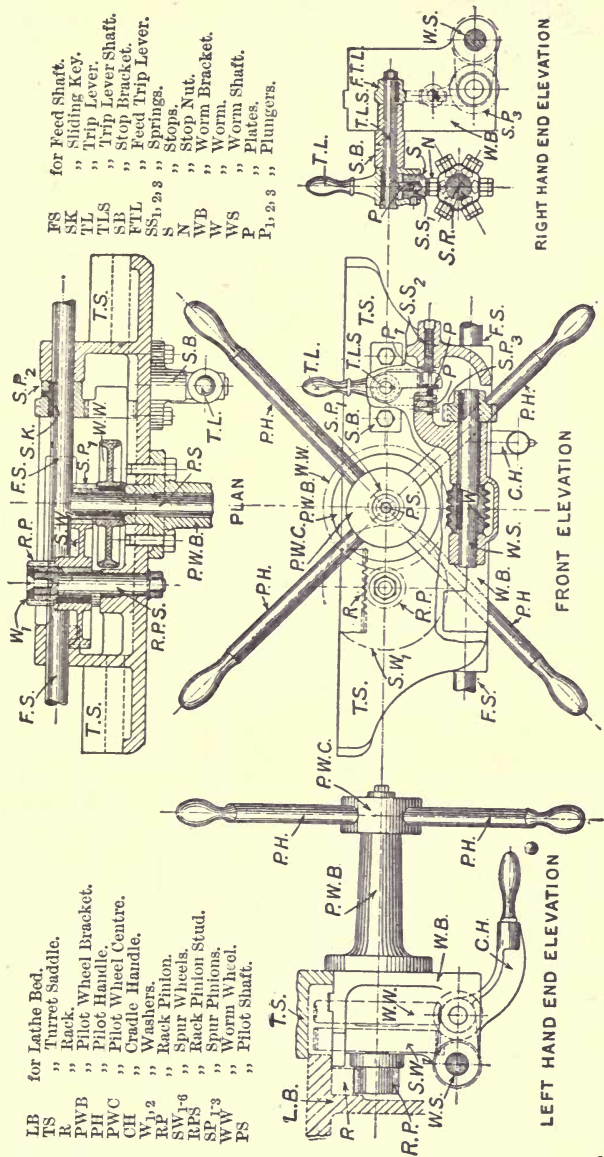


FIG. 3.—GENERAL ARRANGEMENT OF APRON FOR HEXAGON TURRET LATHE.

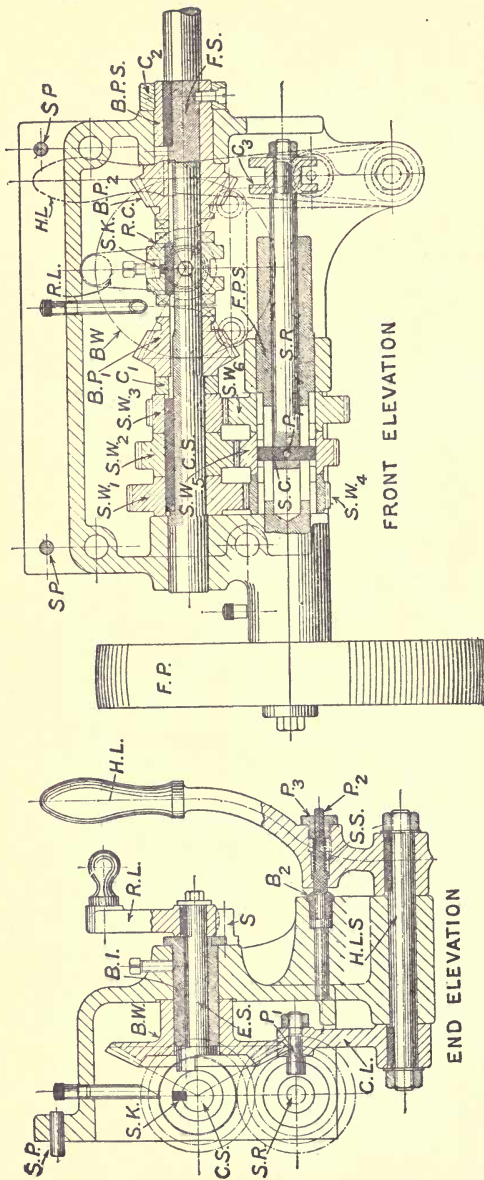


FIG. 4.—GENERAL ARRANGEMENT OF FEED MOTION FOR HEXAGON TURRET LATHE.

INDEX TO PARTS.

END ELEVATION.

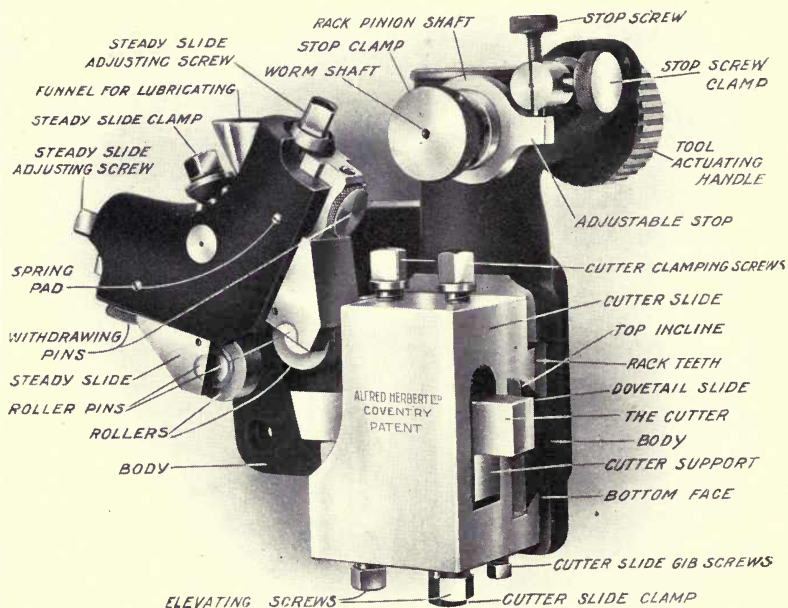
- SP for Steadying Pins.
- BW Bevel Wheel.
- B<sub>1</sub><sup>2</sup> Bushes.
- ES Eccentric Shaft.
- RL Reversing Lever.
- S Stops.
- P<sub>1</sub><sup>2</sup><sup>3</sup> Pins, Plunger and Plug.
- CL Clutch Lever.
- SS Spring.
- HL Hand Lever.

FRONT ELEVATION.

- FP for Feed Pulley.
- SW<sub>1</sub>-SW<sub>6</sub> Spur Wheels.
- C<sub>1</sub><sup>2</sup><sup>3</sup> Collars.
- BP<sub>1</sub>, BP<sub>2</sub> Bevel Pinions.
- RC Reversing Clutch.
- BPS Bevel Pinion Sleeve.
- SK Sliding Key.
- SC Steel Cotter.
- FPS Feed Pulley Shaft.
- SR Sliding Rod.



# HEXAGON TURRET LATHE.



## THE PATENT ROLLER STEADY TURNER.

The following improvements have been made in this lathe:—

1.—The motor is mounted on a hinged base plate, by which it can be elevated and depressed, and it drives the headstock by means of a belt. The swinging movement of the motor serves to adjust the tension of the belt.

2.—The geared headstock provides sixteen speeds within itself, thus enabling a constant speed motor to be used, and renders the lathe equally applicable to direct or alternating-current motors.

3.—The driving pulley of the lathe contains an epicyclic reverse gear for giving the spindle speeds in every direction.

4.—The feed motion now gives *nine* feeds instead of *three* as formerly.

5.—The turning tool holders are fitted with roller steadies, enabling much higher cutting speeds to be used than formerly.

6.—A scale and adjustable pointer are fitted to the bed to enable direct measurements to be made without using a rule.

of 8 B.H.P. It is a semi-enclosed motor with speed variation obtained by inserting resistance in the field coils from a 60-point shunt controller. Consequently any desired speed is easily got between 1150 and 1850 revolutions per minute. The motor M and spindle MS is attached to the rawhide pinion  $P_1$ . This pinion drives by means of the compound gear train  $F_1, P_2, F_2$  with  $P_3$ , the first shaft FBS. As will be seen from the sectional plan the shaft FBS carries two spur pinions  $P_3$  and  $P_5$ , either of which can be put in or out of gear with follower wheels  $F_3$  and  $F_5$  by means of the friction clutches  $CS_1, CS_2$ . The levers  $L_3, L_5$  with central spindle SS actuate the friction clutches  $CS_1, CS_2$ . Hence, for every speed of the motor this shaft FBS can give two different speeds to the sleeve or followers  $F_3$  and  $F_5$  on the lathe spindle LS which is driven from shaft FBS. These follower wheels correspond to the cone pulley on an ordinary lathe, and drive the spindle LS direct through the friction ring carrier  $RC_3$ , which slides along the key  $K_2$  fixed in the lathe spindle LS; or, through double gearing  $P_4, F_4, BS, P_6, F_6$  and friction ring carrier  $RC_1$  connected to lathe spindle LS by key  $K_2$ , the change from the one to the other is made by means of the lever  $L_1$ . The friction clutches on the shaft FBS are of the expanding ring type operated by means of wedge surfaces P and rollers  $R_1, R_2$ , whilst the clutches  $RC_1, RC_2$  on the spindle LS are also of the expanding ring type, but actuated by toggles T. (See Index for toggle joints.)

The lathe shown by the frontispiece is of a later type. It is driven by a constant speed motor mounted on a hinged base plate. This base plate provides for the necessary tightening of an endless belt.

**Chucks.**—The lathe may be fitted either with an automatic or universal chuck. The former chuck is recommended where the bars to be worked are practically straight and cylindrical, whilst the latter is used on bars which are badly out of round or not straight.

Looking at the front outside view of lathe, it will be seen how that the bar to be turned is gripped by an automatic chuck worked by the longer handle parallel to and beside  $L_1$ .

The automatic chuck may be opened and closed whilst the machine is running, and it has the advantage of holding finished work without bruising.

**Turret-Head and Saddle.**—It will be seen from the frontispieces and the accompanying views, Figs. 2 and 3, that the saddle can be moved either by hand or power, as in the ordinary lathe. In the former method of working, the saddle is moved by means

of the pilot handle PH acting through wheels  $SP_1$ ,  $SW_1$ ,  $RP$  to the rack  $R$ ; whilst, by the second method, motion is taken from the

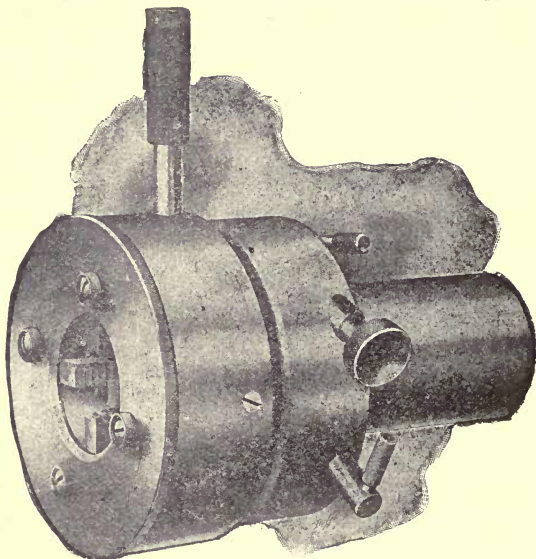


FIG. 5.—THE “COVENTRY” SELF-OPENING DIX HEAD—WITH ROUGHING AND FINISHING ATTACHMENT.

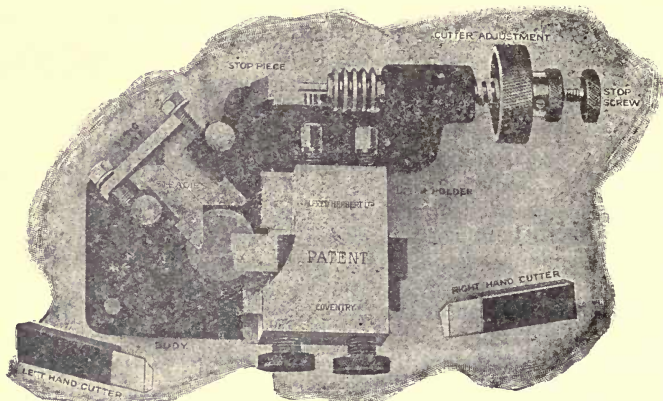


FIG. 6.—TURNING TOOLS—VIEW SHOWING RIGHT AND LEFT-HAND CUTTERS.

feed or traverse shaft  $FS$  through the spur wheels  $SP_3$  and worm  $W$ , worm wheel  $WW$ , and wheels  $SP_1$ ,  $SW_1$ ,  $RP_1$  to the rack  $R$ .

The worm wheel is carried in a cradle CH, hinged round the shaft WS, and held in position by the stop or trip lever TL and spring SS<sub>2</sub>. Whenever the hand traverse is to be used, the traverse worm W is released by the cradle dropping due to its own weight, after releasing the catch or trip lever TL.

When the handle TL is pressed to the right, the bar connected to it turns on a fulcrum, and causes the worm cradle handle CH to be dropped, thus stopping any further traverse of the saddle along the bed of the lathe. This traverse may also be stopped automatically by means of the trip lever TL coming into contact with the stop S, which is set at the desired position on the stop rod SR, and actuating the feed trip lever FTL. The hexagon head can be rotated into any position by turning the handle H, plunger pinion PP, and plunger P<sub>2</sub>, so that P<sub>1</sub> is freed from the cast steel bush B.

Bolted to the faces of the hexagon head are the specially shaped tools, while boring bars may be passed through the circular holes shown in the hexagon head and held firmly by bolts.

**Change Wheel Gear.**—It will be seen from the illustrations in Fig. 4 that the traverse or clutch shaft CS is rotated from the mandril through the feed pulley FP, and the pairs of wheels SW<sub>1</sub>—SW<sub>6</sub>. Any pair of wheels may be put into gear by means of the hand lever HL, which acts upon the internal sliding rod SR, and slides a cotter key SC so as to fix SW<sub>4</sub>, SW<sub>5</sub> or SW<sub>6</sub> to the shaft SR. The handle or reversing lever RL puts the claw clutch RC in gear with bevel pinions BP<sub>1</sub> or BP<sub>2</sub> to give either a forward or backward rotation or stoppage of the shaft CS. Consequently, three variations of speed for cutting purposes may be obtained by altering the position of the vertical hand lever HL, while the direction of rotation of the feed shaft may be changed by turning the reversing lever RL.

**Turning Tool Holders** as supplied with the lathe are one of its most important features. A general idea of the different tools on the turret or hexagon head is obtained from the frontis-plate. When another tool is required to be fetched up to the work, this is effected by turning the short handle H seen near top of saddle downwards, thus releasing the turret, which can now be turned as desired, afterwards bringing the handle back to its normal, horizontal position, and firmly fixing the hexagon head. Separate views of the turning tool holder, cut-off tool rest and self-opening die head for the hexagon head are also given, see Fig. 6 on page 194 and plate facing page 193.

**Rapid Turning with High Speed Steel.**—The student will be interested to learn something regarding the recent im-

provements which have been made in rapidly turning out work with the most modern electric-driven lathes, and the new kinds of high speed steel. Consequently, we have made a short abstract from three papers read recently before Engineering Societies where these matters were specially dealt with and discussed.\*

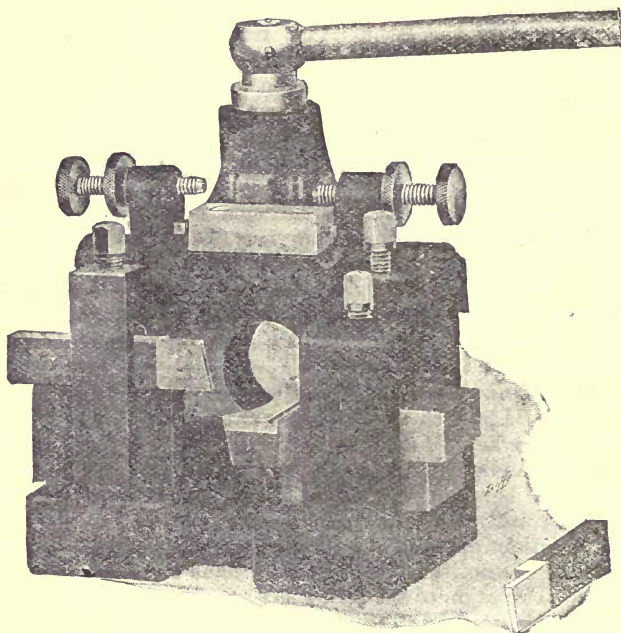


FIG. 7.—CUT-OFF TOOL REST SHOWING STEPPED CUT-OFF TOOL.

**Description of Lathe.**—The lathe on which all the tests were made was supplied by Armstrong, Whitworth & Co., and was one of their 15" centre screw-cutting lathes, taking in 9' 6" between the centres, but for these experiments 18" headstocks were fitted. The fast headstock had both double and treble back

\* See Proc. Inst. Eng. and Ships. in Scotland, vol. xlvii. 1904, for Mr. Charles Day's Paper on "Experiments with Rapid Cutting Steel Tools." Also see Mr. J. M. Gledhill's paper on "High Speed Tool Steel," read before the Coventry Engineering Society in March 1904, and Mr. P. V. Vernon's paper on "Speeds of Machine Tools," read before the Manchester Association of Engineers on November 14, 1903.

gears, the ratios being 14.9 to 1 and 42.5 to 1. The headstock was specially fitted with a 3-step cone suitable for a 6" belt. The lathe was driven by a direct current shunt-wound motor of 120 E.H.P., with a large air-cooled rheostat. The speed of the motor could be varied between 150 and 300 revolutions per minute at no load on the lathe, and from 60 to 300 revolutions with heavy cuts by means of the rheostat. The lathe was driven by two intermediate countershafts having 10" belts.

**Results.**—The diagrams show the maximum cutting speeds successfully used in each experiment made with rapid cutting steel tools, and the curves show the average speeds during each set of trials. Figs. 8 and 9 show the average results obtained from those tools which finished in such condition as to warrant attention, whilst the dotted lines in the figures show the maximum results obtained with any tool which finished in a satisfactory condition.

It was found that no single make of steel proved to be superior to all others in every respect, but it would appear that the average curves are those which may be taken as standards of all round comparison for use in general engineering shops. The following empirical formulæ give approximately the cutting speeds which may be adopted for different areas of cut upon different materials, and the curves show the results obtained therefrom :

$$\text{For soft steel} \quad S = \frac{1.96}{A \times 0.013} + 12$$

$$,, \text{ medium steel} \quad S = \frac{1.823}{A \times 0.015} + 5$$

$$,, \text{ hard steel} \quad S = \frac{1.77}{A \times 0.027} + 5$$

$$,, \text{ soft cast iron} \quad S = \frac{2}{A \times 0.02} + 20$$

$$,, \text{ med. } ,, \quad S = \frac{1.5}{A \times 0.23}$$

$$,, \text{ hard } ,, \quad S = \frac{1.4}{A \times 0.35}$$

Where:— $S$  = Cutting speed in ft. per minute.

$A$  = Area of section of cut in sq. inches, *i.e.*, traverse in inches multiplied by the depth of cut in inches.

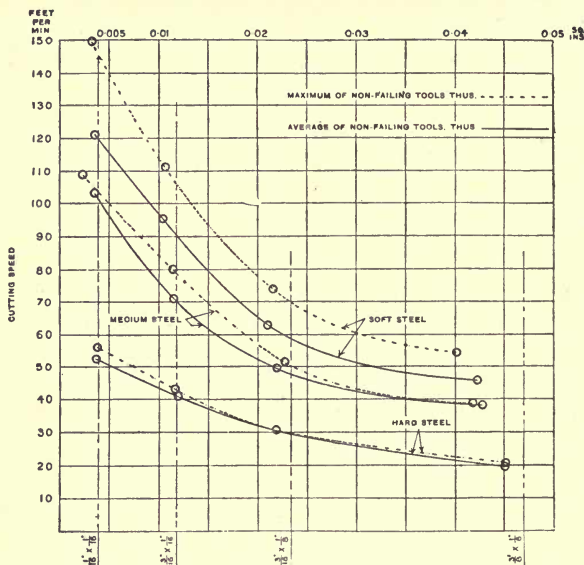


FIG. 8.—VARIATION OF CUTTING SPEED WITH AREA OF CUT STEEL.

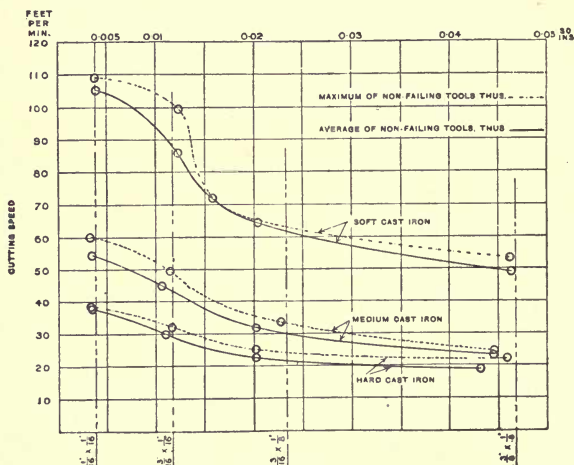


FIG. 9.—VARIATION OF CUTTING SPEED WITH AREA OF CUT CAST IRON.

**Power Records.**—Very careful records were taken of the power used for the various cuts and spaces. Also, data were obtained of the forces brought to bear on the cutting tools.

**Cutting Speed Results.**—Table I. gives the average results obtained from the tools which finished in good condition. The horse-powers given are the gross horse-powers as calculated from the readings of electrical instruments attached to the motor, and include the motor losses with any countershaft friction. The net horse-powers required to overcome the resistance to cutting are not given in the Table. These are only required for determining the cutting force on the tool point.

**Endurance Results.**—Table II. gives the average results obtained from soft forged steel and medium cast iron, which maintained their average cutting edge in fair condition for 60 minutes or longer.

**Comparison of Results.**—For the purposes of comparing results which may be obtained with the new steels against those obtainable with ordinary Mushet steel and ordinary water-hardened steel, tools made of these materials were tested, and the average results are also given in Table II. It will be noted that the new steels give decidedly improved results, and that with them the cutting speed can be about twice as fast as with ordinary Mushet steel, and three or four times as fast as with ordinary water-hardened steel.

An item of interest which may be mentioned here is that the ordinary Mushet steel can be very greatly improved by treating it in the same manner as the new steels when tempering. This is a point of value, as it enables greatly improved results to be obtained from existing tools.

It will be seen from an inspection of Table I. that, when much metal has to be removed, this may be done not only more quickly, but also with a less expenditure of energy per lb. of material removed, if a heavy cut is taken at a comparatively low speed in preference to a lighter one at a high speed.

**Cutting Forces and H.P. for Lathes.**—The figures showing the cutting force on tool points should prove of great service to machine-tool designers.

The information regarding the horse-powers is worthy of special attention, for it is this element which perhaps will form the greatest difficulty in the way of using existing lathes efficiently with the new high-speed cutting tools. A lathe on which a cut of  $\frac{3}{8}$ "  $\times$   $\frac{1}{8}$ " on soft steel can be taken is by no means



TABLE II. ENDURANCE TRIALS—AVERAGE OF RESULTS OBTAINED.

Material operated on.	Description of Tool Steel	Actual Cutting Speed per min. in ft.	Actual Cut.			Duration of Trial in mins.	Area Machined per min. in sq. ft.	Weight removed per min. in lbs.	Cutting Force on point of Tool.	
			Depth ms.	Traverse ms.	Area sq. ms.				Actual force in lbs.	Tons per sq. in.
Soft steel	...	92.6	0.187	0.0625	0.0117	120	0.479	3.69	2417	97.4
"	Ordinary Mushet	43.0	0.192	0.0625	0.0119	78.8	0.226	1.74	3100	116
"	Ordinary water hardened	23.1	0.182	0.0625	0.0114	18	0.120	0.871	4920	192
Medium Cast Iron	High speed air hardened	34.8	0.182	0.0625	0.0114	63.7	0.181	1.21	2934	115
"	Ordinary Mushet	19.5	0.189	0.0625	0.0118	60	0.106	0.69	2620	99.5
"	"	22.5	0.188	0.0625	0.0117	5.5	0.115	0.68	2050	78
"	Ordinary water hardened	11.2	0.189	0.0625	0.0118	6.2	0.058	0.19		

an abnormal one, and this duty can be carried out on most good lathes of, say, 12" centres, but the driving cones, the countershaft, and the belts connected with few such lathes would be suitable for 24 h.p. Further than this, the line shafts in most engineering shops are too light to drive many lathes using 20 h.p. each, or anything approaching that figure.

**Example.**—As a small example of what the Herbert Turret Lathe (*see* previous figs.) has done with this new high-speed steel, we quote the following from the Proceedings of the Coventry Engineering Society, as found in the paper read by Mr. J. M. Gledhill on March 4, 1904:

Sample No. 1 was a 1" bolt 6" long in shank, with  $1\frac{7}{8}$ " round head,  $1\frac{3}{16}$ " deep, and with the point screwed for 2".

This was finished complete in 5 minutes 28 seconds, using "A. W." tools. The following are the details:

Reducing $1\frac{7}{8}$ in. bar to 1 in., 6in.			
long, 96 cuts per inch	.	.	4 min. 0 secs.
Screwing	.	.	40 "
Cutting-off	.	.	28 "
Idle movements, etc.	.	.	20 "
Total			<u>5 min. 28 secs.</u>

The above time does not include the facing of the back of the head, which would require  $\frac{1}{2}$  minute.

This bolt, made in the ordinary way with ordinary tool-steel, with a good operator, requires 15 minutes.

**Forging and Hardening the Tools.**—In forging, annealing, and hardening crucible steels it is essential that the most suitable temperatures should be found for all of these processes, and then accurate means be taken to ensure such temperatures being actually obtained as near as practically possible. This can only be effected by the skilful use of pyrometers or other scientific heat-recorders, for to work on the old-fashioned lines of judging by the eye is no criterion of actual temperature, and is no longer advisable. It is now known that every composition of steel has its own definite temperature that is best suited for obtaining from it the most satisfactory results, and the nearer this can be worked to the better, any deviation from the correct temperature, up or down, involving a corresponding difference in the efficiency of the steel.\*

\* Students may refer here to the illustrated descriptions of these Pyrometers in the Author's Elementary and Advanced Steam Books.

Having obtained a bar of, say, the Armstrong-Whitworth or "A. W." brand of tool steel, it is necessary to cut off the required lengths; and this must be done at a forging heat. The lengths must not be broken off cold, as this tends to cause cracks in the bars. For forging, the steel should be placed in the fire, and slowly but thoroughly heated, taking care that the heating has penetrated to the centre of the bar, and then forged at a bright red heat. Whilst forging, the bar should not be allowed to get lower than a good red. After the tool is forged it should be laid down in a dry place and allowed to cool slowly. To harden the tool, the nose only should be raised to a white melting heat and then cooled with an air blast.

To obtain the maximum efficiency from this "A. W." steel it is essential that the nose of the tool shall be raised to a white melting heat as described, for if during this heating the point of the nose becomes fused or melted, no harm whatever has been done. The tool is then ready for use after grinding on a wet stone.

Another method which may be described of preparing the tools is as follows:

Forge the tool as before, and when cold roughly grind to shape on a dry stone or dry emery wheel. The tool then requires heating to a white heat, just short of melting, and cooling in the air blast. This method also lends itself for cooling the tools in oil; before which the temperature has to be lowered from the white heat to a good red heat (about 1600° F.) either by the air blast or in the open, and the tool then quenched in oil. Tools hardened by the latter method are specially good where the retention of a sharp edge is a desideratum, as in finishing tools, capstan and automatic lathe tools, brass workers' tools, &c. Nothing has yet been found to be so good for air hardening steels as the wet sandstone.

**Specific Gravity Test.**—Where engineering firms buy large quantities of various steels, it is advisable to have samples cut off, to ascertain the various specific gravities and tabulate them. The specific gravity of the steel of any one particular *brand* and *maker* is found to be fairly constant. Consequently the firm will be able to identify from amongst any number the manufacturer of the steel which they find by trial to be most suitable for their purpose by this method.

## LECTURE XVI.—QUESTIONS.

1. Sketch the fast headstock of a double-gear lathe, and explain the contrivance for increasing or diminishing the speed of the mandril. In the headstock of a lathe a pinion of 20 teeth drives a wheel of 60, and a second pinion of 20 drives another wheel of 60; compare the rates of rotation of the first driving pinion and of the mandril of the lathe. *Ans.* 9 : 1.

2. Why is a lathe often back-gear? Sketch a section through the headstock showing the arrangement. If the two wheels have 63 and 63 teeth respectively, and each pinion has 25 teeth, find the reduction in the velocity ratio of the lathe spindle due to the back-gear. *Ans.* 6·35 : 1.

3. Make a vertical longitudinal section through the *movable* or loose headstock of a lathe, showing precisely the manner in which a screw and nut are applied to produce the necessary movement of the centre which supports the work. Name the materials of which the several parts are made.

4. What is the use of the guide-screw in a lathe? Where is it usually placed? Show by sketches the precise manner in which the slide rest is connected with or disengaged from the guide-screw.

5. Describe and show by sketches the means by which the slide rest of a lathe may be connected with the leading screw. If the slide rest traverses the bed at the rate of  $1\frac{1}{2}$  feet when the leading screw makes 56 revolutions, what is the pitch of the screw thread? *Ans.*  $\frac{1}{4}$  inch.

6. Sketch and describe the mechanism by which the saddle of a screw-cutting lathe can be made to travel automatically in either direction along the lathe bed while the speed pulleys run always in the same direction.

7. How is the *copying principle* applied in a screw-cutting lathe? Describe a method of throwing a self-acting screw-cutting lathe in and out of gear, and of reversing it by means of a belt and overhead pulleys. (See Fig. 5 in Lecture XI.)

8. Explain the use of the quadrant for change wheels in a screw-cutting lathe by making a sketch showing it in its position on a lathe with the wheels in gear. (See the general and the end views of the 6" screw cutting-lathe bed, and "Index to Parts" for the part marked CP.)

9. Explain the mode in which *change wheels* are employed in a screw-cutting lathe. The leading screw being of  $\frac{1}{4}$ -inch pitch, give a sketch of the arrangement of change wheels required for cutting a screw of 15 threads to the inch, marking the numbers of the teeth on each wheel.

10. Sketch and describe the mechanism for cutting a screw with five threads to the inch in a lathe where the guide screw has three threads to the inch. Assign suitable numbers to the wheels which you would employ.

11. The leading screw of a lathe is  $\frac{1}{4}$ -inch pitch and *right-handed*. Sketch and describe the arrangement whereby you would employ the lathe for cutting a screw of  $\frac{1}{4}$ -inch pitch, and *left-handed*.

12. Describe the operation of cutting a screw in a lathe, showing the wheels required, and how they are placed to cut a right-handed screw with eight threads to the inch in a lathe whose leading screw is of  $\frac{1}{4}$ -inch pitch.

13. Explain the use of change wheels in a screw-cutting lathe. It is desired to cut a screw of  $\frac{5}{8}$ -inch pitch in a lathe with a leading screw of four threads to the inch, using four wheels. If both screws be right-handed, what wheels would you employ?

14. The leading screw in a self-acting lathe has a pitch of  $\frac{1}{2}$  inch; show an arrangement of change wheels for cutting a screw of  $\frac{3}{8}$ -inch pitch.

15. You are required to cut a left-handed screw of five threads to the inch in a lathe fitted with a right-handed guide screw of  $\frac{1}{2}$ -inch pitch. Show clearly by the aid of sketches the change wheels which you would employ for the purpose, indicating how they would be respectively carried, and the number of teeth in each wheel.

16. What do you understand by a single-gear, a double-gear, and a treble-gear lathe? Give such sketches as will show clearly the arrangement of the headstock in each of these cases.

17. Given a screw-cutting lathe with a right-handed leading screw with four threads per inch—sketch an arrangement for cutting a left-hand thread of eleven threads per inch. What gear wheels would be required?

18. A driving shaft runs at 100 revolutions per minute, and carries a pulley 22 inches in diameter from which a belt communicates motion to a pulley 12 inches in diameter carried upon a counter-shaft. On the counter-shaft is also a cone pulley having steps, 8, 6, and 4 inches in diameter respectively, which gives motion to another cone pulley with corresponding steps on a lathe spindle. Sketch the arrangement in front and end elevation, and find the greatest and least speeds at which the lathe spindle can revolve. *Ans.* 366.6 revs. per minute, and 91.6 revs. per minute.

19. Describe and sketch the arrangement of the mechanism by which the saddle of a lathe is traversed by hand along the bed.

If the slide rest of a screw-cutting lathe when in gear with the leading screw moves along the bed for a distance of 14", while the leading screw makes 56 revolutions, what must be the pitch of the thread on the leading screw? *Ans.*  $\frac{1}{4}$  inch.

20. Give free-hand sketches of the front and back outside views of the Hexagon Turret Lathe described in this Lecture with index to parts.

21. Give a sectional plan with index to parts of the electrically-driven Hexagon Turret Lathe, showing the detailed arrangement of motor and gears with friction clutches for the fixed headstock.

22. Give a plan with front and end elevations, together with an index to parts, showing the general arrangement of Saddle and Turret for the Hexagon Lathe.

23. Give a plan with front and end elevations of the general arrangement of the apron for the Hexagon Turret Lathe. Indicate each part by letters and an index.

24. Give a front and end elevation with index to parts of the general arrangement of the feed motion of the Hexagon Turret Lathe.

25. What is meant by "rapid turning high speed tool steel"? Indicate the variations of cutting speed with area of cut—steel and cast-iron by diagrams plotted on squared paper with an example of the h.p. required under certain conditions.

26. Describe the process of forging and hardening rapid-cutting tool steel, and state how it differs from ordinary tool steel.

27. Describe, with sketches, the mechanism for giving an automatic feed to the cutting tool of a lathe or shaping machine, and how it is put in or out of action, and the amount of feed varied. (B. of E., 1902.)

28. On the headstock spindle of a lathe is keyed a speed cone, the greatest and least diameters of which are 10 ins. and  $5\frac{1}{2}$  ins. respectively. It is driven from a similar speed cone keyed to a counter-shaft which makes 350 turns a minute. The back gearing is of the usual type, the spur-wheels concentric with the headstock spindle having 62 and 30 teeth, gearing with wheels having 18 and 50 teeth respectively on the back spindle. Sketch and describe the arrangement, and find the greatest and least speeds at which the headstock spindle can run. (C. & G., 1903, O., Sec. A.)

*Ans.* Without gearing: 636.36 revs. per minute, and 192.5 revs. per minute.

With gearing: 110.7 revs. per minute, and 33.5 revs. per minute.

29. The gearing of a capstan engine is arranged as follows: Fixed on the crank shaft is a double-threaded worm, which gears with a worm-wheel of 50 teeth; keyed to the worm-wheel shaft is a spur-wheel (A) of  $22\frac{1}{4}$  ins. diameter pitch circle, and very approximately  $3\frac{1}{2}$ -in. pitch, driving another wheel of 40 teeth, which is keyed to the same shaft as the holder round which the cable passes. Find the number of teeth in the wheel A; and if the effective diameter of the cable-holder is 24 ins., find the number of revolutions the engine must make to heave in 90 ft. of cable.

(C. & G., 1904, O., Sec. A.)

*Ans.* Number of teeth on wheel A = 20. Revs. of engine required to heave in 90 ft. of cable = 715.

30. It is required to transmit a velocity ratio of 80 to 1 by a train of toothed wheels; no pinion of this train of wheels is to have less than 16 teeth, and no wheel is to have more than 90 teeth. Determine a suitable train of wheels, and the number of teeth in each wheel.

(B of E., 1905.)

$$\text{Ans. : } e = \frac{64 \times 80 \times 64}{16 \times 16 \times 16} = \frac{80}{1}.$$

31. In the gear for an electrically-driven turret, the motor shaft is provided with a single-threaded worm gearing with a worm-wheel of about  $23\frac{1}{2}$  ins. diameter, and  $1\frac{1}{2}$ -in. pitch, keyed to a spindle, which carries a wheel of 12 teeth, which gears with a wheel of about  $30\frac{1}{2}$  ins. diameter and 4-in. pitch. Keyed to the spindle of the latter is a spur-wheel of 15 ins. diameter, gearing with a circular rack on the turret, and which is 15 ft. diameter. It is found that the shortest and longest times to turn the turret through  $270^\circ$  are 52 seconds and 21 minutes respectively. Find the corresponding revolutions per minute of the motor.

(C. & G., 1905, O., Sec. A.)

*Ans.* Number of teeth on worm-wheel = 49;

" " " spur-wheel = 24;

Revs. of motor per minute, 1018 and 42.

## LECTURE XVII

**CONTENTS.**—Hydraulics—Definition of a Liquid—Axioms relating to a Liquid at Rest—Transmission of Pressure by Liquids—Pascal's Law—"Head" or Pressure of a Liquid at Different Depths—Total Pressure on a Horizontal Plane immersed in a Liquid—Lord Kelvin's Wire-testing Machine—Total Pressure on any Surface immersed in a Liquid—Examples I. II.—Questions.

**Hydraulics.**—Hitherto the student's attention has been confined to solid bodies, which were supposed to remain perfectly rigid and unchanged when acted upon by forces. We shall now direct his consideration to the properties and applications of another great division of matter—viz., liquids—which possess the marked opposite character of mobility under the action of forces. In nature we do not meet with either perfectly solid or perfectly liquid bodies; and consequently the practical engineer, when applying the formulæ of the physicist to his machines and hydraulic works, has to make certain allowances according to circumstances, with the aid of constants predetermined by experience and experiment.

The most common and the most useful liquid with which the engineer has to deal is that of water. Hence the term "hydraulic engineer," as applied to persons who direct and guide the action of waters, as in the case of the water supply for a town, or for navigation purposes, or for the transmission of force and power. The term hydraulics, therefore, comprehends *hydro-statics*, which is the science of liquids in equilibrium, and *hydro-kinetics*, the science of liquids in motion. We shall only have space in this manual for an elementary inquiry into the former of these two divisions of hydraulics.\*

**Definition of a Liquid.**—*A liquid is a collection of particles which are perfectly movable about each other.* In consequence of this property, a liquid requires some external force or resistance to keep its particles together, such as the sides of a vessel; for its molecules can be displaced by the smallest force, and are readily divided from each other in any direction.†

\* For Viscous Fluids, see p. 224.

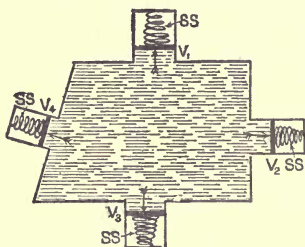
† The late Prof. Clerk Maxwell distinguished solids from liquids in the following manner:—"Bodies which can sustain longitudinal pressure,

**Axioms relating to a Liquid at Rest.**—It follows directly from the above definition, that when equilibrium exists—

- (1) *The free surface of a liquid at rest is horizontal ;*
- (2) *Any surface of a liquid at rest is everywhere perpendicular to the force which acts upon it ;*
- (3) *A liquid at rest acted on by a force presents a surface which is everywhere perpendicular to the direction of the force ;*
- (4) *A surface supporting a liquid at rest reacts everywhere perpendicularly to the pressure of the liquid ;*
- (5) *In all cases of pressure on or from liquids at rest, action and reaction are equal and opposite.*

If such were not the case, equilibrium could not exist, and motion of the liquid would take place.

**Transmission of Pressure by Liquids.**—Take a tight vessel



TRANSMISSION OF PRESSURE  
BY LIQUIDS.

(Horizontal Section.)

other valves had been of different areas from valve  $V_1$ , their springs would have registered pressures corresponding with the ratio of their areas to the area of valve  $V_1$ . Or the pressure per square inch on valve  $V_1$  is communicated throughout the liquid to the other valves, and to every square inch of the internal surface of the vessel, with undiminished effect.

**Pascal's Law.**—*Fluids transmit pressure equally and in all directions.\** In the case of solids pressure is only transmitted

*however small that pressure may be, without being supported by lateral pressure, are called solids, and those which cannot are termed liquids." A perfect liquid is therefore one in which there is absolutely no resistance to a change of shape, although there may be practically an infinite resistance to change of volume. We say practically because, although liquids are more or less compressible to a very small extent, yet the amount is so small as to be negligible in the case of most engineering problems.*

\* Here the word fluid has been used instead of liquid, as being more general, since the term fluid includes both liquids and gases. Refer to p. 2, Lecture I., for the distinction between a liquid and a gas.

along the line of its action, and therefore we have in this law an exemplification of the fundamental distinction between solids and fluids. In Lecture XIX. we will explain several machines that depend upon the principle enunciated by Pascal's law for their action.

**Head or Pressure of a Liquid at Different Depths.**—Imagine a very small horizontal area,  $a$  (for instance, a *square inch*), situated at a depth or height,  $h$ , inches from the free surface of a liquid, and that the *vertical* column from,  $a$ , to the surface becomes solidified without in any way disturbing equilibrium. It is evident that the horizontal and the vertical forces on the solid column must be separately in equilibrium, otherwise motion would ensue. But the only vertical forces are the weight of the column downward and the pressure of the surrounding liquid upwards on the base,  $a$ . Therefore,

The pressure upwards = weight of the prism.

$$\text{Or, } p = haw.$$

Where,  $w$ , is the weight of every inch of its height or the weight of a cubic inch of the column. But the area,  $a$ , and the weight,  $w$ , are constant quantities for any particular unit of area and kind of liquid. Hence—

*Pressure varies directly as the depth from the free surface.*

$$\text{Or, } p \propto h.$$

The technical term "*head*" expresses the above fact in a single word. For, when speaking of the working pressure per square inch due to a supply of water for a mill wheel or turbine, we say it has 10 or 20 or 30 feet of head, meaning thereby the pressure due to a difference of level of so many feet, from the free surface of the water as it enters the supply pipe to the free surface of the tail race or discharge pipe. Since every foot of "head" of water gives in round numbers a pressure of  $\frac{1}{2}$  lb. per square inch, we might have said that the pressure was 5 or 10 or 15 lbs respectively per square inch. Consequently,

*Pressure varies directly as the head.*

**Total Pressure on a Horizontal Plane immersed in a Liquid.**—Take a vessel of any shape having a horizontal base, and fill it with a liquid to any known height. Then from the above rule it follows that,

$$\text{The Total Pressure on the base} = \begin{cases} \text{height in inches from base to sur-} \\ \text{face} \times \text{area of base in square} \\ \text{inches} \times \text{weight of a cubic} \\ \text{inch of the liquid.} \end{cases}$$

For, pressure per sq. in.,  $p = haw$ , when,  $a = 1$  square inch.

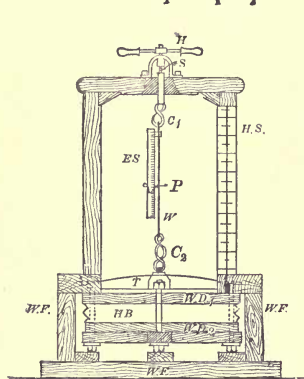
Consequently, if the total area of the horizontal plane be equal to,  $a$ , square inches, instead of 1 square inch.

*The Total Pressure =  $haw$ .*

This shows that the shape of the vessel containing the liquid, and the total weight of water in the vessel, *do not* in any way affect the total pressure on the base. For, it depends solely on the difference of level between the base (or immersed plane) and the free surface, on the area immersed, and on the weight per unit volume or specific gravity of the liquid.

This property results in what used to be termed the *hydrostatic paradox*, which is very well illustrated by Lord Kelvin's apparatus for testing the tensile strength and percentage elongation of the sheathing wires used for covering and protecting the insulated conductors of submarine cables.

**Lord Kelvin's Wire-testing Machine, or Hydrostatic Paradox.**— $W$  represents the wire to be tested, which is fixed to the clips  $C_1$   $C_2$ .  $HB$  is a circular hydrostatic bellows,



THOMSON'S HYDROSTATIC  
WIRE-TESTING MACHINE.

$3'$  diameter, with india-rubber sides.  $WD_2$  is the bottom wooden disc attached by bolts to an iron tripod  $T$ , which is connected at its centre to the clip  $C_2$ ; while  $WD_1$  is an upper wooden disc rigidly fixed to the wooden framing  $WF$ .  $H$  is a handle keyed to the screwed spindle  $S$ .  $HS$  is a hydrostatic scale, fixed behind the vertical glass tube which is fitted into a short brass cylinder passing through  $WD_1$  and into  $HB$ .  $ES$  is the scale for measuring the percentage elongation. The upper end of this scale is fixed to the wire  $W$ , and the lower end is free. There is a clip pointer  $P$  which is affixed to

each wire before testing it, and moved up or down until it is opposite to the zero of the scale  $ES$ .

*Method of Testing Wire by this Machine.*—(1) Turn the handle  $H$  backwards until  $C_1$  is as far down as it can get. (2) Fix wire in clips, and attach the pointer  $P$  so as to be opposite the zero of scale  $ES$ . (3) Turn the handle  $H$  forward, thus lifting  $WD_2$ , and stretching the wire, by forcing water up the glass tube in front of  $HS$ . This gives the necessary "head,"  $h$ , or pressure due to the difference in level between the free surface in the glass tube and the bottom of the wooden base  $WB_2$ . The area in square inches

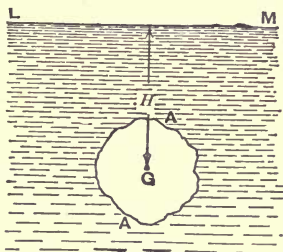
of this base gives,  $a$ , and hence the total pull on the wire is  $=haw$ .  
 (4) Note the elongation by the scale ES, and the total tensile stress by the scale HS, at the moment the wire breaks. WD<sub>2</sub> falls upon stops, so as not to injure the india-rubber hydrostatic bellows HB.

This machine was used in 1872-73 by the Author and others in testing all the sheathing wire for the Western and Brazilian Company's cables. The homogeneous wire gave an average of 55 tons per square inch.

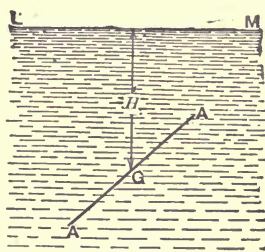
In this machine we see that, owing to the *quaque versus* principle enunciated above a few pounds weight of water can produce a stress of many hundreds or even thousands of pounds by simply giving it "head," through a small tube in connection with an enlarged area.

When the sides of a vessel taper towards the top, as in the case of a wine bottle, the liquid pressing vertically upwards upon them produces a reaction on the base, which makes up for the want of weight of liquid which would be naturally due to direct vertical pressure in the case of a cylindrical vessel.

**Total Pressure on any Surface immersed in a Liquid.**  
 —Let a surface of *any shape* be immersed in a liquid of *any kind* to *any depth*, as illustrated by the following figures. Then, by applying the previous proofs, and a property of the "centre of



END VIEW.



SIDE VIEW.

#### PRESSURE ON ANY SURFACE IMMERSED IN A LIQUID.

gravity" (which affirms that the *mean perpendicular distance from any plane, is equal to the distance from the c.g. of the surface to that plane*), we find, that the *total pressure* on the immersed surface is represented by the following equation :

$$P = HAW.*$$

Where  $P$  = Pressure (total) in lbs.

„  $H$  = Height from *c.g.* to free surface in feet.\*

„  $A$  = Area in square feet.\*

„  $W$  = Weight of a cubic foot of the liquid.\*

\* The student will observe that we have suddenly jumped from heights

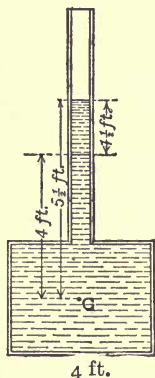
**EXAMPLE I.**—Find the total pressure on the bottom of a cubical tank having a bottom  $4' \times 4'$  and filled with water to a depth of  $4'$ .

**ANSWER.**—By the above formula—

$$P = HAW.$$

$$P = 4' \times (4' \times 4') \times 62.5 \text{ lbs.} = 4000 \text{ lbs.} = 1.8 \text{ tons.}$$

**Note.**—We may here remark that  $62.3 \text{ lbs.}$  is the weight of a cubic foot of fresh water at  $65^\circ \text{ F}$  (see *Appendix* for Useful Constants), whereas  $62.5 \text{ lbs.}$  is the value at the maximum density of water, or  $39^\circ \text{ F}$ .



**EXAMPLE II.**—A rectangular tank for holding water has a vertical side whose dimensions are 3 feet vertical by 4 feet horizontal. An open pipe is inserted into the cover of the tank, and water is poured in until the level in the pipe is 7 feet above the base of the tank. Find the pressure on the vertical side and the reduction of pressure when the water in the pipe is allowed to sink  $1\frac{1}{2}$  feet. (The weight of a cubic foot of water =  $62.5 \text{ lbs.}$ ) (S. and A. Exam. 1890.)

**ANSWER.**—In the first case,

Height from *c.g.* of side to free surface =  $H_1 = 5.5'$ .

Area of this vertical side in sq. ft. =  $A = 3' \times 4' = 12 \text{ sq. ft.}$

Weight of a cubic foot of water =  $W = 62.5 \text{ lbs.}$

By the above formula,

The total pressure  $P_1 = H_1AW$ .

$$\therefore P_1 = 5.5' \times 12 \times 62.5 = 4125 \text{ lbs.}$$

In the second case, when the free surface is lowered by  $1\frac{1}{2}$  ft., everything remains the same except the  $H$ , which is now reduced from  $H_1$  to  $H_2 = 4'$ .

By the formula,

$$P_2 = H_2AW.$$

$$\therefore P_2 = 4 \times 12 \times 62.5 = 3000 \text{ lbs.}$$

Consequently, the reduction in pressure is the difference between these pressures.

$$\text{Or } (P_1 - P_2) = 4125 \text{ lbs.} - 3000 \text{ lbs.} = 1125 \text{ lbs.}$$

in inches to those in feet, areas in square inches to those in square feet, and weights of cubic inches to those of cubic feet. This is because the usual units of measurement in hydraulics are feet, square feet, and cubic feet. Before attempting the more difficult questions on page 213, he should study a few pages of the next Lecture.

## LECTURE XVII.—QUESTIONS.

1. Define the terms liquid, hydro-statics, hydro-dynamics, and hydraulics.
2. Give the chief properties of a liquid, stating wherein it differs from a solid and a gas.
3. Describe and illustrate any experiment, other than the one referred to in this Lecture, to prove the law of transmission of pressure by liquids. State Pascal's law.
4. Describe the nature of fluid pressure. A mass of stone when in water appears to be lighter than when it is situated in the open air. Will you explain the cause of this fact, and state the difference of weight per cubic foot of water displaced?
5. What is meant by "head" in relation to water supplies for developing power? Give an example.
6. Explain how the pressure on the base of a vessel depends solely upon the area of the base and its depth from the free surface. Illustrate your remarks by showing a series of connected vessels of very different shapes, but with each of their bases of the same size and on the same level, and filled with water to the same height.
7. Sketch and describe Sir Wm. Thomson's wire-testing machine, and explain how such a great force is obtained thereby from such a small quantity of water.
8. How is the pressure of water on a given area ascertained? A tank, in the form of a cubical box, whose sides are vertical, holds 4 tons of water when quite full; what is the pressure on its base, and what is the pressure on one of its sides? *Ans.* 4 tons; 2 tons.
9. A water tank is 13 feet square and 4 feet 6 inches deep; find the pressure upon one of the sides when the tank is full. *Ans.* 8226.56 lbs.
10. State approximately the increase of pressure to which a diver would be exposed when working at a depth of 50 feet below the surface of fresh water. *Ans.* About 22 lbs. per square inch.
11. In the vertical plane side of a tank holding water, there is a rectangular plate whose depth is 1 foot and breadth 2 feet, the upper edge being horizontal, and 8 feet below the surface of the water; find the pressure on the plate. *Ans.* 1062.5 lbs.
12. The base of a rectangular tank for holding water is a square, 16 square feet in area. The sides of the tank are vertical, and it holds 250 gallons of water when quite full. Find the depth of the tank and the pressures on each side and on the base when quite filled with water. *Ans.* 2.5 feet; 781.25 lbs.; 2500 lbs.
13. A rectangular tank for holding water has a vertical side whose dimensions are 4 feet vertical by 5 feet horizontal. An open pipe is inserted into the cover of the tank, and water is poured in until the level in the pipe is 10 feet above the base of the tank. Find the pressure on the vertical side and the reduction of pressure when the water in the pipe is allowed to sink 2 feet. *Ans.* 10,000 lbs.; 2500 lbs.
14. A gauge in a water pipe indicates a pressure of water equal to 40 lbs. on the square inch. What is the depth of the point below the free surface? Sketch and explain the action of some form of gauge suitable for the above purpose. *Ans.* 92.16 ft.

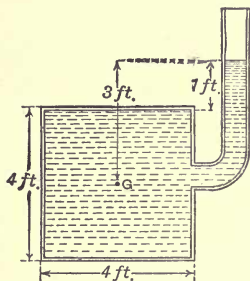
## LECTURE XVIII.

**CONTENTS.**—Useful Data regarding Fresh and Salt Water—Examples I. II. III. IV.—Centre of Pressure—Immersion of Solids—Law of Archimedes—Floating Bodies—Example V.—Atmospheric Pressure—The Mercurial Barometer—Example VI.—Low Pressure and Vacuum Water Gauges—Example VII.—The Siphon—Distinction between Solids, Liquids and Gases—Definitions of perfect, viscous, and elastic Fluids—Cohesion—Questions.

**Useful Data regarding Fresh and Salt Water.**—We will commence this Lecture by giving some useful data regarding the weights, &c., of fresh and salt water, and then work out a few more examples for the pressures on immersed surfaces, finishing with the immersion of solids in fluids, &c.

**FRESH WATER** { Specific gravity  $\ast = 1$ .  
 1 cubic foot weighs 62.5 lbs., or 1000 oz.  
 1 gallon weighs 10 lbs., or 160 oz.  
 1 ton occupies 35.84 cubic feet.  
 1 atmosphere = 14.7 lbs. per sq. in. = 29.92 in. mercury = 33.9 (say 34) ft. head of water.  
 1 foot of head = .43 lb. on sq. in.  
 1 lb. on the sq. in. = 2.308 ft. head.  
 H.P. in a waterfall = cubic ft. per minute  $\times$  head  $\div 33,000$ .

**SALT WATER** { Specific gravity  $\ast = 1.026$ .  
 1 cubic foot weighs 64 lbs.  
 1 gallon weighs  $10\frac{1}{4}$  lbs.  
 1 ton occupies 35 cubic ft., or  $218\frac{1}{4}$  gallons.



**EXAMPLE I.**—A cubical box or tank with a closed lid, the length of a side of which is 4 feet, rests with its base horizontal, and an open vertical pipe enters one of its sides by an elbow. The tank is full of water, and the pipe contains water to the height of 1 foot above the top of the tank. What are the pressures of water on the top, bottom, and sides of the tank? (Given the weight of a cubic foot of water =  $62\frac{1}{2}$  lbs.) (S. and A. Exam. 1887.)

$\ast$  Specific gravity is the ratio of the weight of a given bulk of a substance, to the weight of the same bulk of pure water.

ANSWER.—(1) For the pressure on the *top*—

The depth of *c.g.* of the top from free surface =  $H = 1'$ .

∴ Total pressure on top =  $HAW = 1' \times (4' \times 4') \times 62.5 \text{ lbs.} = 1000 \text{ lbs.}$

(2) For the pressure on the *bottom*—

The depth of *c.g.* of the bottom from the free surface =  $H = 5'$ .

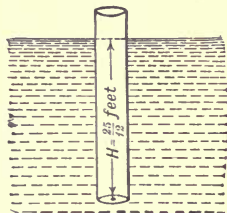
∴ Total pressure on bottom =  $HAW = 5' \times (4' \times 4') \times 62.5 \text{ lbs.} = 5000 \text{ lbs.}$

(3) For the pressure on each of the *sides*—

The depth of *c.g.* of *each side* from the free surface =  $H = 3'$ .

∴ Total pressure on each side =  $HAW = 3' \times (4' \times 4') \times 62.5 \text{ lbs.} = 3000 \text{ lbs.}$

EXAMPLE II.—A cylindrical vessel, 30 inches long and 6 inches in diameter, is sunk vertically in water, so that the base, which is horizontal, is at a depth of 25 inches below the surface of the water. Find the upward pressure in pounds on the base of the vessel. The weight of a cubic foot of water is  $62\frac{1}{2} \text{ lbs.}$ , and  $\pi = 3.1416$ . (S. and A. Exam. 1889.)



ANSWER.—The depth of *c.g.* of the base from the free surface is  $H = \frac{25}{12} = 2\frac{1}{12}' = 2.08\bar{3} \text{ feet.}$

The area of the base =  $A = \frac{\pi d^2}{4} = \frac{22}{7 \times 4} (.5' \times .5') = .196 \text{ sq. ft.}$

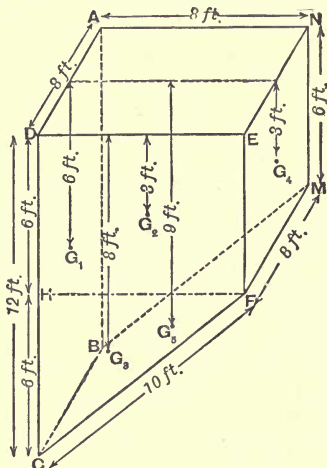
The weight of a cubic foot =  $W = 62.5 \text{ lbs.}$

∴ The total pressure on base =  $HAW = 2.08\bar{3} \times .196 \times 62.5 = 25.5 \text{ lbs.}$

EXAMPLE III.—A water tank, 8 feet long and 8 feet wide, with an inclined base, is 12 feet deep at the front and 6 feet deep at the back, and is filled with water. Find the pressure in lbs. on each of the four sides and on the base; water weighing  $62\frac{1}{2} \text{ lbs.}$  per cubic foot.

ANSWER.—In answering a question of this kind the student will find it best to draw a figure representing the water tank and the positions of the centres of gravity of *each side* and of the base in the manner shown by the accompanying illustration. The only point that presents any difficulty is the *c.g.* of the side DEFC and of the correspondingly opposite one. This might be done by first finding the *c.g.* of the  $\square$  DEFH, viz.,  $G_2$ ; second, of the  $\triangle$  HFC, viz.,  $G_3$ ; third, by joining these two points with a line  $G_2G_3$ ,

and taking a distance along it from  $G_2$  towards  $G_3$  inversely proportional to the areas of the  $\square$  DEFH and the  $\triangle$  HFC; this would give a point  $G_6$  the *c.g.* of the whole side = 4'6" from surface. But it will evidently be easier to treat the pressures on the  $\square$  and  $\triangle$  separately, and then to add them together in order to obtain the total pressure on the whole side DEFC.



SHOWING POSITIONS OF THE CENTRES OF GRAVITY.

$G_1$	represents	centre of gravity of area	ABCD
$G_2$	"	"	DEFH
$G_3$	"	"	HFC
$G_4$	"	"	ENMF
$G_5$	"	"	BCFM.

Let  $H_1$ ,  $H_2$ , &c., represent depths of  $G_1$ ,  $G_2$ , &c.

Then  $H_1 = \frac{1}{2}DC = 6'$ ;  $H_2 = \frac{1}{2}EF = 3'$ .

$G_3$  is  $\frac{1}{3}$  of HC below the line HF (see Lecture III., *re* position of *c.g.* of certain areas).

$\therefore H_3 = 6 + \frac{6}{3} = 8'$ ;  $H_4 = \frac{1}{2}EF = 3'$ .

$G_5$  is at a depth below the surface = the mean between the edges BC and FM of the base BCFM.

$\therefore H_5 = \frac{1}{2}(DC + EF) = \frac{1}{2}(12 + 6) = 9'$ .

Total pressure on area—

$$ABCD = H_1 A_1 W = 6' \times (12' \times 8') \times 62.5 = 36,000 \text{ lbs.}$$

$$DEFH = H_2 A_2 W = 3' \times (6' \times 8') \times 62.5 = 9,000 \text{ lbs.}$$

$$HFC = H_3 A_3 W = 8' \times \left(\frac{6'}{2} \times 8'\right) \times 62.5 = 12000 \text{ lbs.}$$

$$DEFC = DEFH + HFC = 9000 + 12000 = 21000 \text{ ,,}$$

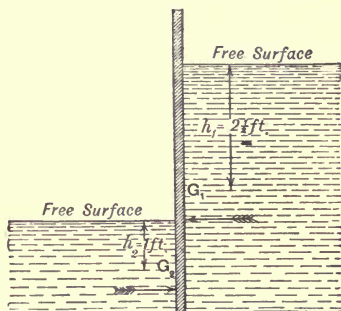
$$ENMF = H_4 A_4 W = 3' \times (6' \times 8') \times 62.5 = 9000 \text{ ,,}$$

$$BCFM = H_5 A_5 W = 9' \times (10' \times 8') \times 62.5 = 45000 \text{ ,,}$$

**EXAMPLE IV.**—A sluice gate is 4 feet broad and 6 feet deep, and the water rises to a height of 5 feet on one side and 2 feet on the other side. Find the pressure in pounds on the gate.

**ANSWER.**—The net pressure on the sluice gate is evidently equal to the difference of the pressures on the two sides.

Total pressure on—



NET PRESSURE ON SLUICE GATE.

$$\text{Back side} = H_1 A_1 W = 2.5' \times (4' \times 5') \times 62.5 = 3125 \text{ lbs.}$$

$$\text{Front side} = H_2 A_2 W = 1' \times (4' \times 2') \times 62.5 = 500 \text{ ,,}$$

$$\left. \begin{array}{l} \text{Subtracting the front from the back} \\ \text{pressure we get the net pressure} \end{array} \right\} = 2625 \text{ lbs.}$$

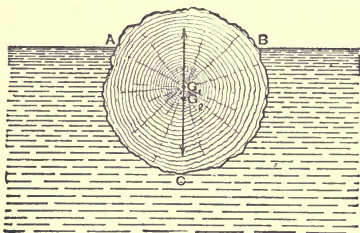
**Centre of Pressure.**—In the case of a plane area immersed in a liquid, the “*centre of pressure*” is the point at which the resultant of all the pressures of the fluid acts. If the plane be horizontal, the resultant naturally acts at the centre of the figure, and therefore the *centre of pressure* agrees with the *centre of gravity* of the figure. In the case of a vertical rectangle, having one of its edges in the surface of liquid, like a dock-gate or a sluice, the *centre of pressure* will be at a point  $\frac{2}{3}$  of the depth from the free surface and at the middle of the breadth of the immersed portion. We will have to prove this in our Advanced Course, and perhaps refer to the position of the centre of pressure in other cases.

**Immersion of Solids.**—**Archimedes’ Discovery.**—If a solid be immersed in any fluid (whether liquid or gas), it displaces a quantity of that fluid equal to its own volume. This is evident from the principle of impenetrability—viz., “*two bodies cannot occupy the same space at the same time.*”

Hence we have a simple method of determining the volume of

any irregular body by plunging it into a liquid, and noting the cubic contents of the liquid displaced, by letting it run into a measure of known volume, such as a graduated jar. This principle was first discovered by Archimedes, a philosopher of Syracuse, in the year 250 B.C. The story of this discovery is related by Vitruvius, who states that Hero, a king, sent a certain weight of gold to a goldsmith to be made into a crown. Suspecting that the workman had kept back part of the gold, he weighed the crown, but found that it was the same as the weight of the gold previously sent by him to the goldsmith. He was, however, not satisfied with this test, so he consulted Archimedes, and asked him whether he could find out if the crown was adulterated. Not long afterwards the philosopher, on going into his bath (which happened to be full of water), observed that a quantity of the water was displaced. He immediately conjectured that the water which ran over must be equal to the volume of the immersed part of his body. He was so overjoyed at the discovery that he jumped out of the bath and ran naked to the king, exclaiming, *Εὕρηκα! εὕρηκα!* (I have discovered! I have found out!) He then began to experiment with the crown by taking a quantity of pure gold of the same weight, and observed its displacement in water. Next he ascertained by the same process the volume of the same weight of silver, and finally the volume of the crown, which actually displaced more water than its equivalent weight of pure gold. In this interesting manner the fraud of the artificer was detected, to his great astonishment and chagrin, and a Law of Nature was discovered.

**Floating Bodies.**—A body is said to float in a fluid when it is entirely supported by the fluid. In order that a body may float,



CONDITIONS OF EQUILIBRIUM IN THE  
CASE OF A FLOATING BODY.

the forces acting upon it must be in equilibrium. Now, as may be seen from the case illustrated by the accompanying figure, there are only two forces to be considered—viz., the weight of the body acting *vertically downwards* through its centre of gravity  $G_1$ , and the pressure of the fluid acting *vertically upwards* through the centre of gravity  $G_2$  of the

displaced fluid. The horizontal pressures of the fluid on the body are in equilibrium by themselves, and simply tend to compress it so that they do not affect the question. The upward

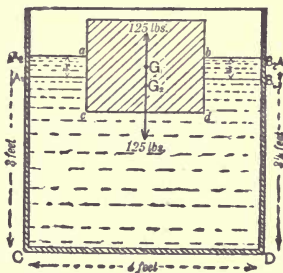
pressure of the fluid must be equal to the weight of the body, otherwise the body would rise or sink.

Also, the weight of the body must be equal to the weight of the fluid displaced. This will be evident when we remember that the total upward pressure of the fluid on the surface ACB is equal to the weight of the fluid which formerly filled that space. But, since equilibrium still exists when the body is floating, it is clear that the weight of the body must also be equal to the weight of the fluid displaced. If the body be wholly immersed it will be pressed upwards with a force equal to the weight of the fluid which it displaces. Hence the statement known as the—

**Principle of Archimedes.**—*When a body is wholly or partially immersed in a fluid it is pressed vertically upwards by the fluid with a force equal to the weight of the fluid which it displaces and this force may be taken to act at the c.g. of the displaced fluid.*

As a natural deduction from the above proof we conclude that a body cannot float in a liquid of less specific gravity than itself. A solid glass or metal ball will float in mercury, but not in water. If the specific gravity of a body be the same as that of a liquid, it will float totally submerged. If the body and the liquid are each *incompressible*, the body will float indifferently at any depth. If the body be incompressible, but be placed in a compressible fluid, such as air, the body will rise or fall until it finds a place where its mean specific gravity is the same as that of the displaced gas. This is exemplified by the case of a balloon filled with a gas lighter than air. It rises until it arrives at a height from the earth where the combined weight of the machine and the gas contained therein are equal to the weight of the same volume of air.\*

**EXAMPLE V.**—A rectangular tank, 4 feet square, is filled with water to a height of 3 feet. A rectangular block of wood, weighing 125 lbs., and having a sectional area of 4 square feet, is placed in the tank, and floats with its sides vertical and with this section horizontal. How much does the water rise in the tank, and what is now the pressure on one vertical side of the tank? (S. and A. Exam. 1892.)



**ANSWER.**—Let A<sub>1</sub>B<sub>1</sub> be the original surface of the water in the tank before the block was immersed, A<sub>2</sub>B<sub>2</sub> the surface after immersion of the block,

\* We must leave the subject of metacentres, &c., to our Advanced Course.

Let  $V_1$  = volume represented by  $A_1B_1CD$ .

„  $V_2$  = „ „ „ „  $A_2B_2CD$ .

„  $v$  = „ of water displaced by block,  
represented by  $abcd$ .

„  $x$  = amount by which the water rises in the tank  
when the block is immersed.

Then clearly,  $V_2 = V_1 + v$ .

Or,  $V_2 - V_1 = v$ .

Now  $V_2 - V_1$  = volume represented by  $A_2B_2B_1A_1$ ,  
= cross sectional area of tank  $\times x$ ,  
=  $4^2 \times x = 16x$  cub. ft.

$$\therefore 16x = v \text{ or } x = \frac{v}{16} \text{ ft.}$$

But, by the *principle of Archimedes* we know that

*The weight of water displaced by block = The weight of the block.*

$$\therefore v \times 62\frac{1}{2} \text{ lbs.} = 125 \text{ lbs.}$$

$$\therefore v = \frac{125}{62\frac{1}{2}} = 2 \text{ cub. ft.}$$

$$\therefore x = \frac{v}{16} = \frac{2}{16} = \frac{1}{8} \text{ ft.} = 1\frac{1}{2} \text{ inches.}$$

Next, we have to find *the pressure on one of the vertical sides of the tank*. Here the depth of the centre of gravity of the area of the side subjected to pressure below the free surface of the water is

$$H = \frac{1}{2}A_2C = \frac{1}{2} \left(3' + \frac{1}{8}'\right) = \frac{25}{16} \text{ feet.}$$

$$\therefore \text{Total pressure on side} = P = HAW$$

$$\therefore P = \frac{25}{16} \times (4' \times 3\frac{1}{8}') \times 62\frac{1}{2} \text{ lbs.}$$

$$\text{Or, } P = 1220.7 \text{ lbs.}$$

**Atmospheric Pressure.**—Surrounding the earth's surface there is a deep belt of air, which gets rarer and lighter the higher we rise from the earth. If we consider the case of a complete vertical column of this air, we find that it produces an average pressure on the earth's surface of about 15 lbs.; or, in other words, we say that the atmosphere produces an average pressure of 15 lbs. on the square inch, for we find that it will balance a vertical column of mercury of about 30 inches, or a vertical column of water of 34 feet. We do not experience any inconvenience from this normal pressure of the atmosphere, because we are so constituted as to be able to resist it. Should we, however, enter the closed compressed air-chamber of the underground workings of a railway tunnel (such as those in operation

at the present time for the construction of the London Tube Railway), or the caissons of a great bridge while they are being sunk (as in the case of the Forth Bridge), or go down into the sea in a diving-dress or diving-bell, then we do feel a *most uncomfortable* sensation in our ears, eyes, &c. Or, if we climb a very high mountain, or rise far into the air in a balloon, we have a somewhat similar sensation, but due to an opposite effect—viz., a diminution from the normal pressure to which we are accustomed.

**The Mercurial Barometer.**—The pressure of the atmosphere is usually measured by a mercurial barometer, which consists of a vertical tube of glass about 33 inches long, of uniform calibre, hermetically sealed at the top end, into which has been carefully introduced mercury freed from air. The lower end dips into an open dish containing a quantity of that liquid metal. Consequently the pressure of the atmosphere acting on the mercury in the open dish forces it up inside the tube to a height directly proportional to its pressure, since there is supposed to be a perfect vacuum between the upper surface of the mercury and the closed end of the glass tube.

**EXAMPLE VI.**—Suppose the height of mercury as registered by a mercurial barometer is 30 inches, and that the specific gravity of mercury be taken as 13·6, what would be the height in feet of a water column which would support the same atmospheric pressure?

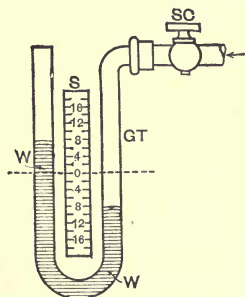
**ANSWER.**—  $1 : 13\cdot6 :: 30 \text{ inches} : x$   
 $\therefore x = 30 \times 13\cdot6 = 408'' = 34 \text{ feet.}$

**Low Pressure and Vacuum Water Gauges.\***—It is often necessary for the engineer to measure *low pressures* or *vacuums* of gases. For example, in the supply of illuminating gas to a town, or in the pressure of air feeding a boiler furnace by natural or forced draught, or the vacuum produced by a chimney-stalk; or, in the case of the vacuum in a coal mine produced by a furnace below the earth, or by a guibal fan situated near the upcast shaft, &c. In such cases, as well as in many others where low pressures have to be observed, the force is not reckoned by pounds per square inch, or by inches of mercury sustained in a vertical column, but by the number of inches of water which the pressure will support or which the vacuum will detract from the atmospheric pressure.

The accompanying figure illustrates the apparatus usually employed in determining such low pressures. It consists of a

\* For a description of mercurial pressure and vacuum gauges, as well as Bourdon's high-pressure and vacuum gauges, refer to the Author's *Elementary and Advanced Books on "Steam and Steam Engines."*

simple bent **U** glass tube with a scale between the vertical legs of the **U**, divided into inches and tenths of an inch, so that either the pressure or the vacuum may be read off in inches of water pressure, according as the forward pressure from the point of supply is positive or negative in respect to the pressure of the atmosphere. For example, let the leg of the **U** tube next the cock be connected to the gas pipe of a house, then the pressure of the gas supply acts on the water in the right-hand leg of the tube, and forces it downwards, whilst the water in the other leg rises correspondingly. The reading observed on the scale **S**, below or above the zero or equilibrium line, has of course to be doubled in order to ascertain the exact total pressure in inches of water. If the **U** tube be connected to a vacuum or negative pressure, then the water rises in the inner leg of the **U** tube, owing to the greater pressure of the atmosphere on the outer limb, and the inches of water representing the amount of the vacuum are accordingly read off in the same way. For example, if the apparatus be connected to the base of



GAS PRESSURE GAUGE.

#### INDEX TO PARTS.

SC	represents	Steam or gas cock.
GT	"	Glass tube.
S	"	Scale.
W	"	Water.

a steam boiler chimney, or to the inlet of a guibal fan creating a draught in a coal mine, then the suction produced forms a vacuum which requires the supply of atmospheric air, and consequently the air presses on the open water of the outer limb of the **U** tube, and forces it downwards. The vacuum is therefore observed and recorded by adding the inches of water below and above the zero line.

**EXAMPLE VII.**—A difference of level is observed of 4 inches between the outer and inner limbs of a **U** tube water-gauge. What is the pressure of the gas supply in lbs. per square inch?

**ANSWER.**—A vertical column of 34 feet of water corresponds to 15 lbs. pressure on the square inch. Consequently,

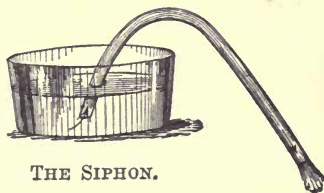
$$(34' \times 12'') : 4'' :: 15 \text{ lbs.} : x$$

$$x = \frac{15 \times 4}{34 \times 12} = \frac{5}{34}, \text{ or nearly } \frac{1}{7} \text{ of a lb. per sq. in.}$$

**The Siphon** is simply a bent tube for withdrawing liquids from a higher to a lower level by aid of the atmospheric pressure. It is used in chemical laboratories and works for emptying acids

from carboys, in breweries and distilleries for extracting beer from vats and spirits from casks, in the crystal glass tube of Lord Kelvin's recorder for conveying ink from the ink-pot to the telegraph message-paper; and on a large scale for draining low-lying districts, such as the fens of Lincolnshire.

The conditions for the successful working of a siphon are, that—



THE SIPHON.

1. The liquid shall be carried by the outer limb of the tube to a lower level than the surface of the supply.

2. The vertical height from the free surface of the liquid being drained to the top of the bend of the siphon *shall not be greater than* the height of the water barometer at the time—say only 30 feet—on account of the necessary deduction of 3 or 4 feet to be made from the full height of 34 feet, due to having to overcome the friction of the pipe.

3. The end of the siphon dipping into the liquid to be drained, shall *not become uncovered*.

To start the siphon, either the tube must be filled with liquid, the ends closed, and the siphon inverted, with the shorter limb under the fluid to be drained, before uncovering the ends; or, whilst the end of the shorter limb is in the liquid a partial vacuum must be formed in the siphon tube by extracting the air from the end of the longer leg.

*The principle upon which the siphon acts is as follows:—*

A vacuum having been formed in the tube, the pressure of the atmosphere acting on the free surface of the liquid to be drained, forces it up the shorter limb, and having turned the highest point of the  $\cap$  the liquid descends the longer limb by the action of gravity with a velocity proportional to the  $\sqrt{\text{difference of levels}}$  between the outlet and the free surface of the source of supply. The outflowing liquid is always acting as a water-tight piston at the bend of the  $\cap$ , and in this way keeping up the vacuum there, until either the inlet and the outlet free surfaces come to a level (when the siphon stops for want of "head"), or, when the difference of level between the free surface of the supply and the top of the bend exceeds the height supportable by the atmosphere, when it stops for want of breath or atmospheric pressure.

**Distinction between Solids, Liquids, and Gases.**—At the very commencement of this book we referred to the fact that Matter exists under three conditions.

(1) Solids ; (2) Liquids ; (3) Gases. We shall now define and distinguish concisely between the three states of matter.

(1) *A Solid* is matter in such a condition, that the molecules cannot move freely amongst themselves, and consequently it retains its shape and volume unless acted upon by a force.

(2) *A Liquid* is a collection of inter-mobile particles of matter, which offer great resistance to change of volume, but little to change of shape.

(3) *A Gas* is matter in its most subdivided state, and which readily yields to the slightest force tending to change its shape or its volume.

We thus see that the chief characteristic distinctions between these three states of matter are, that—

(1) *A Solid* resists both change of shape and of volume.

(2) *A Liquid* only resists change of volume.

(3) *A Gas* resists neither change of shape nor of volume.

(4) *A Fluid* may be either a liquid or a gas.

(5) *A Viscous Fluid* is a liquid which offers more or less resistance to motion amongst its particles, *e.g.*, treacle, tar, and heavy oils, &c.

(6) *An Elastic Fluid* is a gas whose volume will increase indefinitely.

(7) *Cohesion* is a property of matter common to both solids and liquids. It causes more or less resistance to the separation of the molecules of matter.

## LECTURE XVIII.—QUESTIONS.

1. What are the respective specific gravities and the weights per cubic foot and per gallon of fresh and of salt water?

2. A cylindrical vessel, 120 inches long and 10 inches in diameter, is sunk vertically in water, so that the base, which is horizontal, is at a depth of 100 inches below the surface of the water. Find the upward pressure in pounds on the base of the vessel. *Ans.* 284.2 lbs.

3. A cubical box or tank with a closed lid, the length of a side of which is 5 feet, rests with its base horizontal, and an open vertical pipe enters one of its sides by an elbow. The tank is full of fresh water, and the pipe contains water to the height of 10 feet above the top of the tank. What are the pressures of water on the top, bottom, and sides of the tank? *Ans.* 15,625 lbs.; 23,437.5 lbs.; 19,531.25 lbs.

4. A water tank 10' long, 10' wide, with an inclined base 10' deep at one end and 5' at the other end, is filled with fresh water. Find the pressure in pounds on each of the four sides and on the base. *Ans.* 31,250 lbs.; 7,812.5 lbs.; 18,229.16 lbs.; 52,500 lbs.; 10,421.8 lbs.\*

5. A lock gate is 12 feet wide, and the water rises to a height of 8 feet from the bottom of the gate. What pressure in pounds does it sustain? The weight of a cubic foot of water is  $62\frac{1}{2}$  lbs. *Ans.* 24,000 lbs.

6. A vertical rectangular sluice gate, measuring 2 feet horizontal by 3 feet vertical, is immersed so that its upper side is 4 feet below the surface of the water pressing on it. Find the pressure on the gate: you are required to explain the reasoning on which your calculation is founded. *Ans.* 2062.5 lbs.

7. What is meant by the "centre of pressure" in the case of a plane surface immersed in a liquid? If the plane be a horizontal circle, where does the centre of pressure act? If it be a vertical rectangle 10 feet wide and 6 feet deep, immersed in water so that the upper edge of the rectangle is flush with the surface of the water, where does the "centre of pressure" act? *Ans.* at the centre of the circle; 4 feet below surface of water.

8. State the law discovered by Archimedes, and the conditions for a body in equilibrium floating in a liquid. A cylinder 10 feet long and 2 feet in diameter floats in fresh water, with 2 feet projecting from the surface; find the weight of the cylinder. *Ans.* 1571 lbs.

9. A rectangular tank, 5 feet square, is filled with water to a height of  $7\frac{3}{4}$  feet. A rectangular block of wood, weighing 312.5 lbs., and having a sectional area of 5 square feet, is placed in the tank, and floats with its sides vertical and with its section horizontal. How much does the water rise in the tank, and what is now the pressure on one vertical side of the tank? *Ans.* 2.4 inches; 9875.4 lbs.

10. The mercurial barometer registers 31"; calculate the height of columns of fresh and of salt water that will balance the corresponding pressure. *Ans.* 35.13 ft., 34.24 ft.

11. Sketch and describe a mercurial barometer. State how it is made, and how it acts as a register of the pressure of the atmosphere.

12. Describe some simple form of gauge which would enable you to measure the pressure at which gas is supplied, and explain the principle on which it is constructed.

\* In the answers given,  $\sqrt{125}$  is assumed to be 11.2.

13. Sketch and explain the action of the siphon, and give a few practical examples of its use. Also state under what circumstances it fails to work.

14. The bottom of a water-tank measures 7' in length and 3' 4" in width. When the tank contains 900 gallons of water, what will be the depth of the water, and what would be the pressure on the bottom, on each side and end of the tank respectively? One gallon of water weighs 10 lbs. One cubic foot weighs 62.3 lbs. *Ans.* Pressure on bottom of tank = 9000 lbs. Pressure on each side = 8382 lbs. Pressure on one end of tank = 3991 lbs. Depth of centre of pressure is 6.2 feet.

15. Draw the diagram of water-pressure on the side of a tank with vertical sides, 12 feet high, and filled with water. Deduce the vertical depth of the centre of pressure below the top edge of the tank.

16. Name the chief physical properties of a liquid, and show in what respect a liquid differs from a gas and from a solid. How is the pressure of water on the vertical sides of a tank calculated?

A water-tank is 10' long, 10' wide, and 10' deep. When it is filled with water, what will be the force with which the water acts on one side of the tank? *Ans.* 31,250 lbs.

17. Describe how you would carry out an experiment to determine the discharge of water through a sharp edged circular orifice on the side of a water tank. (B. of E., 1904.)

## LECTURE XIX.

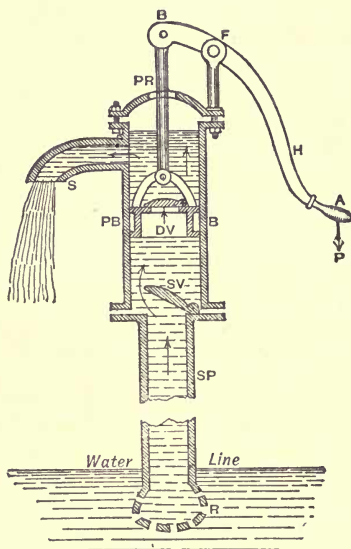
CONTENTS.—Hydraulic Machines—The Common Suction Pump—Example I.—The Plunger, or Single-acting Force Pump—Example II.—Force Pump with Air Vessel—Continuous-delivery Single-acting Force Pump without Air Vessel—Combined Plunger and Bucket Pump—Double-acting Force-Pump—Example III.—Centrifugal Pumps—Example IV.—Questions.

**Hydraulic Machines.**—The Common Suction Pump consists of a bored cast-iron barrel PB, terminating in a suction pipe, SP, fitted with a perforated end or rose R, which dips into the well from which the water is to be drawn. The object of the rose is to prevent leaves or other matter getting into the pump, that might clog and spoil the action of the valves. At the junction between the barrel and suction pipe there is fitted a suction valve SV, of the hinged clack type faced with leather. The piston or bucket B is worked up and down in the barrel of the pump by the force P, applied to the end of the handle H, being communicated to it through the connecting link of the hinged piston-rod PR. In the centre and at the top of the bucket is fixed the clack delivery valve DV, which is also faced with leather in order to make it water-tight. The bucket is sometimes packed with leather; but, as shown by the figure, a coil of tightly woven flax rope wrapped round the packing groove would be more suitable in the present instance.

*Action of the Suction Pump.*—(1) Let the barrel and the suction pipe be filled with air down to the water-line, and let the bucket be at the end of the down stroke. Now raise the bucket to the end of the up-stroke by depressing the pump handle. This tends to create a vacuum below DV; therefore the air which filled the suction pipe opens SV, expands, and fills the additional volume of the barrel. Consequently, according to Boyle's law, its pressure must be diminished in the *inverse ratio* to the enlargement of its volume.\* This enables the pressure of the atmosphere

\* The student may refer to Lecture XII. of the Author's Elementary Manual on "Steam and the Steam Engine," for an explanation and demonstration of Boyle's law; where it is shown that if  $p$  = the pressure of a gas and  $v$  = its volume, then at a uniform temperature  $pv = a \text{ constant}$ , or  $p$  varies as  $\frac{1}{v}$ .

(which acts constantly on the surface of the water in the well) to force a certain quantity of water up the suction pipe, until the weight of this column of water and the pressure of the air (between it and the delivery valves) balance the pressure of the outside atmosphere.



COMMON SUCTION PUMP.

INDEX TO PARTS.

H represents	Handle.	SP represents	Suction pipe.
P	" Push or pull at A.	R	" Rose.
F	" Fulcrum of H.	SV	" Suction valve.
PR	" Plunger rod.	B	" Bucket or piston.
B	" Pump barrel.	DV	" Delivery valve.
S	" Spout.		

(2) In pressing the bucket to the bottom of the barrel by elevating the handle, the suction valve closes and the delivery valve opens, thereby permitting the compressed air in the barrel to escape through the delivery valve into the atmosphere.

(3) Raise and depress the piston several times so as to produce the above actions over again, and thus gradually diminish the volume of the air in the pump to a minimum. Then water will have been forced by the pressure of the atmosphere up the suction pipe and into the pump, if the bucket and the valves are tight,

and if the delivery valve when at the top of its stroke be not more than the height of the hydro-barometric column above the water line of the well.\*

(4) The bucket now works in water instead of in air. In fact, the machine passes from being an air-pump to be a water one. During the down-stroke water is forced through the delivery valve. During the up-stroke this water is ejected through the spout; at the same time more water is forced up through suction pipe and valve to supply the place of the vacuum created by the receding piston. The water is therefore discharged *only* during the up-stroke in the case of the pump illustrated by the figure. Should it, however, be fitted with an air-tight piston-rod and pump cover, and should the pump handle be moved rapidly, more water will be taken into the barrel than can escape from the spout during the up-stroke. Consequently, the compression of the pent-up air between the surface of the water in the barrel and the cover, will cause the water to flow out in a more or less continuous stream during the down-stroke. In other words, the top cover and the portion of the pump above the spout may be converted into an air vessel, the precise action of which will be explained later on.

EXAMPLE I.—If the cross area of the bucket of a suction pump be 20 sq. in. and if water be raised 24 ft. from its surface in the well, what is the pull on the pump rod?

ANSWER.—The pull  $P$  on the pump rod is evidently equal to the weight of a column of water of height  $H = 24$  ft., and the area of the bucket in sq. ft.  $= A = 20 \div 144$ . Therefore, by the formula employed for the pressure of a liquid on a surface in Lectures XVII. and XVIII.—

$$P = HAW,$$

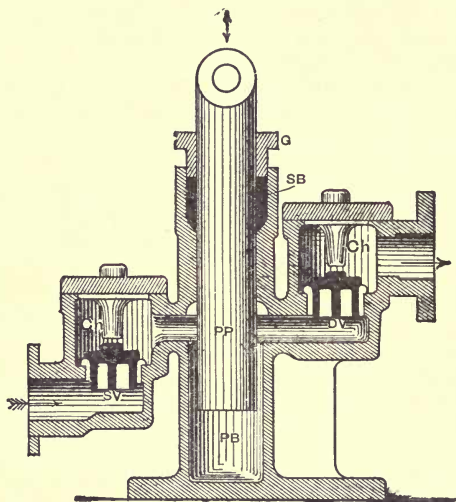
$$P = 24' \times \frac{20'}{144} \times 62.5 = 208 \text{ lbs.}$$

**The Plunger, or Single-acting Force Pump.**—The upper or outer end of the barrel of this pump is provided with a stuffing-box and gland, through the air-tight packing of which the solid pump plunger works.

During the up or outward stroke of the plunger a vacuum is

\* Theoretically, such a pump should be able to lift water from a depth of 34 feet below the highest part of the stroke of the delivery valve, but practically, owing to the imperfectly air-tight fitting of the piston and the valves, it is not used for withdrawing water from wells more than 20 to 25 feet below this position of the delivery valve. In fact, such a pump frequently requires a bucket or two of water to be poured into it above the delivery valve in order to make it work at all, if it should have been left standing for some time without being worked.

created in the pump barrel, and consequently air is expanded into it from the suction pipe. This pipe is attached to the flange of the suction valve-box. During the down or inward stroke the suction valve closes, and the pent-up air in the barrel is forced through the delivery valve. This action goes on precisely in the manner just explained in the case of the suction pump, until the water rises into the barrel. Then the inward stroke of the plunger drives water through the delivery valve to any desired height (or against any reasonable back pressure, as in the case of a feed



THE PLUNGER FORCE PUMP.

INDEX TO PARTS.

SV represents	Suction valve.	PB	represents	Pump barrel.
DV	„ Delivery valve.	PP	„	Pump plunger.
Ch	„ Checks for valves.	SB and G	„	Stuffing box and gland.

pump for a steam boiler) consistent with the strength of the pump and the power applied.

The eye of the plunger may be attached to a connecting-rod actuated by a hand lever, as in the case of the suction pump, or it may be worked from one eccentric or crank revolved by a steam engine or other motor.

By whichever way it is worked, the force applied to the plunger must be sufficient to overcome the friction between the plunger

and the packing, the resistance due to sucking the water from the source of supply, and of driving the same up to the place where it is delivered.

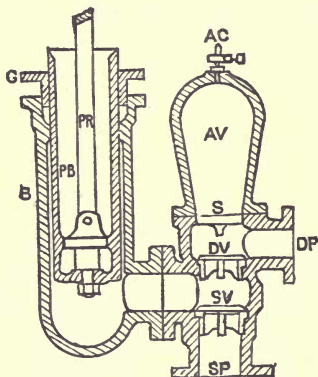
With this pump (as in the case of the suction pump), the water is only delivered during one out of every two strokes of the plunger, and consequently, in an intermittent or pulsating fashion. In order to make the supply continuous we have to use one or other of the devices about to be described.

**EXAMPLE II.**—In a single-acting plunger force pump the cross area of the plunger is 10 sq. in., and its distance from the surface of the water in the well, when at the end of its outward or suction stroke, is 20 ft. During the inward stroke the water is pumped up to a height of 100 ft. above the end of the plunger. What forces are required to move the pump plunger during (1) an "out," and (2) an in-stroke (neglecting the forces to overcome friction).

ANSWER.—(1)  $P_1 = H_1 A W = 20' \times \frac{10}{144} \times 62.5 = 86.8$  lbs. pull.

(2)  $P_2 = H_2 A W = 100' \times \frac{10}{144} \times 62.5 = 434$  lbs. pressure.

**Force Pump with Air Vessel.**—In the following figure of a force pump the only points of difference worth noticing *between*



FORCE PUMP WITH AIR VESSEL.

INDEX TO PARTS.

SP represents	Suction pipe.	DV represents	Delivery valve.
SV	" Suction valve.	S	" Stop for DV.
B	" Barrel of pump.	AV	" Air vessel.
PB	" Plunger barrel.	AC	" Air cock.
G	" Packing gland.	DP	" Delivery pipe.
PR	" Plunger rod.		

it and the previous one are :—(1) The plunger, instead of being solid, is a hollow trunk or barrel, with the connecting rod fixed to an eye-bolt at its lower end.

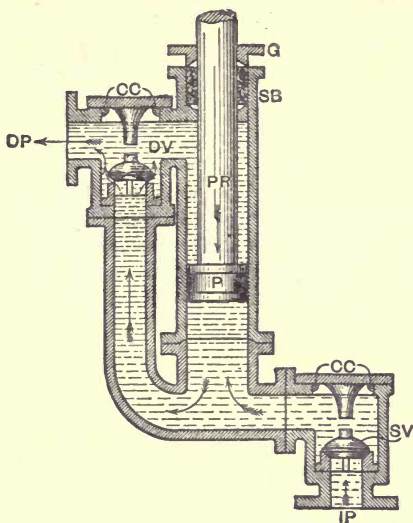
(2) The suction and the delivery valves are both at one side, instead of being fixed on opposite sides of the pump.

(3) There is an air vessel.

*Action of the Air Vessel.*—During the inward or delivery stroke of the plunger, part of the water forced from the barrel goes up the delivery pipe, and the remainder enters the air vessel, and consequently compresses the air in AV. During the outward or non-delivery stroke of the plunger the compressed air in the air vessel presses the rest of the water into the delivery pipe. In this simple way a continuous flow of water is maintained in the delivery pipe, and with far less shock, jar, and noise than in the previous case. Where very smooth working is required, an air vessel is also put on to the suction side of the pump. Should the air in the air vessel become entirely absorbed by the water, the fact will be noticed at once, by the noise and the intermittent delivery. Then the pump should be stopped, the air cock AC opened, and the water run out. When the air vessel is full of air, the air cock should be shut and the pump started again.

**Continuous-delivery Pump without Air Vessel.**—A fairly continuous delivery may be obtained by making the plunger of the piston form, and the pump rod exactly half its area, as shown by the accompanying figure. During the down stroke, half the water expelled by the piston from the under side of the pump barrel goes up the delivery pipe, and the other half is lodged above the piston, to be in turn sent up the delivery pipe during the up-stroke. Where very high pressures are required, such as in the filling of an accumulator ram, pumps working on this principle, but of the following form, are frequently used. The action is precisely the same as in the one just described, and the same index letters have been used, so that the student will have no difficulty in understanding the figure; more especially as the directions of motion of the piston and of the ingoing and outflowing water have been marked by straight and feathered arrows respectively. Where sea or acid water is used it may be necessary to fit the pump barrel, PB, with a brass liner, L, to prevent corrosion.

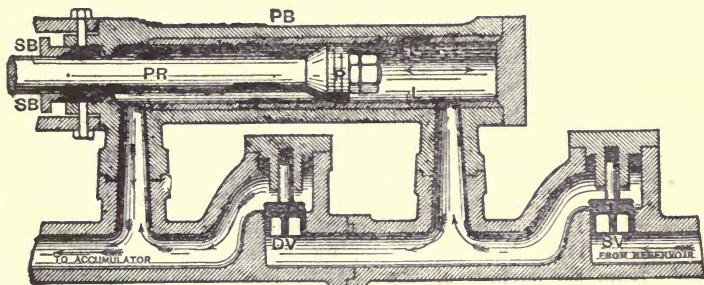
In accumulator and other kinds of high-pressure work it is not advisable to use air vessels, because you cannot prevent the water which enters the vessel absorbing air and carrying the same with it to the hydraulic machines, where its presence would be most objectionable, and because with, say, 750 to 1000 or more lbs.



CONTINUOUS-DELIVERY FORCE PUMP WITHOUT AIR VESSEL.

INDEX TO PARTS.

IP represents	Inlet pipe.	DV represents	Delivery valve.
SV	" Suction valve.	CC	" Cover and check to
CC	" Cover and check to		DV.
	SV.	DP	" Discharge pipe.
P	" Piston.	SB	" Stuffing-box.
PR	" Pump-rod.	G	" Gland.



CONTINUOUS-DELIVERY FORCE PUMP.

As used in Connection with the Armstrong Accumulator.

(See Indexes to previous Figures.)

pressure per square inch, you would require a very large and very strong air vessel before it could be of any service. If a pressure of only 750 lbs. per square inch were used, then, since the normal pressure of the atmosphere is 15 lbs. per square inch, the air in the air vessel would be compressed to  $\frac{15}{750}$ , or  $\frac{1}{50}$ th of its original volume, in accordance with Boyle's law. Consequently, with an air vessel of 50 cubic feet internal volume, there would be only 1 cubic foot of air in it, when the pump was in full action.

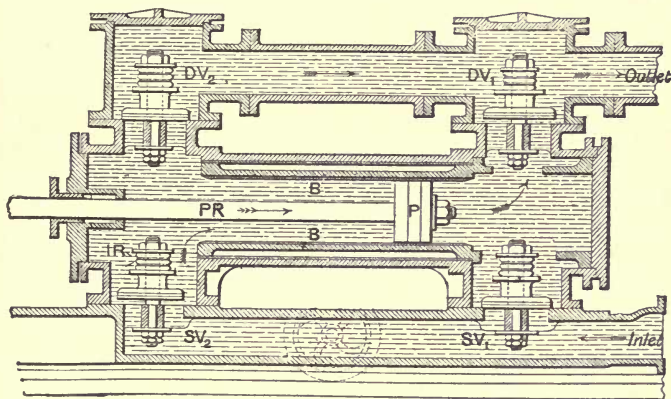
**Combined Plunger and Bucket Pump.**—We have already seen that a suction pump discharges water during the outward stroke, and that a plunger pump discharges water during the inward stroke; consequently, by combining these two kinds, we get a double-acting pump. By making the cross area of the plunger half that of the barrel, half the water raised by the bucket during the up-stroke goes into the delivery pipe, whilst the other half fills the space left by the receding plunger. During the down-stroke the plunger forces the latter half up the delivery pipe. We do not happen to have a figure with which to illustrate these remarks, but if the student will first of all sketch a complete vertical section of a suction pump like that shown by the first figure in this lecture, and then draw a solid plunger, with stuffing-box and gland, like that in the second figure, in place of the pump rod and open cover in the suction pump, it will form a useful exercise in the designing of such a pump.

**Double-acting Force Pump.**—The pumps which we have hitherto considered are all single-acting in this sense, that they do not both suck and discharge water during every stroke. This can, however, be accomplished by having two sets of suction and delivery valves placed at each end of the pump barrel, as shown by the accompanying figure. Then, during the outward stroke of the piston the pump draws water from the source of supply through the inlet pipe and suction valve  $SV_1$ . At the same time the piston forces the water in front of it through the delivery valve  $DV_2$  and outlet pipe. During the inward stroke, suction takes place through  $SV_2$  and discharge through  $DV_1$ , all as clearly shown by arrows in the drawing. The valves are provided with india-rubber cushions, IR, to ease the shock and minimise the jarring noise due to their reaction and natural reverberation when they are suddenly opened and closed.

**EXAMPLE III.**—In a double-acting force pump the vertical height from the surface of the well to the point of delivery is 100 feet. If the area of the piston equal 1 square foot, what is the stress on the piston-rod during each stroke?

**ANSWER.**—Here we need not distinguish between the force required during suction and delivery, for both actions take place

during each stroke. We have only to deal with the net force required to elevate a column of water to a height of 100 feet.



DOUBLE-ACTING FORCE PUMP.\*

## INDEX TO PARTS.

SV <sub>1</sub> , SV <sub>2</sub> represent	Suction valves.	B represents	Barrel (liner).
DV <sub>1</sub> , DV <sub>2</sub> "	Delivery valves.	P "	Piston (solid).
IR "	India-rubber cushions.	PR "	Piston-rod.

Neglecting friction, the stress on the piston rod will therefore be the weight of a column of water of height 100' and cross area = 1 sq. ft.

$$\therefore P = HAW = 100' \times 1' \times 62.5 = 6250 \text{ lbs. pull and push.}$$

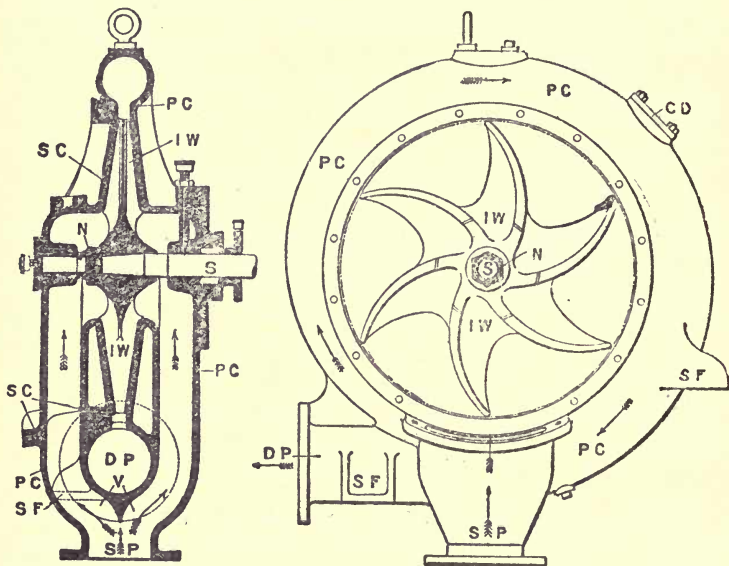
If 30 per cent. of the force applied be spent in overcoming friction, what will then be the stress on the pump-rod. Here 6250 is only 70 per cent. of the whole stress, for 30 per cent. of the whole is lost force.

$$\therefore 70 : 100 :: 6250 : x$$

$$x = \frac{6250 \times 100}{70} = 8928.5 \text{ lbs. pull and push.}$$

\* We are indebted for the above figure to Professor H. Robinson's book on "Hydraulic Machinery," published by Messrs. Charles Griffin & Co. Students should refer to Lecture XXIV. of the Author's Elementary Manual on "Steam and the Steam Engine" for detailed illustrations and description of the air and circulating pumps of the SS. "St. Rognvald."

**Centrifugal Pumps.**—The following illustration shows one of these pumps or reversed water turbines. They are often used in preference to the reciprocating pumps previously described, when large quantities of water have to be quickly elevated through a small height, such as emptying graving docks and holds of vessels,



THE BON-ACCORD CENTRIFUGAL PUMP.

Designed and made by Drysdale & Company, Glasgow.

#### INDEX TO PARTS.

SF for Supporting flanges.  
 PC „ Pump casing.  
 OP „ „ door.  
 SP „ Suction pipe.  
 V „ Suction water guide.

IW for Impeller wheel.  
 S „ „ shaft.  
 N „ „ nut.  
 DP „ Delivery pipe.  
 → „ Direction of flow.

or circulating the cooling water through the condenser tubes of steam-engines, as well as for dredging soft-bottomed rivers.

The original type of centrifugal pump had straight radial blades, but it has been found, that if these are curved in the direction and manner shown by the accompanying figure, there is less shock due to the quick flowing water and greater efficiency. They may be driven by belts, direct coupled steam-engines, turbines, or electric motors as preferred.

*Details.*—As will be seen from the vertical cross-section and side view, the chamber consists of a snail-like outer pump casing P C, supported upon two flanged feet S F, connected to a suction pipe S P, and a delivery pipe D P. In the centre is fitted the shaft S, which carries an impeller wheel I W, that rotates between the tapered inside faces of the pump casing P C, and a removable side cover S C.

*Action when circulating cold water through a steam condenser.*—Should this pump be situated below the level of the supply water, the air is driven out of the pump and its pipes by this head of water. In such a case, the pump can be started straight away by its motor. But, where the pump is situated above the suction supply, then the mere rotation of the impeller wheel I W does not produce a sufficient vacuum to make the water rise into it; and consequently, the pump casing has either to be filled with water through the nipple hole (beside the lifting eye-bolt) or a steam ejector with a sluice valve are added in certain cases. Supposing that the pump is fairly started, then the mere rotation of I W inside the water-tight casing not only gives the kinetic energy and pressure to the water contained therein, to force the same right through the delivery pipe D P and the condenser tubes, but also to keep up the necessary vacuum in the pump, so as to ensure a continuous feed of water through the suction pipe. It will be observed, that the incoming water is divided by the sharp, knife-edged portion of cast iron, at the volute V, and that it flows equally up each side to the centre of the wheel, whereby the same is subjected to balanced side pressures.

Should the interior of the pump require to be inspected, the attendant may first open the cleaning door C D, but if he finds that any adjustment is required, then he can take off the side cover S C. When this is removed, he will obtain a clear view of the whole of I W, and he may remove the same from the tapered end of the shaft S, by unscrewing the nut N.

EXAMPLE IV.—A centrifugal pump is to lift 6.2 cubic feet of water per second to a height of 7 feet; how much horse-power must be supplied to it if its efficiency is 45 per cent?

It is direct-driven by a continuous current electro-motor which works at 200 volts. How many amperes of current must be supplied to the motor, if its efficiency is 85 per cent? (B. of E., 1904.)

$$\left. \begin{array}{l} \text{Useful work done} \\ \text{by pump in lifting} \\ \text{the water} \end{array} \right\} = \begin{array}{l} \text{Weight of water in lbs.} \times \text{distance in ft. through} \\ \text{which it is raised.} \end{array}$$

$$\text{ " " } = 6.2 \times 62.5 \times 7 = 2712.5 \text{ ft.-lbs.}$$

$$\left. \begin{array}{l} \text{But, Efficiency of} \\ \text{Pump, } \eta_p \end{array} \right\} = \frac{\text{Useful work done by centrifugal pump in ft.-lbs.}}{\text{Total work done in driving the centrifugal pump.}}$$

$$\left. \begin{array}{l} \frac{45}{100} \end{array} \right\} = \frac{2712.5 \text{ ft.-lbs.}}{\text{Total work done.}}$$

$$\therefore \left. \begin{array}{l} \text{Total work done} \\ \text{in driving Cen-} \\ \text{trifugal Pump} \end{array} \right\} = \frac{2712.5 \times 100}{45} \text{ ft.-lbs.}$$

And this work is done *per second*.

$$\therefore \left. \begin{array}{l} \text{H.P. required} \\ \text{to drive the Cen-} \\ \text{trifugal Pump} \end{array} \right\} = \frac{2712.5 \times 100}{45 \times 550} = 10.96 \text{ H.P.}$$

Also 1 H.P. = 746 watts, and 1 watt = 1 volt  $\times$  1 ampere.

$$\left. \begin{array}{l} \text{Hence, Watts} \\ \text{given out by Motor} \end{array} \right\} = 10.96 \times 746. \text{ watts.}$$

$$\left. \begin{array}{l} \text{And, Efficiency of} \\ \text{Motor} \end{array} \right\} = \frac{\text{Watts given out.}}{\text{Watts taken in.}}$$

$$\left. \begin{array}{l} \frac{85}{100} \end{array} \right\} = \frac{10.96 \times 746}{\text{Watts taken in.}}$$

$$\therefore \left. \begin{array}{l} \text{Watts taken in} \\ \text{by Motor} \end{array} \right\} = \frac{10.96 \times 746 \times 100}{85} \text{ Watts.}$$

$$\left. \begin{array}{l} \text{But, Watts taken} \\ \text{in by Motor} \end{array} \right\} = \text{Current in Amperes} \times \text{Pressure in Volts.}$$

$$\therefore \left. \begin{array}{l} \text{Current in am-} \\ \text{peres} \end{array} \right\} = \frac{\text{Watts taken in by Motor.}}{\text{Pressure in Volts.}}$$

$$\left. \begin{array}{l} \text{ " " } \end{array} \right\} = \frac{10.96 \times 746 \times 100}{85 \times 200}$$

$$\left. \begin{array}{l} \text{ " " } \end{array} \right\} = \frac{4088}{85} = 48.1 \text{ Amperes.}$$

## LECTURE XIX.—QUESTIONS.

1. Explain the manner in which the pressure of the atmosphere is made serviceable in the case of the common suction pump. Sketch and explain by an index the details of this pump.

2. Describe, with a sketch, an ordinary suction or lifting pump, and explain its action. If the diameter of the bucket is 4", and the spout is 20' above the free surface of the well, what is the tension on the pump-rod in the up-stroke? *Ans.* 109 lbs.

3. Sketch and describe a force pump, drawing a section so as to show the packing of the plunger and the construction of the valves. How is an air-vessel applied to such a pump? Why is the air-vessel dispensed with when pumping water into an accumulator?

4. Explain the use of an air-vessel in connection with a force pump. Sketch a section through a double-acting force pump, showing the valves and the connection of the pump with the air-vessel, and explain the action of the pump. Water is forced up to 100 feet above the air-vessel; what proportion of the volume of the air-vessel is occupied with water, and what is the pressure of the air therein? *Ans.* 74·6 per cent.; 43·35 lbs. per sq. in. above the atmospheric pressure.

5. The leverage to the end of the handle of a common force pump is five times that to the plunger, and the area of the plunger is 5 square inches; what pressure at the end of the lever handle will produce a pressure of 45 lbs. per square inch on the water within the barrel? *Ans.* 45 lbs.

6. A force pump is used to raise water from a well to a tank. The piston has a diameter of 1·6", and is 20' above the free surface of the water in the well, and 40' below the mouth of the delivery pipe leading into the tank. Find the force required to work the pump—(1) Neglecting friction; (2) when 30% is spent in overcoming friction; (a) when sucking, (b) when forcing, (c) what is the work put in and got out per double stroke of 6"? *Ans.* (a) (1) 17·45 lbs.; (2) 24·93 lbs.; (b) (1) 34·9 lbs.; (2) 49·86 lbs.; (c) 37·39 ft.-lbs.; 26·17 ft.-lbs.

7. What is the difference between a double-acting and a single-acting pump? The area of the plunger of a force pump being 3 square inches, find the pressure upon it when water is forced up to a height of 20'. *Ans.* 26·04 lbs.

8. Describe, with a sketch, some form of pump which will deliver half the contents of the barrel at each respective up-stroke and down-stroke of the pump-rod. Name the valves.

9. Sketch and describe a "double-acting force pump." If the diameter of the piston be 12", the stroke 3', the distance from pump to well 20', from pump to position for delivering the water 40', and if the number of strokes per minute be 40, what is (1) the theoretical horse-power required to work the pump, (2) the actual, if 30 per cent. of the power be spent against friction. *Ans.* (1) 10·71; (2) 15·3.

10. What is the difference between a single and a double acting pressure pump? Sketch in section a double-acting force pump for working at high pressure, showing the arrangement of valves, and indicate of what material the several parts should be constructed.

11. Sketch and describe the construction and action of some form of pump by which you could raise water from a well where the level of the

water is 45 feet below the surface of the ground. Explain fully where you would fix the pump, and give reasons for the arrangement which you propose to adopt.

12. Sketch in section and describe the action of the ordinary lifting pump. In such a pump the pump-rod is  $\frac{3}{4}$  inch in diameter, and the pump barrel is 5 inches in diameter, while the spout at which the water is delivered is 20 feet above the surface of the pump bucket when the latter is at its lowest point; what would be the maximum tension on the pump rod in the upstroke of the pump, neglecting the weight of the pump rod and the pump bucket, also the weight of water below bucket in suction pipe (the weight of a cubic foot of water is 62.5 lbs.)? *Ans.* 166.6 lbs.

13. Describe the construction and action of an ordinary suction-pump for raising water from a well. If 200 gallons of water are raised per hour from a depth of 20 feet, and if the efficiency of the pump is 60 per cent., what horse-power is being given to the pump? *Ans.* .034.

## LECTURE XX

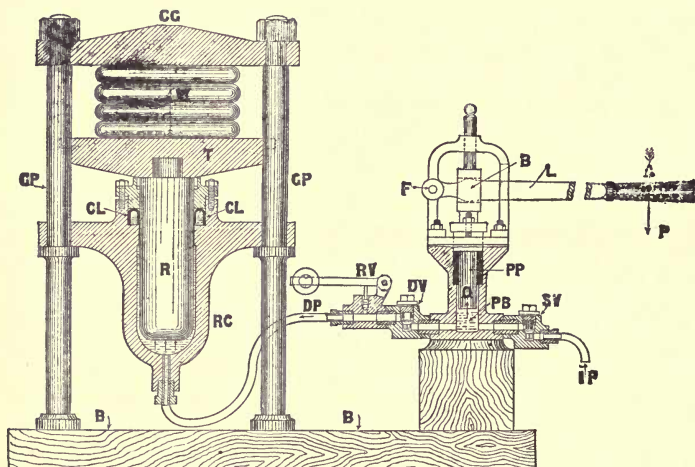
**CONTENTS.**—Bramah's Hydraulic Press—Bramah's Leather Collar Packing—Examples I. II.—Large Hydraulic Press for Flanging Boiler Plates—The Hydraulic Jack—Weem's Compound Screw and Hydraulic Jack—Example III.—The Hydraulic Bear or Portable Punching Machine—The Hydraulic Accumulator—Example IV.—Questions.

**Bramah's Hydraulic Press.**—This useful machine was invented by Pascal, but he could not make the moving parts watertight. Bramah, about the year 1796, discovered a means by which this difficulty was effectually overcome; and thus the instrument has been handed down to us under his name. As may be seen from the following figure, it consists of a single-acting force pump in connection with a strong cylinder containing a plunger or ram, which is forced outwards from the cylinder through a tight collar by the pressure of the water delivered into the cylinder from the force pump.

From what was said in Lecture XIX. about force pumps, we need not particularise about this part of the machine, except to say that the suction and delivery valve boxes can be disconnected from the pump, and the valve cover-checks removed at any time for the purpose of examining the parts, or of regrounding the valves into their seats. The plunger extends through a stuffing-box and gland filled with hemp packing, and is guided by a centrally bored bracket bolted to the top flange of the pump. The lever fits through a slot in this guide-bar, whereby it has an easy free motion, when communicating the force applied through it to the pump plunger. The relief-valve RV has a loaded lever, adjusted like the lever safety valve in Lecture IV., so as to rise and let the water escape when the pressure exceeds a certain amount. It may also be used for taking the pressure of the object under compression, or for lowering the ram R by simply lifting the little lever and pressing down the table T, when the water flows easily from the cylinder, and out of DP by the relief valve. The delivery pipe DP is made of solid drawn brass, and the ram cylinder is carefully rounded at the bottom end, instead of being flat, in order that it may be naturally of the strongest shape.\*

\* In the case of large cylinders for very great pressures, the lower or

The guide pillars are securely bolted to the base B by nuts and iron washers, not shown. The cup leather packing CL deserves special attention, because it formed the chief improvement by



VERTICAL SECTION OF A BRAMAH HYDRAULIC PRESS,  
Made in the Engineering Workshop of The Glasgow Technical College.

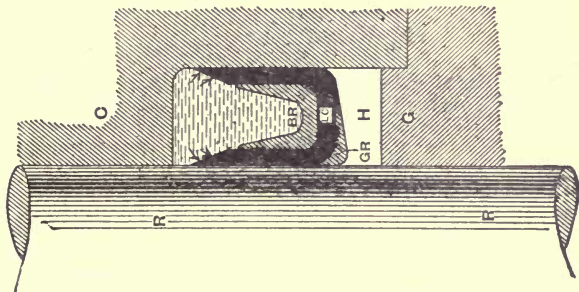
#### INDEX TO PARTS.

L represents	Lever.	DV represents	Delivery valve.
P	" Pressure on L at A.	RV	" Relief valve.
F	" Fulcrum of L.	DP	" Delivery pipe.
B	" L's connection with plunger's guide- rod.	RC	" Ram cylinder.
		R	" Ram or plunger.
PP	" Pump plunger.	CL	" Cup leather packing.
Q	" Reaction or stress on plunger PP.	T	" Top, table, or T piece.
PB	" Pump barrel.	W	" Weight lifted, or total pressure on R.
IP	" Inlet pipe.	CG	" Cross girder.
SV	" Suction valve.	GP	" Guide pillars.
		BB	" Base block.

inner end of the cylinder should be carefully rounded off, both inside and outside. For, if left square, or nearly square, the crystals formed in the casting of the metal naturally arrange themselves whilst cooling in such a manner as to leave an initial stress, and consequent weakness, inviting fracture along the lines joining the inside to the outside corners of the cylinder end. The severe shocks and stresses to which this weak line of division is subjected during the working of the press would sooner or later force out the end of the cylinder, in the shape of the frustum of a cone, unless the cylinder had been made unnecessarily thick and strong at the bottom end.

Bramah on Pascal's press. It consists of a leather collar of **U** section, placed into a cavity turned out of the neck of the cylinder, and kept there by the gland of the cylinder cover. The following figure shows an enlarged section of Bramah's packing suitable for a huge press, where the desired shape of the leather collar **LC** is maintained by an internal brass ring, **BR**, and an outside metal guard ring **GR**, resting on a bedding of hemp **H**. It will be observed at once, from an inspection of this figure, that the water which leaks past the easy fit between the plunger or ram **R**, and the cylinder **C**, presses one of the sharp edges of the leather collar against the ram, and the other edge against the side of the bored cavity in the neck of the cylinder, with a force directly proportional to the pressure of the water in the cylinder. By this simple automatic action, the greater the pressure in the cylinder the tighter does the leather collar grip the ram and bear on the cylinder's neck.

**Bramah's Leather Collar Packing.**—This collar is made from a flat piece of new strong well-tanned leather, thoroughly soaked in water, and forced into a metal mould of the requisite



ENLARGED VIEW OF BRAMAH'S LEATHER COLLAR FOR A  
BIG HYDRAULIC PRESS.

#### INDEX TO PARTS.

<b>R</b> represents Ram.	<b>BR</b> represents Brass ring.
<b>C</b> "     Cylinder.	<b>GR</b> "     Guard ring.
<b>G</b> "     Gland of C.	<b>H</b> "     Hemp bedding.
<b>LC</b> "     Leather collar.	

size and shape until it has assumed the form of a **U** collar. The central or disc portion of the leather is then cut out, and the circular edges are trimmed up sharp in the bevelled manner shown by the above figure.

**Formula for the Pressure on the Ram of a Bramah Press.**

—Referring again to the first figure in this Lecture, it will be found that by taking moments about the fulcrum at F, we obtain the pressure or reaction Q on the plunger of the force pump. Therefore, neglecting weight of lever and friction, we get—

$$P \times AF = Q \times BF. \quad \therefore Q = \frac{P \times AF}{BF}$$

Further, by Pascal's law for the transmission of pressure by liquids, enunciated in Lecture XVII., we know that the statical pressure Q is transmitted with undiminished force to every corresponding area of the cross section of the ram.

Or,  $Q : W :: \text{area of plunger} : \text{area of ram}.$

$$\therefore W \times \text{area of plunger} = Q \times \text{area of ram}.$$

$$W \times \pi r^2 = Q \times \pi R^2$$

Where  $r$  = radius of plunger, and  $R$  = radius of ram, both in the same unit. Substituting the previous value for  $Q$ , and dividing each side of the equation by  $\pi$ , we get—

$$W \times r^2 = \frac{P \times AF}{BF} \times R^2$$

$$\therefore W = \frac{P \times AF}{BF} \times \frac{R^2}{r^2}$$

Since the radius of a circle is directly proportional to its diameter, we may write the formula thus, where  $D$  is the diameter of the ram and  $d$  the diameter of the plunger, both in the same unit—

$$W = \frac{P \times AF}{BF} \times \frac{D^2}{d^2}$$

**EXAMPLE I.**—In a small Bramah press,  $P = 50$  lbs.,  $AF = 20$  in.,  $BF = 2$  in., area of plunger = 1 sq. in., whilst area of ram = 14 sq. in. Find  $W$ , neglecting friction and weight of lever.

**ANSWER.**—By the above formula—

$$W = \frac{P \times AF}{BF} \times \frac{\pi R^2}{\pi r^2}$$

Substituting  
values, we get—

$$W = \frac{50 \times 20}{2} \times \frac{14}{1} = 7000 \text{ lbs.}$$

**EXAMPLE II.**—In Bramah's original press at South Kensington the plunger is 3" in diameter, and it acts at a distance of 6 inches from the fulcrum, which is at one end of a lever 10 feet 3 inches long, carrying a loaded scale-pan at the other end. What should be the pressure of the water in the press in order to lift a weight of 3 cwt. in the scale-pan, neglecting the weight of the lever? Make a diagram of the arrangement. (S. and A. Exam. 1892.)

ANSWER.—Here  $d = 3$  in., consequently the area of the plunger  $= \frac{\pi d^2}{4} = 7.854 \times 3'' \times 3'' = 7$  sq. in.;  $BF = 6''$ ;  $AF = 10' 3'' = 123''$ ;  $P = 3$  cwt.  $= 3 \times 112 = 336$  lbs.; and we have to find the pressure per sq. in. on the ram that will balance  $P$ , acting with the stated advantage, since the area of the ram is not given. By the formula—

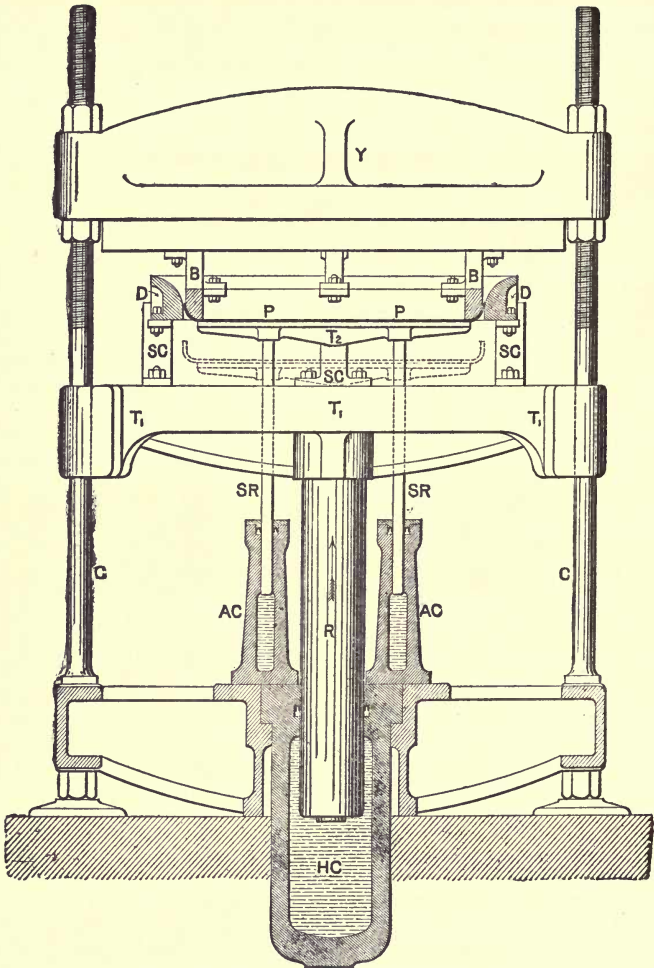
$$W = \frac{P \times AF}{BF} \times \frac{\text{area of 1 sq. in.}}{\text{area of plunger}} = \frac{336 \times 123''}{6''} \times \frac{1 \text{ sq. in.}}{7 \text{ sq. in.}} = 984 \text{ lbs.}$$

$\therefore$  Pressure per sq. inch on ram of press  $= 984$  lbs.

### Large Hydraulic Press for Flanging Boiler Plates, &c.—

As an example of the practical application of the Bramah press to modern boiler-making, the accompanying illustration shows the form which it takes when worked by a high-pressure water supply derived from a central accumulator, which may at the same time be used to work cranes, punching, riveting, and other similar machine tools, in the same works.

The operation of flanging, say the end tube-plates of the cylindrical barrel of a locomotive boiler, is carried out in the following manner:—The ram  $R$  is lowered to near the bottom of the hydraulic cylinder  $HC$ , thus leaving room to place the boiler plate (which has been heated all round the outside edge) on the movable table  $T_1$ . High-pressure water is then admitted from the central accumulator to the auxiliary cylinders  $AC$ , thus forcing the side rams  $SR$ ,  $SR$ , with their table  $T_2$ , and the plate  $P$ , vertically upwards, until the upper surface of the plate bears hard against the bearers  $B$ ,  $B$ , or internal part of the dies. Water from the same source is now admitted into the hydraulic cylinder  $HC$ , which forces up the ram  $R$ , with its table  $T_1$ , supporting columns  $SC$ ,  $SC$ , and the external part of the dies  $D$ ,  $D$ , until the latter has quietly and smoothly bent the hot edge of the plate round the curved corner of the internal bearer  $B$ ,  $B$ . The ram  $R$  is now lowered, carrying with it the table  $T_1$  and dies  $D$ , by letting out water from  $HC$ , and then the table  $T_2$ , with the flanged plate, are lowered by letting out water from  $AC$ . The plate is removed from its table, allowed to cool, placed in position in the barrel of the boiler, marked off for the rivet holes, drilled and riveted in the usual manner. The student will now understand what a useful and powerful servant a hydraulic press is to the engineer in the hands of a skilful workman, for it can be made to do work in the manner indicated above in far less time, and with far greater certainty of uniformity and exactitude, than the boiler-smith could turn out, with any number of hammermen to help him. It is fast replacing, the steam-hammer for crossing work, and the steam or belt-driven punching and riveting



LARGE HYDRAULIC PRESS FOR FLANGING BOILER PLATES.\*

\* The above figure is a reduced copy of one from Prof. Henry Robinson's book on "Hydraulic Machinery," published by Messrs. Charles Griffin & Co., but it has been indexed according to the Author's style of symbols, and described in an elementary manner.

## INDEX TO PARTS.

HC represents	Hydraulic cylinder.
R	" Ram of HC.
C, C	" Columns supporting Y.
Y	" Yoke or cross-head.
BB	" Bearers of the internal die ring.
P	" Plate to be flanged.
DD	" Dished die or external die ring.
SO	" Supporting columns for DD.
T <sub>1</sub>	" T-piece or movable table for DD.
T <sub>2</sub>	" T-piece or movable table for P.
SR	" Side rams for T <sub>2</sub> .
AC	" Auxiliary cylinders.

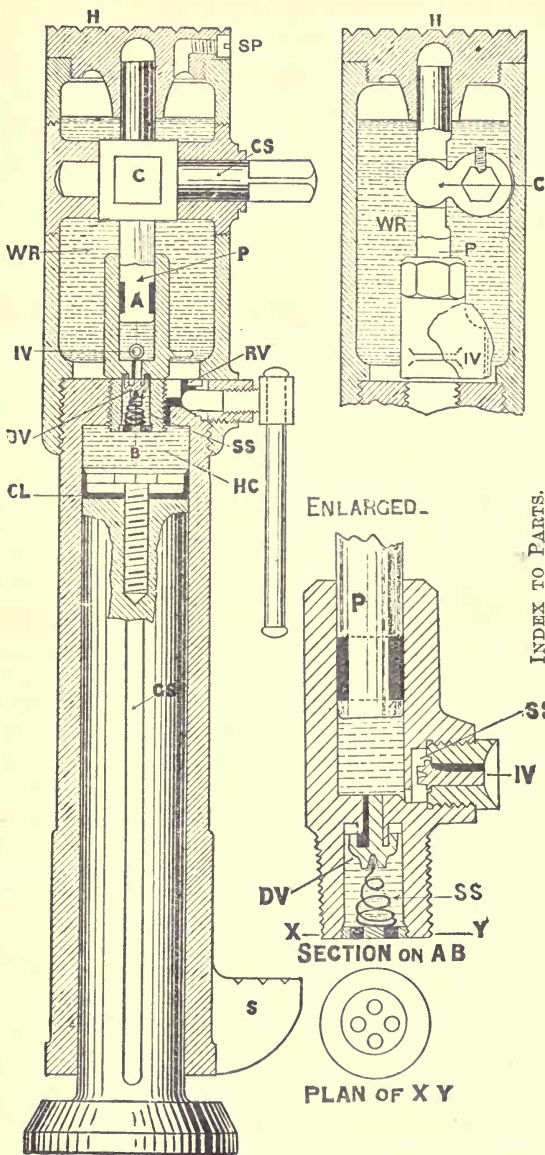
machines, the steam screw and wheel-gear worked cranes, screw and wheel-gear hoists, as well as the screw press for making up bales of goods mentioned in Lecture XV. For with it, you can bring to bear a force of a few pounds on the square inch or as many tons, by merely turning the handle of a small cock, and with a certainty of action unattainable by any other means.

The Hydraulic Jack is a combined hydraulic press and force pump, arranged in such a compact form as to be readily portable, and applied to lifting heavy weights through short distances. It therefore effects the same objects as the screw-jack described in Lecture XV., but with less manual effort or greater mechanical advantage.

The base on which the jack rests is continued upwards in the form of a cylindrical plunger, so as to constitute the ram of the hydraulic cylinder HC. Along one side of this ram there is cut a grooved parallel guide slot GS, into which fits a steel set pin, screwed through the centre of a nipple cast on the side of the cylinder (not shown in the drawings) for the purpose of guiding the latter up and down without allowing it to turn round. The top of the ram is then bolted with a water-tight cup leather CL, by means of a large washer and screw-bolt.

The action of this cup leather is precisely the same as the leather collar in the cylinder of the Bramah press already described; but it has only to be pressed by the water in one direction—viz., against the sides of the truly-bored cast-steel cylinder, instead of against both the ram and the cylinder neck, as in the previous case. The head H and upper portion of the machine is of square section, and is screwed on to the hydraulic cylinder in the manner shown by the figure. It contains a water reservoir WR, which may be filled or emptied through a small hole by taking out the screw-plug SP.\* In the centre line of the head-

\* This screw plug SP is slackened back a little to let the air in or out



INDEX TO PARTS.

H represents Head for placing under a load.  
 SP Screw plug for filling and emptying [WR].  
 WR Water reservoir.  
 C Crank.  
 CS Crank shaft.  
 P Pump plunger.  
 IV Inlet valve.

DV represents Delivery valve.  
 SS Spiral springs for IV and DV.  
 RV Relieve valve.  
 HC Hydraulic cylinder.  
 CL Cup leather-packing.  
 CS Guide slot along the length of ram.  
 S Step for placing under a load.

THE HYDRAULIC JACK.

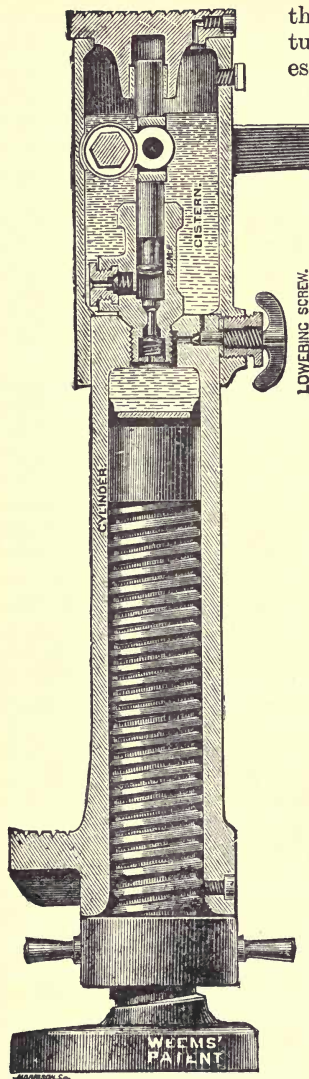
piece there is placed a small force pump, the lower end of which is screwed into the centre of the upper end of the hydraulic cylinder. This pump is worked by the up-and-down movement of a handle placed on the squared outstanding end of the turned crank shaft CS. To the centre of the crank shaft there is fixed a crank C, which gears with a slot in the force-pump plunger P, and thus the motion of the handle is communicated to the pump plunger in a reduced amount, corresponding to the inverse ratio of the lengths of the handle and the crank from the fulcrum or centre of the crank shaft. By comparing the right-hand section of the water reservoir, and the section on the line AB, with the vertical left-hand section of the jack, it will be seen where the inlet and delivery valves IV and DV are situated. On raising the pump plunger P, water is drawn from WR into the lower end of the pump barrel through IV, and on depressing the plunger this water is forced through the delivery valve DV into the hydraulic cylinder, thus causing a pressure between the upper ends of the cylinder and the ram, and thereby forcing the cylinder, with its grooved head H, and footstep S, upwards, and elevating whatever load may have been placed thereon. Both the inlet and outlet valves are of the kind known as "mitre valves." They have a chamfer cut on one or more parts of their turned spindles, so as to let the water in and out along these channels. The valves are assisted in their closing action by small spiral springs SS, bearing in small cups or hollow centres, as shown more clearly in the case of DV by the enlarged section on AB.

**Weems' Compound Screw and Hydraulic Jack.**—This is a jack combining some of the advantages of the ordinary screw-jack with those of the hydraulic one. It is often desirable to be able to bring the head or footstep into trial contact with the load before applying the water pressure. This can easily be done by turning the nut at the foot of the screw, cut on the ram of the jack. The arrangement will at once be understood from the figure. It will be observed that the load may also be lowered by turning this nut, or by the screw-tap which permits water to flow from the cylinder back into the cistern, as in the previous case. The bottom nut may be screwed hard up to the foot of the hydraulic cylinder, so as to sustain the whole load, and thus prevent overhauling through leakage of the water.

When it is necessary to lower the load or the head of the jack,

of the top of the water reservoir when working the jack. There is generally another and separate screw plug opening (as will be seen by the following figure of Weems' patent jack) for filling or emptying the water reservoir, quite independent of the above-mentioned one, which is used in this case for both purposes.

the relief valve or lowering screw, is turned so as to permit the water to escape from the hydraulic cylinder



back into the water reservoir, as clearly shown by the drawing. This may be done very gently by simply giving this screw a very small part of a complete turn; in other words, by throttling the passage between the hydraulic cylinder and the water reservoir. Or it may be done quickly by turning it through one or more revolutions. This passage can then be closed by screwing the plug home on its seat.

Mr. Croydon Marks, in his book on "Hydraulic Machinery," illustrates and describes another method of lowering the jack-head (first introduced by Mr. Butters, of the Royal Arsenal, Woolwich), where, by a particular arrangement, the inlet and delivery valves are acted upon by an extra depression of the handle, and consequent movement of the pump plunger. He also gives the main dimensions, with a drawing, of the standard 4-ton pattern as used by the British Government, where the ram has a diameter  $D = 2''$ , the pump plunger a diameter  $d = 1''$ ; and the ratio of the leverage of the handle to the crank is 16 to 1. Therefore from the previous formula we find that,

*The Theoretical Advantage =*

WEEMS' COMPOUND SCREW AND  
HYDRAULIC JACK.

$$\frac{W}{P} = \frac{AF}{BF} \times \frac{D^2}{d^2} = \frac{16}{1} \times \frac{2^2}{1^2} = \frac{64}{1}$$

And he instances two trials by Mr. W. Anderson, the Inspector-general of Ordnance Factories, to determine the efficiency of these jacks, where, with a pressure on the end of the working handle of 76 lbs., the theoretical load should have been 76 lbs.  $\times$  theoretical advantage =  $76 \times 64 = 4864$  lbs., instead of which it was only 3738 lbs.;

$$\therefore \quad . \quad . \quad 4864 \text{ lbs.} : 3738 \text{ lbs.} : 100 : x$$

$$\text{Or,} \quad . \quad . \quad x = \frac{3738 \times 100}{4864} = 77 \% \text{ efficiency}$$

In a second trial, a load of 1064 lbs. required a pressure of 22 lbs. on the handle, and consequently the efficiency at this lighter load, as might be expected, was less, or only 74 %.

EXAMPLE III.—With a hydraulic jack of the dimensions given above, and of 77 % efficiency, it is desired to lift a load of 4 tons; what force must be applied to the lever handle?

ANSWER.—By the previous theoretical formula,

$$W = \frac{P \times AF}{BF} \times \frac{D^3}{d^3}$$

$$\therefore P = \frac{W \times BF}{AF} \times \frac{d^3}{D^3}$$

$$P = \frac{4 \times 2240 \times 1}{16} \times \frac{1^3}{2^3} = 140 \text{ lbs.}$$

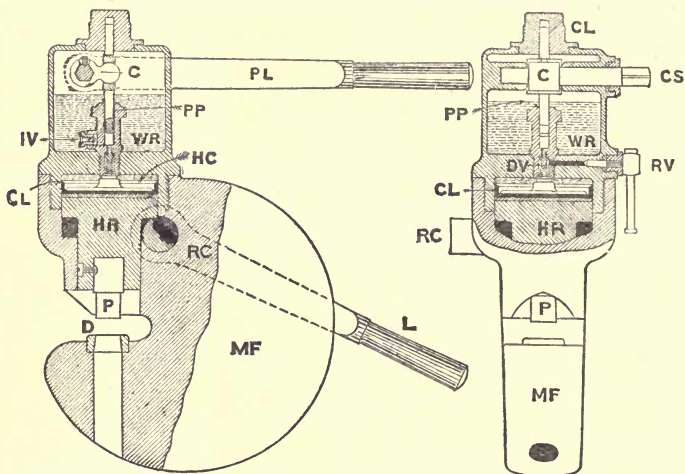
But the efficiency of the machine is only 77 %: consequently 140 lbs. is 77 per cent. of the force required—

$$\therefore 77 : 100 :: 140 \text{ lbs.} : x \text{ lbs.}$$

$$x = \frac{140 \times 100}{77} = 181.81 \text{ lbs.}$$

**The Hydraulic Bear, or Portable Punching Machine.**—This is another very useful application of the hydraulic press and force pump. It is used in every iron or steel shipbuilding-yard and bridge-building works. By comparing the drawing with the index to parts, and taking into consideration the fact that its construction and action are so very similar to the hydraulic jack already described in full detail, we need say nothing more than direct the student's attention to the action of the raising cam, and to the means by which the apparatus is lifted and suspended. In order to raise the punch for the admittance of a plate between it and the die D, the relief valve RV must first be turned backwards, and the lever L depressed. This causes the corner of the raising cam RC to force the hydraulic ram HR upwards, and the water from the hydraulic cylinder HC back into the water

reservoir WR. The relief valve may now be closed and the plate adjusted in position. Then the pump lever can be worked up and down until the punch P is forced through the plate, and the punching drops through the die D and the hole in the metal frame MF, on to the ground, or into a pail placed beneath to receive it.



SIDE VIEW AND SECTION.

END VIEW AND SECTION.

### THE HYDRAULIC BEAR, OR PORTABLE PUNCHING MACHINE.

#### INDEX TO PARTS.

PL represents	Pump lever.	HC represents	Hydraulic cylinder.
CS	„ Crank shaft.	CL	„ Cup leather.
C	„ Crank.	HR	„ Hydraulic ram.
PP	„ Pump plunger.	RC	„ Raising cam.
WR	„ Water reservoir.	L	„ Lever for RC.
IV	„ Inlet valve.	P	„ Punch.
DV	„ Delivery valve.	D	„ Die ring.
RV	„ Relief valve.	MF	„ Metal frame.

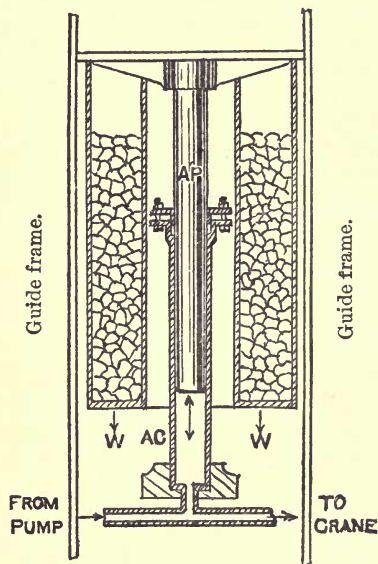
The whole bear is suspended by a chain (worked by a crane or other form of lifting tackle) attached to a shackle, whose bolt passes through a cross hole in the back of the metal frame MF, just above, but a little to the front of the centre of gravity of the machine. This hole and shackle are not shown in the drawing, but the student can easily understand that the hole would be bored a little above where the letters RC appear on the side view,

and that the chain would pass clear of the pump lever, since it works well to the right-hand side of the bear.

**The Hydraulic Accumulator.**—The demand for hydraulic power to work elevators, cranes, swing bridges, dock gates, presses, punching and riveting machines, &c., being of an intermittent nature—at one moment requiring a full water supply at the maximum pressure, and at another a medium quantity, whilst in many cases all the machines may be idle—it is evident that if an engine with pumps were devoted to supplying this demand in a direct manner, the power thereof would have to be equal to the greatest requirements of the plant, and would have to instantly answer any and every call from the same. In the case of a low-pressure supply, as for lifts, this difficulty is best overcome by placing one tank in an elevated position at the top of the hotel or building where the lift is required, and another tank below the level of the lowest flat. Then a small gas engine working a two- or three-throw pump, or a Worthington duplex steam pump, may be used to elevate the water more or less continuously from the lower to the higher tank. The “head” of water in the elevated tank will, if sufficient, work the lift at the required speed, and the discharged water from the hydraulic cylinder will enter the lower tank, to be again sent round on the same cycle of operations. Should the lift be stopped for any considerable time, then a float in the upper tank, connected by a rope or chain with the shifting fork for the belt-driven pumps (in the case of the gas engine) will force the belt over on to the loose pulley, or shut off the steam from the Worthington pump. And when the water falls in the upper tank, the float will cause a reverse movement of the rope and shift the belt to the tight pulley, or open the steam valve, and so start the pumps. When the pressures required are great, such as for cranes, &c., where 700 lbs. on the square inch is considered a very medium pressure, an elevated tank would be out of the question, for it would have to be fully 1600 feet high in order to exert this force and to overcome friction. Under these circumstances recourse is had to a very simple and compact arrangement called an accumulator, of which a lecture diagram is herewith illustrated, without any details of cocks or valves, and automatic stopping and starting gear. A steam engine or other motor works a continuous delivery pump, of the combined piston and plunger type, without the aid of an air vessel, as illustrated by the fourth and fifth figures in Lecture XIX. The water from the pump enters the left-hand branch pipe leading into the foot of the accumulator cylinder, and forces up the accumulator ram with its cross head or top T piece, and the attached weight or dead load, until the ram has reached nearly to the end of its stroke. Then

the top of the T piece or a projecting bracket on the side of the wrought-iron cylinder containing the dead load, engages with and lifts a small weight attached to a chain passing over a pulley fixed to the guide frame or to the wall of accumulator house. This chain is connected directly to the throttle valve of the steam

engine supply pipe, or to the belt shifting gear (if the pump is driven by belt gearing), and being provided with a counter-weight, the motor and pump are automatically stopped by the raising of the weight and the chain in the accumulator house. Should the water which has been forced into the accumulator cylinder be now used by a crane or other machine, the load on the ram causes it to follow up and keep a constant pressure per square inch on the water. The starting weight naturally falls as the receding T piece or bracket descends, thus pulling the starting chain, and opening the steam engine throttle valve, or shifting the belt from the loose to the fixed pulley, and again setting the pump to work. Should the hydraulic machines be working continuously, then the pump is kept going, for the water from it passes directly on to the machines, and only the surplus water finds its



THE HYDRAULIC ACCUMULATOR.

#### INDEX TO PARTS.

- AC for Accumulator cylinder.
- AP „ Accumulator plunger or ram.
- W „ Weight or load contained in an annular cylinder of wrought iron and suspended from the top of T-piece or crosshead.

way into the accumulator cylinder if the pump's supply exceeds the demand of the machines for water.

The annular cylinder of wrought iron is generally filled with scrap iron, iron slag, or sand, or other inexpensive weighty material. The accumulator cylinder AC has a stuffing-box and gland at its upper end. A coil of hemp woven into a firm rectangular section and smeared with white lead is placed in the bottom of the stuffing-box. The gland is screwed down on the top

of this packing until the normal pressure of the water in the cylinder cannot leak past it. Cup leather packing is seldom used for this simple form of accumulator; just the ordinary packing that would be used for pump rods is found to answer all requirements. This is the simplest form of accumulator which we have described, but it requires the greatest load for a certain hydraulic pressure per square inch. There are several other forms of accumulators, and several most interesting appliances such as capstans, cranes, bridges, punching and riveting machines, &c., are worked by them, which we would have liked to have described here, but the limits of our space and the complexity of their construction necessitate our deferring this pleasure to our Advanced Course.

EXAMPLE IV.—Describe and sketch in section a hydraulic accumulator, showing how the ram is kept tight in the cylinder. A hydraulic press, having a ram 16 inches in diameter, is in connection with an accumulator which has a ram 8 inches in diameter and is loaded with 50 tons of ballast; what is the total pressure on the ram of the press? (S. and A. Exam. 1892.)

ANSWER.—The first part of the question is answered by the previous figure and by the text.

By Pascal's Law the pressure *per square inch* in the accumulator is equal to the *pressure per square inch in the hydraulic press*. Consequently—

$$\frac{\text{Total Pressure on Press}}{\text{Total Load on Accumulator}} = \frac{\text{Cross Area of Press}}{\text{Cross Area of Accumulator}}$$

$$\frac{P}{50 \text{ tons}} = \frac{\pi}{4} \times 16^2 \bigg/ \frac{\pi}{4} \times 8^2 = \frac{16^2}{8^2}$$

$$\therefore P = \frac{50 \times 16 \times 16}{8 \times 8} = 200 \text{ tons.}$$

## LECTURE XX.—QUESTIONS.

1. Draw a section through a hydrostatic press, showing the cylinder, ram, and force pump, together with the valves. Why is the base of the cylinder of a large press rounded instead of being flat as in a steam cylinder? If the diameter of the ram is 9 times that of the force pump, and if  $Q$  be the pressure on the pump, what is the pressure exerted by the ram, neglecting friction? *Ans.*  $81Q$ .

2. Explain by aid of a sketch the mode of packing the ram of a hydraulic press and explain how it acts. The force which actuates the force pump is applied at the end of a lever giving a mechanical advantage of 14 to 1, and the area of the plunger of the pump is 1 square inch. What pressure must be applied to the end of the lever to produce a pressure of 1 ton per square inch on the water enclosed in the press? *Ans.* 160 lbs.

3. In the force pump of a press the area of the plunger is  $\frac{1}{4}$  of a square inch, the distance from the fulcrum of the lever handle to the plunger is 2 inches, and the distance from the fulcrum to the other end of the lever is 2 feet; what pressure per square inch is exerted on the water underneath the plunger, when a weight of 20 lbs. is hung at the end of the lever handle? *Ans.* 720 lbs. per square inch.

4. In what way do you estimate the theoretical advantage gained by the use of the hydraulic press? In a small press the ram is 2 inches and the plunger  $\frac{1}{2}$  inch in diameter; the length of the lever handle is 2 feet, and the distance from the fulcrum to the plunger is  $1\frac{1}{2}$  inches. Find the pressure exerted on the ram when 10 lbs. is hung at the end of the lever. *Ans.* 2560 lbs.

5. In an hydraulic press with two pumps the plungers are  $2\frac{1}{2}$  and 1 inch in diameter, and each is worked by a similar lever, which is acted on by the same force. When the larger pump alone is at work the pressure on the ram is 40 tons; what will it be when the smaller plunger is only working? *Ans.* 250 tons.

6. An hydraulic press, which is used for making lead pipes, has a ram 20 inches in diameter, while the ram which presses the lead is 5 inches in diameter. Find the pressure per square inch on the lead when the hydraulic gauge indicates 1 ton per square inch. Sketch a sectional elevation of the press, and show the packing of the hydraulic ram. *Ans.* 16 tons.

7. How is the pressure taken off the object under compression when required, in a hydraulic press? Sketch the arrangement. What is the proportion of the diameters of the plunger and ram when the theoretical advantage gained thereby is 100 to 1, neglecting friction? *Ans.* 1 to 10.

8. Make a rough sketch, and write a short description of the hydraulic lifting jack. It may be arranged on any system that you are acquainted with. Show clearly how the valves act and how the jack is lowered.

9. Sketch and describe the hydraulic bear or portable punching machine. Explain how the punch is raised and how the tool is handled.

10. Sketch and describe the construction of a vessel suitable for storing up a supply of water under pressure, and intended for actuating hydraulic machinery. If the plunger of this vessel be 17 inches in diameter, what load will bring the pressure of the water to 700 lbs. per square inch? *Ans.* 158,950 lbs.

11. Sketch and describe the hydraulic accumulator for storing up water

under pressure. If the ram of the accumulator be 6 inches in diameter, what load will be required to produce a water pressure of 500 lbs. on the square inch? To what head of water would this pressure correspond?

*Ans.* 14,142·8 lbs. and 1152 feet.

12. A hydraulic accumulator, with a ram of 16 inches in diameter, is connected with a hydraulic press whose ram is 26 inches in diameter. The load on the accumulator is 80 tons; what force would the press exert? Make a vertical section through the accumulator, showing its construction. *Ans.* 211·25 tons.

13. Make a sectional sketch of a hydrostatic press suitable for giving a pressure of 100 tons, showing the valves and pump and by what contrivance the leakage of water is prevented.

The pump for such a press has a cylindrical plunger 1 inch in diameter with a lever of 10 to 1, what should be the least diameter of the ram which would give 100 tons pressure when a force of 56 lbs. was applied at the end of the pump lever? What form is most suitable for the base of the ram cylinder, and for what reason is a special form adopted?

*Ans.* 20 inches.

14. Sketch and describe any tool used by riveters and worked by water pressure. (S. E. B. 1902.)

15. The pressure of water in a hydraulic company's main is 750 lbs. per square inch, and the average flow is 25 cubic feet per minute. What horse-power does this represent? If the charge for the water is twopence per 100 gallons, what is the cost per horse-power hour? (S. E. B. 1902.) *Ans.* 81·8; 2·3d.

16. Distinguish between the velocity ratio and the mechanical advantage of a machine. In a hydraulic lifting jack the ram is 6" in diameter, the pump plunger is  $\frac{7}{8}$ " diameter, the leverage for working the pump is 10 to 1. What is the velocity ratio of the machine? Experimentally we find that a force of 20 lbs. applied at the end of the lever lifts a weight of 8500 lbs. on the end of the ram. What is the mechanical advantage of the machine?

*Ans.* 470; 425.

17. A hydraulic crane is supplied with water at a pressure of 700 lbs. per square inch, and uses 2 cubic feet of water in order to lift 4 tons through a height of 12 feet. How much energy has been supplied to the crane? and how much has been converted into useful work?

*Ans.* 201,600 ft.-lbs.; 107,520 ft.-lbs.

18. Sketch and describe the construction and working of any hydraulic accumulator with which you are acquainted. If an accumulator has a ram 20" diameter with a lift of 15', and the gross weight of the load lifted is 130 tons, what is the pressure of water per square inch and the maximum energy in ft.-lbs. stored in the accumulator, neglecting friction? (S. E. B. 1900.) *Ans.* (1) 927 lbs. (2) 4,368,000 ft.-lbs.

19. A single-acting hydraulic engine has three rams, each of 3 inches diameter; common crank 3 inches long; pressure of water above that of exhaust 100 lbs. per square inch; 100 revolutions per minute; no slip of water. What is the horse-power? If this engine does 2·15 horse-power usefully by means of a rope, what is the efficiency? (S. E. B. 1901.) *Ans.* 3·2 horse-power, and efficiency ·67.

20. Water at a pressure of 700 lbs. per square inch is supplied to a hydraulic crane, and 11 cubic feet are used in lifting 15 tons through a height of 18 feet. How much energy has been given to the crane? How much energy has been wasted? (B. of E. 1903.)

*Ans.* Energy given to crane = 1,108,800 ft.-lbs.

Energy which is wasted = 504,000 ft.-lbs.

21. The ram of a hydraulic accumulator is 4 inches in diameter; what is the total weight of the ram and the load upon it in lbs. if the desired water pressure in the accumulator is  $1\frac{1}{2}$  tons per square inch, neglecting friction? If, owing to the friction of the ram against the cup leathers, 5 per cent. of the load is wasted, what load would be necessary to produce the required pressure? (B. of E. 1905.)

*Ans.* (i.) 18·85 tons; (ii.) 19·8 tons.

22. Convert a Horse-Power-Hour into foot-pounds per minute.

If water under the pressure of 700 lbs. per sq. inch acts upon a piston or ram 1 square foot in area through a distance of 1 foot, what work is done? What work is done per gallon of water?

If the Hydraulic Company charges 18 pence for 1000 gallons of such water, how much is this per horse-power-hour? (B. of E. 1904.)

*Ans.* 1 H.P. = 33,000 ft.-lbs. per minute. ∴ 1 H.P.-hour =  $33,000 \times 60$  ft.-lbs. Work done on ram = 100,800 ft.-lbs. Work done per gallon of water = 16,128 ft.-lbs., and cost per H.-P.-hour = 2·23 pence.

## LECTURE XXI.

CONTENTS.—Motion and Velocity—Uniform, Variable, Linear, and Angular Velocity—Unit of Velocity—Acceleration—Unit of Acceleration—Acceleration due to Gravity—Graphic Representation of Velocities—Composition and Resolution of Velocities—Newton's Laws of Motion—Formulae for Falling Bodies—Formulae for Linear Velocity—with Uniform Acceleration—Atwood's Machine with Experiments—Results and Formulae—Galileo's and Kater's Pendulum Experiments—The Path of a Projected Body—Centrifugal Force due to Motion in a Circle—Centrifugal Force Machine—Experiments I. II. III.—Example I.—Balancing High-speed Machinery—Centrifugal Stress in the Arms of a Fly-wheel—Example II.—Energy—Potential Energy—Kinetic Energy—Accumulated Work—Accumulated Work in a Rotating Body—The Fly-wheel—Radius of Gyration—Example III.—The Fly Press—Example IV.—The Energy Stored in a Rotating Fly-wheel—Motion on Bicycle and Railway Curves—Momentum—Examples VI. to IX.—Questions.

**Motion and Velocity.**—(1) Motion is the opposite of rest, for it signifies change of position.

(2) *Velocity* is the rate at which a body moves, or rate of motion. It is considered *absolute* when it is measured from some fixed point, and *relative* if it refers to another body in motion at the same time.

(3) *Uniform Velocity* takes place when the rate of motion does not change—i.e., when the body moves over equal distances in equal times.

(4) *Variable Velocity* takes place when the rate of motion changes—i.e., when a body moves with either a constantly increasing or decreasing velocity. For example, a stone pitched into the air rises with a gradually *decreasing* velocity, but falls with a gradually increasing rate of motion..

(5) The *Unit of Velocity* is the velocity of a body which moves through unit distance in unit time. The British unit of velocity is therefore 1 foot in 1 second. In physical problems velocity is generally expressed in *feet per second*, but for convenience the engineer reckons the piston speed of engines in *feet per minute*, and the public speak of the speed of a man walking, of a horse trotting, or of a train, in miles per hour.

(6) *Linear Velocity* is the rate of motion in a straight line, and is measured, as we have just stated, in feet per second or per minute, or in miles per hour.

If  $v$  = the velocity;  $l$  = the distance; and  $t$  = the time.

$$\text{Then } v = \frac{l}{t}; \text{ or } l = vt; \text{ or } t = \frac{l}{v}.$$

(7) *Angular Velocity* is the rate at which a body describes an angle about a given point—for example, the number of revolutions per minute of a pulley; but angular velocity may also be measured by the feet per second or per minute which a point at a known distance from the centre of motion moves.

(8) *Acceleration*.—In the case of variable velocity, the *rate of change of the velocity* is termed *the acceleration*, and may be either positive or negative—i.e., it may be an increasing or a decreasing rate.

(9) The *Unit of Acceleration* is that acceleration which imparts unit change of velocity to a body in unit time; or in this country it is *an acceleration of 1 foot per second in one second*.

(10) The *Acceleration due to Gravity* is considerably greater than the above unit, and varies at different places on the earth's surface. At Greenwich it is 32.2 feet per second in one second. In Elementary Applied Mechanics questions we will indicate it by the symbol  $g$ , and consider  $g = 32$  feet per second in one second.

**Graphic Representation of Velocities.**—The *linear velocity* of a point (such as the *c.g.* of a body) may be represented in the same way as we have hitherto represented a force. A line drawn from a point with an arrow-head indicates the direction of motion, and the length of the line to scale the magnitude of the velocity. (See p. 3, Lecture I.)

**Composition and Resolution of Velocities.**—Velocities may be compounded and resolved in exactly the same way as we treated forces by the parallelogram and triangle of forces, &c., in Lecture VIII.

**Newton's Laws of Motion.**—I. *A body in motion, and not acted on by any external force, will continue to move in a straight line and with uniform velocity.*

II. *When a force acts upon a body in motion, the change produced in the quantity of motion is the same, both in magnitude and direction, as if the force acted on the body at rest.*

*The change in the quantity of motion is therefore proportional to the force applied, and takes place in the direction of that force.\**

III. *If two bodies mutually act upon each other, the quantities of motion developed in each in the same time are equal and opposite.*

Or, *Action and reaction are equal and opposite.*

These three laws were first stated clearly by Sir Isaac Newton as the result of *inductive* reasoning. Having observed certain facts, he set about investigating what would be the consequence if his conjectures as to these facts were applied to particular

\* Here "quantity of motion" means "momentum," or mass  $\times$  velocity.  
 $\therefore$  Quantity of motion or momentum =  $Wv/g$ .

cases. Finding that his estimate of the probable result came true, he formulated a general law in accordance with his observations and reasonings.

The student has already conceived the truth of the first and third laws in the reasonings and applications of force to matter, treated of in the previous Lectures. We will now give in as brief a form as possible the formulæ for falling bodies, because they naturally lead on to the formulæ for "*centrifugal force*" on a rotating body, and to the "energy stored" up in a moving body, both of which are of great interest and importance to the young engineer. The experimental and algebraical proofs of these formulæ are given in most books on Theoretical Mechanics, so that we might assume that the student had studied these, yet the following will be refreshing.

**Formulæ for Falling Bodies.**—If a body falls freely *in vacuo* under the action of gravity from *rest* through a height  $h$  feet; then (since gravity produces a constant acceleration in the velocity of the body) at the *end* of each successive second the velocity of the body will be increased by  $g$ , or 32 feet. Let  $v$  be the velocity of the body at the end of  $t$  seconds,

$$\text{Then,} \quad . \quad . \quad v = gt; \text{ but } v^2 = 2gh$$

$$\therefore \quad . \quad . \quad h = \frac{v^2}{2g} = \frac{g^2 t^2}{2g} = \frac{1}{2}gt^2$$

$$\text{And,} \quad . \quad . \quad t = \frac{v}{g} = \frac{\sqrt{2gh}}{g} = \sqrt{\frac{2h}{g}}$$

**Formulæ for Linear Velocity with Uniform Acceleration.**—Suppose that instead of the uniform accelerating force of gravity we have any other constant force of  $F$  lbs. acting on a body, and if this force moves the body through a distance of  $l$  feet along a *perfectly smooth horizontal plane*, the above formulæ naturally become\*—

$$\text{Then,} \quad . \quad . \quad v = at; \text{ but } v^2 = 2al$$

$$\therefore \quad . \quad . \quad l = \frac{v^2}{2a} = \frac{a^2 t^2}{2a} = \frac{1}{2}at^2$$

$$\text{And,} \quad . \quad . \quad t = \frac{v}{a} = \frac{\sqrt{2al}}{a} = \sqrt{\frac{2l}{a}}$$

\* We intentionally use the letter  $l$  for length or distance, and  $a$  for acceleration. Most writers use the word "space" for distance and the symbol  $s$ ; but space is of *three* dimensions, and involves the idea of volume. It cannot therefore be, *strictly* speaking, used to represent distance or length, which is only of *one* dimension. The letter  $f$  is also often used for acceleration; but  $f$  naturally represents a force, so we prefer to use,  $a$ , for acceleration, in order to be consistent with our notation.

**Atwood's Machine.\*** *General Description.*—This machine is much used in Physical Laboratories by teachers and students, to prove by experiment the previously mentioned Laws and Formulæ for the relation between time, distance, velocity and acceleration. The results have many important applications to practical mechanics which will be dealt with later on in this Manual, and in the author's more advanced text-book on Applied Mechanics, vol. ii.

In the latest form, (see fig. p. 263) it is provided with both vertical and horizontal adjustments for the scale and plumb-bob, and may be fixed to the wall of a class-room by its brackets. The graduated brass upright is 8 feet long, and carries on its upper end a light, accurately adjusted and balanced aluminium wheel, which runs freely in agate bearings to minimise the friction. The inertia of this wheel is equivalent to that of a known mass situated at its circumference†. Two sliding platforms, one of which is a sliding ring; a fine, strong, flexible silk thread; cylindrical weights with circular overweights of the same diameter  $\bigcirc$ , and riders  $-\bigcirc-$  are also provided.

The release of the left-hand weight is gently effected by aid of a pneumatic pipe and ball, as shown at the foot of the figure. This permits the other right-hand weight with its rider  $-\bigcirc-$  to start falling freely without either jerk, swing or vibration. A correction for the friction of the wheel bearings may be made by putting a little fine wire on the descending weight when two equal cylindrical weights are used, until no slowing down is observable after setting these weights in motion. The weight of the cord or thread passing over the pulley may be balanced by a length of the same kind and size, attached to the bottoms of the right and left weights; or, by hanging therefrom loose, vertical lengths of such thread. Either of these plans enables the experimenter to raise and lower the weights into position for adjustment or starting an experiment.

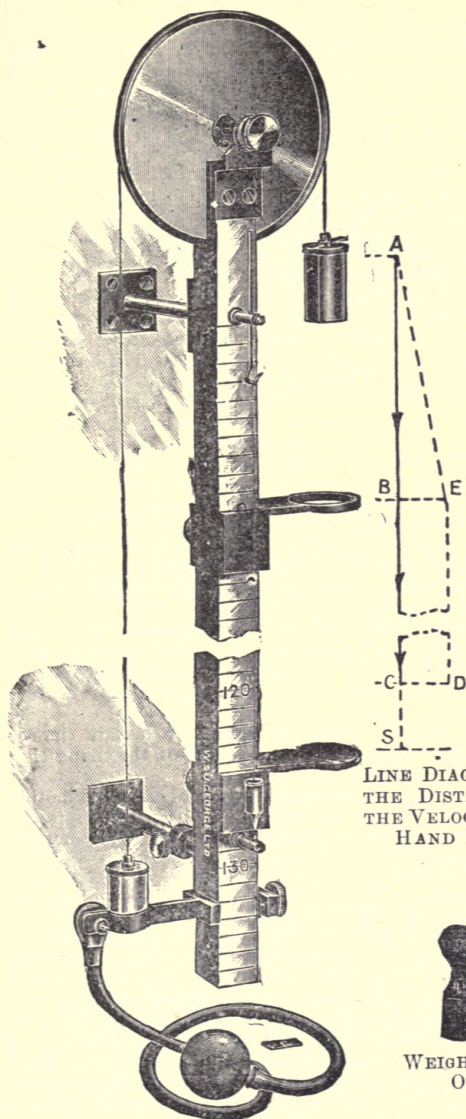
*Method of Using the Machine.*—If the two weights attached to the ends of the cord are equal, then no motion will ensue, since the downward force of gravity acting upon one weight balances its downward force upon the other through their tensions on the connecting thread. The balanced wheel is unaffected by the action of gravity. But, if the right-hand weight be made heavier than the left-hand weight, by placing a very small wire weight to balance the friction of the wheel bearings, plus a rider  $-\bigcirc-$  upon it, then, it will be set in motion, solely due to the action of gravity upon the rider, until the rider is caught by the ring. The ring, although intercepting the rider, permits the weight with the little wire weight to pass clear through it without touching. The stop-stand may be clamped at any desired position on the vertical scale.

Now, referring to the two following figures, let us suppose that only a rider  $-\bigcirc-$  be placed on the right-hand weight whose bottom is level with the

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\* George Atwood was born in 1745, educated at Cambridge, where he became a Fellow and Tutor of Trinity College. He published a few treatises on Mechanics and Engineering, and died in 1807. The author first experimented with the old Atwood's Machine in 1868-9, made by Professor Copeland in 1796, which belonged to the Natural Philosophy Department of Aberdeen University, and again with an identically similar machine at the College of Science and Arts, Glasgow, from 1880-87. He had it fitted with automatic electrical time starting and recording apparatus.

† This mass may be arrived at by trial as explained under the following case VI.



LINE DIAGRAM TO REPRESENT  
THE DISTANCES, TIMES, AND  
THE VELOCITIES OF THE RIGHT-  
HAND FALLING WEIGHT.



WEIGHTS, RIDERS, AND  
OVERWEIGHTS.

zero of scale and top level with A on the line diagram. Then if the ring intercepts this rider at B one second after the weights have begun moving, the motion during that time has been uniformly accelerated, and the velocity is represented to scale by line BE.\* But, when the right-hand weight passes the ring, its motion will thereafter be uniform until its base is arrested by the stop-stand with the velocity which it had acquired when its top *just* passed through the ring, viz., CD which is equal to BE. We may thus find the velocity gained in the first second, and, by changing the mass of the weights and riders, the positions of the ring and of the stop-stand, we can try the following and other experiments, wherein the rider may be replaced by one or other of the circular overweights which pass clear through the ring, and cause uniform acceleration until they come to the stop-stand.

*Noting the Times in Seconds.*—These may be done by aid of a simple pendulum beating distinct seconds, or a laboratory clock or a split seconds stop-watch. It is simplest and best to start the right-hand weight with its lower surface at zero upon the beat of the sixtieth second of a minute; to so set the ring that it catches the rider at an exact number of seconds from the start, and that the base of the right-hand weight is arrested by the stop-stand S, at an exact number of seconds from the time of starting. This saves any confusion and trouble, arising from noting parts of a second, and ensures that the two known distances passed through by the *top* of the right-hand weight, viz., from A to B and from A to C, are accomplished in exact whole seconds. The distance CS is simply equal to the height of the right-hand weight.

I. To prove, for *uniform velocity*, that the velocity is equal to the distance passed through divided by the time; that is,  $v = h/t$ ; or,  $h = vt$ .

II. To prove, that the distances described from rest are proportional to the squares of the times; that is,  $h \propto t^2$ .

III. To prove, that the distances from rest during acceleration are half those described in the same time, after the motion has become uniform.

IV. To prove, that the acceleration (or increase of velocity per second) is equal to the velocity (at any instant) divided by the time from rest; that is,  $a = v/t$ ; or,  $v = at$ .

V. To prove, that the distance moved through in a certain time is equal to half the acceleration multiplied by the square of the time; that is,  $h = \frac{1}{2} a t^2$ ; or,  $a = 2h/t^2$ ; and  $v^2 = 2 a h = 2 a l$ .

VI. To prove, that when a force produces uniformly accelerated motion in a body, or a system of bodies, then the acceleration is directly proportional to the force and inversely proportional to the total mass moved; or,  $a = \frac{F}{M}$ ; but  $F = M a = m g$

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\* The scientific meaning of the term *acceleration* is an increase of velocity per unit of time. And, since velocity is a rate or distance per unit of time, we see that acceleration means an increase of distance passed over by a body per unit of time per unit of time; or, an increase of distance per second per second. Whereas, when traffic managers of railways or shipping companies speak or write of the speeds of their trains or ships being accelerated, they simply mean that their average velocity has been increased so as to cover their normal distance between certain stations or ports in less time than before. It is only during the times of getting up to their full speed that the train or ship has (+) or positive acceleration, and when slowing down that (-) or negative acceleration is experienced.

## RESULTS OF EXPERIMENTS WITH AN ATWOOD'S MACHINE.

No. of Experiment.	Position of Ring B from zero A in ins.	Position of stop-stands from zero A in inches.	Time in seconds to ring B from rest.	Time in seconds to stop-stand S from rest.
1	$1\frac{3}{4}$	$8\frac{3}{4}$	1	3
2	7	21	2	4
3	$15\frac{3}{4}$	$47\frac{1}{4}$	3	6

CASE I.—When accelerating force is removed then the velocity becomes uniform and is equal to the distance passed through  $\div$  the time; that is,

$$v = \frac{h}{t}; \text{ or, } h = vt$$

Exp. (1) Distance from ring to stand =  $8\frac{3}{4} - 1\frac{3}{4} = 7$  inches.  
Time taken to move " " =  $3 - 1 = 2$  seconds.

$$\therefore v_1 = \frac{h_1}{t_1} = \frac{7}{2} = 3.5'' = \text{velocity acquired in 1 second.}$$

Exp. (2) Distance from ring to stand =  $21 - 7 = 14$  inches.  
Time taken to move " " =  $4 - 2 = 2$  seconds.

$$\therefore v_2 = \frac{h_2}{t_2} = \frac{14}{2} = 7'' = \text{velocity acquired in 2 seconds.}$$

Exp. (3) Distance from ring to stand =  $47\frac{1}{4} - 15\frac{3}{4} = 31\frac{1}{2}$  inches.  
Time taken to move " " =  $6 - 3 = 3$  seconds.

$$\therefore v_3 = \frac{h_3}{t_3} = \frac{31\frac{1}{2}}{3} = 10.5'' = \text{velocity acquired in 3 seconds.}$$

CASE II.—The distances described from rest are proportional to the squares of the times.

Exp. (1) Distance in 1st second =  $1\frac{3}{4}$  inches =  $\frac{7}{4} = \frac{7}{4} \times 1$ , or ::  $1^2$

Exp (2) " 2nd " = 7 " =  $7 = \frac{7}{4} \times 4$ , " ::  $2^2$

Exp. (3) " 3rd " =  $15\frac{3}{4}$  " =  $\frac{63}{4} = \frac{7}{4} \times 9$ , " ::  $3^2$

CASE III.—The distances from rest during acceleration are half those described after the motion becomes uniform, in the same time.

Exp. (1) Distance passed through from rest in 1 second =  $(1\frac{3}{4} \times 1)$  inches.

Distance after motion becomes uniform in  $(3 - 1$   
= 2 seconds) is  $8\frac{3}{4} - 1\frac{3}{4}$  " " " " " " = 7 " "

$\therefore$  Distance in 1 second after motion becomes uniform  
=  $3\frac{1}{2}$  inches " " " " " " =  $(1\frac{3}{4} \times 2)$  " "

Exp. (2) Distance passed through from rest in 2 seconds =  $(7 \times 1)$  " "

Distance after motion becomes uniform in  $(4 - 2$   
= 2 seconds) is  $21 - 7$  " " " " " " = 14 " "

Distance in 2nd second after motion becomes uniform =  $(7 \times 2)$  " "

CASE IV.—Uniform acceleration is equal to velocity  $\div$  time; or,  $a = v/t$ .

From the detailed working out of Case I. we see, that the change of velocity =  $(v_1 - v_0) = (v_2 - v_1) = (v_3 - v_2) = 3.5''$  per sec.

Hence, since the change of velocity in 3 seconds is  $10.5''$ ; we get,

$$\frac{v}{t} = \frac{10.5}{3} = 3.5 = a \text{ the acceleration.}$$

CASE V.—Distance moved through =  $\frac{1}{2}$  acceleration  $\times$  time<sup>2</sup>.

Or,  $h = \frac{1}{2} a t^2$ .

Exp. (3) Distance  $h$  moved through in time ( $t$ ) 3 seconds =  $15\frac{3}{4}''$ .

And, from Case IV. the acceleration  $a = 3.5''$  per sec. per sec.

Hence, by substituting known values in the equation—

We get,  $h = \frac{1}{2} a t^2$ .  
 $h = \frac{1}{2} \times 3.5 \times 3^2 = 15\frac{3}{4}$  inches.

Now, from Case IV.  $v = a t$ ,  $\therefore v^2 = a^2 t^2$ ; or,  $t^2 = v^2/a^2$ ,

But from Case V.  $h = \frac{1}{2} a t^2$  .....  $\therefore t^2 = 2h/a$ .

Hence,  $2h/a = v^2/a^2$ .

Or,  $2 a h = v^2$ .

Or generally,  $v^2 = 2 a l$  where  $l$  is the distance passed through by a body having an acceleration  $a$ .

*Corollary to Case V.*—If a body has a certain *initial* velocity  $u$  before it becomes uniformly accelerated, then the final velocity  $v$ , will be the sum of the initial and accelerated velocities.

From the previous definition of uniform acceleration, viz., that it is the increase of velocity per unit of time from the commencement of the acceleration, we see that—

$$a = \frac{v-u}{t} \therefore v = u + a t$$

The *average* or mean velocity will be half the initial and final velocities,

$$\text{Or } \frac{1}{2}[u + (u + a t)] = u + \frac{1}{2} a t.$$

And, the distance  $l$  moved through or height  $h$  through which the body passed will be this mean velocity multiplied by the time of its motion from the instant that the noted acceleration  $a$  began.

That is,  $l = h = (\text{average velocity}) \times (\text{time of motion})$ ;

$$\text{Or, } h = (u + \frac{1}{2} a t) \times t.$$

$$\therefore h = ut + \frac{1}{2} a t^2.$$

The above refers to positive acceleration or getting up speed; but the same applies to negative acceleration or retardation of a body's initial velocity if we apply the  $-$  sign instead of the  $+$  sign.

$$\text{Here, } l = ut - \frac{1}{2} a t^2,$$

$$\text{Hence generally, } l = ut \pm \frac{1}{2} a t^2,$$

And, if  $u = 0$ , or the body's motion is started from rest, we get as in Case IV.

$$l = 0 \times t + \frac{1}{2} a t^2 = \frac{1}{2} a t^2.$$

To prove the above by the Atwood's machine :

*First*, place an overweight and then a rider upon the right-hand weight; and bring lower surface of the latter opposite the zero mark of the scale.

*Second*, set the ring at say  $15\frac{3}{4}''$  below zero A, so that the rider is caught at B, the end of the 3rd second and calculate the velocity with which the whole moving mass is going at the instant the rider is caught by the ring; let this velocity =  $u$  inches per second.

*Third*, note the time  $t$  in seconds and the acceleration  $a$  per second per second which the overweight  $\odot$  now causes the remainder to possess in passing through any convenient height  $h$  inches between the *upper edge* of the ring, and the upper side of the right-hand weight when its bottom is *arrested* by the stop-stand S.

*Fourth*, check the average of an odd number of trials by substituting the values for  $h$ ,  $u$ ,  $t$  and  $a$  in the formula—

$$h = ut + \frac{1}{2} a t^2.$$

CASE VI.—When a constant force produces uniform acceleration of a body or system of bodies, the acceleration is proportional to the force and inversely proportional to the total mass being moved by the force.

In the previous Experiments with the Atwood's machine :

If  $m$  = Mass of the rider =  $\frac{w}{g}$ .

„  $w$  = Weight of the rider =  $\frac{1}{2}$  oz. or  $\cdot 5$  oz.

„  $F$  = Force produced by gravity on rider =  $mg = \cdot 5$  oz.

„  $M$  = Total mass moved = (right + left weight + thread + equivalent mass active at circumference of wheel to overcome its inertia + wire weight to balance friction of wheel bearings + mass of rider) =  $\frac{W + w}{g}$

„  $W$  = Weight due to whole moving mass  $M$  (minus mass  $m$  of weight  $w$ ) = 3 lb.  $6\frac{3}{4}$  oz. = 54.75 oz.

Then „  $W + w$  = Total weight in motion up 'o position of ring.

And „  $a$  = Acceleration produced on Mass  $M$  by gravity acting on  $m = 3\cdot 5$  inches or  $(3\cdot 5 \div 12)$  feet) per second per second.

= Value of the acceleration produced by the force of gravity upon a freely falling body (to be found from the result of the previous experiments).

Hence, from the above statement we see that the following relations should hold good, viz. :—

$$a = \frac{F}{M}; \text{ and } g = \frac{F}{m}$$

Or,  $Ma = F = mg$

$$\therefore \frac{M}{m} = \frac{g}{a}; \text{ or, } g = a \frac{M}{m} = \frac{a(W + w) \div g}{w \div g} = \frac{a(W + w)}{w}$$

Inserting the values obtained from the previous experiments.

We get,

$$g = \frac{a(W + w)}{w} = \frac{(3\cdot 5)(54\cdot 75 + \cdot 5)}{(12) \times \cdot 5} = 32\cdot 23 \text{ ft. per sec. per sec.}$$

It has been proved by the most careful experiments carried out at Greenwich Observatory, that the force of gravity there, when reduced to sea level, produces an acceleration of 32.1912 ft. per sec. per sec. But, Aberdeen, where the above experiments were made, is nearer the centre of gravity of the earth, the value may well be nearer 32.2 which is considered the usual average value, although 32 is often taken when only approximate results are desired.

*Regarding Inertia of the Wheel.*—The value of the small weight  $x$  which forms part of  $W$  in the last equation and which balances the inertia of the aluminium wheel, may be found from two experiments producing two different accelerations  $a_1$  and  $a_2$  by two different riders  $m_1$  and  $m_2$  respectively, and substituting their known values in the previous equation, until it is found that uniform velocity is obtained in each of these trials after the riders  $m_1$  and  $m_2$  have been caught by the ring.

Let,  $W = W + x$ , Where,  $W$  = the combined weight of the left and right-hand equal weights + the small wire on the latter which is required to balance the friction of the axle in its bearings

And  $x$  = the weight required to balance the inertia of the wheel.

Then, since gravity has a constant value we get

$$g = a \frac{W + w}{w} \text{ or in Exp. (1) } g = a_1 \frac{(W + x + w_1)}{w_1}$$

$$\text{and in Exp. (2) } g = a_2 \frac{(W + x + w_2)}{w_2}$$

Hence in Exp. (1)  $\frac{g}{a_1} w_1 = (W + x + w_1)$  and in Exp. (2)  $\frac{g}{a_2} w_2 = (W + x + w_2)$

Or in Exp. (1)  $x = \frac{g}{a_1} w_1 - (W + w_1)$  and in Exp. (2)  $x = \frac{g}{a_2} w_2 - (W + w_2)$

When Experiments (1) and (2) give the same value for  $x$  under the above conditions it balances the *inertia* of the wheel in both cases.

It will now be interesting to find from the foregoing data what should be the ratio of the mass of a *rider* to the *whole mass* moved in order to produce an acceleration of 1 foot per second per second; since the dynamical unit of force which is sometimes called the "British Absolute Unit of Force" called the *Poundal*, is that force which, acting for 1 second upon a mass of 1 lb., imparts to it a velocity of 1 foot per second.

$$\text{Here } M = \frac{(W + w)}{g} = 1 \text{ lb.}$$

$$,, \quad a = 1 \text{ ft. per sec. per sec.}$$

$$,, \quad m = \frac{w}{g}, \text{ and } w \text{ has to be found?}$$

From Case VI. we see that—

$$\frac{M}{m} = \frac{g}{a} = \left( \frac{32.2}{1} \right)$$

But, since the masses are proportional to the weights—

$$\text{We get,} \quad \frac{M}{m} = \frac{W + w}{w} = \frac{32.2}{1};$$

And, since  $(W + w) = 1 \text{ lb.}$

$$\text{We get} \quad w = \left( \frac{1}{32.2} \right) \text{ lb.} = \frac{1}{2} \text{ oz. (nearly).}$$

Or, a rider of  $\frac{1}{32} \text{ lb.}$ , i.e.,  $\frac{1}{2} \text{ oz.}$  would be required to be placed upon the right-hand weight of an Atwood's machine—where the sum of all the moving weights reckoned as before, was equivalent to 1 lb.—in order to produce an acceleration of 1 foot per second per second.

Hence, speaking generally, the value of a *poundal* (or so-called absolute dynamical unit of force) is equal to  $\frac{1}{32.2}$  or  $\frac{1}{g} \text{ lb.}$  in the ordinary engineers gravitation unit of force; and a force of 1 pound weight is equal to 32 or  $g$  poundals. But, we shall not bother the elementary student in this book with poundals, since, as we said before, the gravitation unit of force or force which will sustain a weight of 1 lb. of matter is sufficiently absolute for any particular place and for our purpose.

It will at once be seen from the following proportion, that no rider, however heavy, if it were placed upon any weight, however small (when the latter was balanced against the force of gravity) could produce an acceleration of 32.2 feet per second per second. For, the impressed acceleration  $a$  would also have to be equal to  $g$  or 32.2, when we get

$$\frac{g}{a} = \frac{W + w}{w} = \frac{32.2}{32.2} = \frac{1}{1}$$

That is,  $W$  could have no weight because  $w = 1$ .

The application of the foregoing explanations, formulæ and reasoning to ordinary applied mechanics questions will be seen, when we come to deal with constant forces acting in any direction upon masses of matter for a known time and producing a certain acceleration; such as the acceleration produced upon the piston and piston rod, &c., of a steam engine by the pressure of the steam up to the point of cut off. Or, upon the fly wheel due to the mean force upon the crank pin during so many revolutions; or upon the starting or the stopping of railway trains, centrifugal machines, dynamos, and many other prime movers, or motors where it is advisable to raise their speed as quickly as possible to a permissible maximum or normal velocity and to stop them quickly.

**Galileo's Experiments.**—More than 100 years before Atwood was born, the famous Italian philosopher Galileo (about 1583–1630) experimented upon falling bodies of different kinds and weights by letting them fall simultaneously but separately from the top of the leaning tower at Pisa and found that they reached the ground at the same instant, or from the same height in practically the same time. He also let spherical balls roll down a straight, smooth, inclined plane (where the friction between the ball and the plane was a minimum) and found that the distances through which the ball passed in successive seconds varied directly as the square of the times from rest; and that the velocities acquired by the ball varied directly as the times from rest; or,

Times in seconds varied as

$$1 : 2 : 3 : t$$

Distances passed varied as

$$1^2 : 2^2 : 3^2 : \frac{1}{2}gt^2$$

Speeds acquired varied as

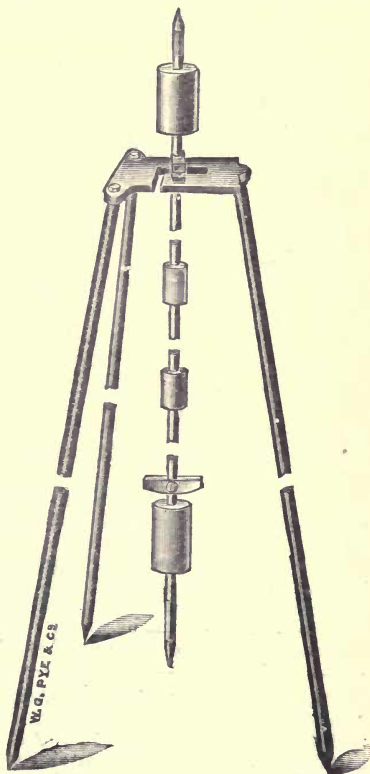
$$1 : 2 : 3 : gt.$$

### Pendulum Experiments.

—Galileo (about 1583), Huyghens (about 1650–70), and Captain Kater, an Englishman (about 1813–32), experimented with pendulums to determine not only the length of the seconds pendulum but also to ascertain the value of the force of, and acceleration due to gravity at different places. The investigation of this subject must be deferred until *harmonic motion* has been explained. It may, however, be stated that for a seconds pendulum—

$$\text{The length } l = \left( \frac{g \times t^2}{\pi^2} \right)$$

= 39.13983 inches at Greenwich (reduced to sea level) where  $g = 32.1912$  feet per second per second.



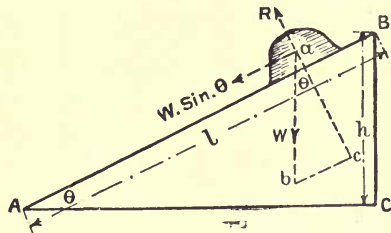
KATER'S REVERSIBLE PENDULUM.

**Kater's Reversible Pendulum.**—This pendulum consists of a vertical bar carrying a brass weight near each end, together with two similar boxwood weights, between the brass weights which are so attached as to compensate for the air displacement of the brass ones. The pendulum may be hung from one or other of two movable steel knife edges working on agate planes. We now test to see if the lower weight has been correctly placed, because, the time taken by the pendulum to perform say 100 swings about the top movable steel knife edge should be equal to the time taken to perform 100 swings about the other knife edge. If the times are not quite equal then one of the brass weights or its movable knife edge must be readjusted until the time of a swing when suspended from either end must be exactly equal. The length of an isochronous, simple equivalent pendulum is precisely equal to the final adjusted distance between the knife edges. Therefore, knowing the length of the pendulum and its time of oscillation, the acceleration due to gravity can be found from the

previously stated formula  $l = \frac{gt^2}{\pi^2}$ . With proper care Kater's pendulum can be made to give a very accurate result for the value of the acceleration due to gravity.

**The velocity attained by a body sliding down any straight inclined plane varies directly as the square root of the height.\***

Let AB be a smooth inclined plane of length  $l$  making an angle  $\theta$  with the horizontal, as in the figure. Resolve the weight of the smooth-faced



TO PROVE THAT THE VELOCITY ATTAINED BY A BODY SLIDING DOWN ANY INCLINED PLANE DEPENDS UPON THE HEIGHT OF THE PLANE.

body  $W$  as represented to scale by the dotted vertical line  $ab$ , into two components, one  $cb$  parallel to the plane and the other  $ac$  perpendicular to it.

Now, the latter component force  $ac$  has no effect on the motion of the body, since its surface and that of the plane are supposed to be smooth, and therefore the former component  $cb$  is the only force which causes the body to move down the plane.

The magnitude of this force  $F = ab \sin \theta = W \sin \theta$ .

---

\* It has been proved in the Author's more Advanced Text-Book on Applied Mechanics and Mechanical Engineering, Vol. II., Lecture XXII., that although the kinetic energy of a spherical ball, or of a cylinder of a certain mass, after it falls or slides or rolls down any path through a definite height is of the same value; yet the *velocity* and *momentum* are of different values for the cases of rolling and of sliding.

Also, it has been shown from Case VI. in connection with the Atwood's machine, &c., that the acting force = mass moved  $\times$  acceleration attained by body; that is,  $F = \frac{W}{g} \times a$

$$\text{Or, } W \sin \theta \times g \dots\dots\dots = \frac{W}{g} a$$

$$\therefore g \sin \theta \dots\dots\dots = a.$$

Further, it was shown in connection with Case V. that  $v^2 = 2 a l$ .

Now, substituting the values for  $a$  and  $l$  in this equation and figure, we get  $v^2 = 2 g \sin \theta \times BA$ .

$$\text{But, } \frac{BC}{BA} = \sin \theta, \text{ or } \dots\dots\dots \frac{BC}{\sin \theta} = BA.$$

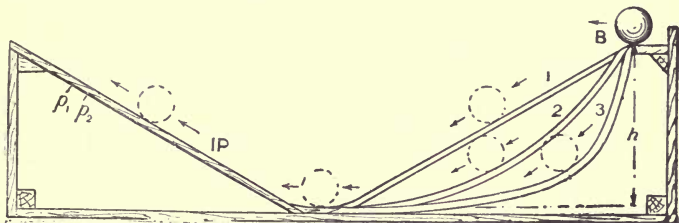
$$\therefore v^2 = 2 g \sin \theta \times \frac{BC}{\sin \theta} = 2 g BC \dots\dots\dots = 2 gh.$$

That is,  $v^2 \propto h$  the vertical height of plane. Or,  $v \propto \sqrt{h}$ .

(Then multiplying both sides by  $W$  and dividing them by  $2g$ .)

$$\text{We get, } \frac{Wv^2}{2g} \text{ (the kinetic energy or stored work) } = Wh \text{ (potential energy).}$$

**The Velocity, the Momentum, and the Kinetic Energy attained by a body in passing from a given height along any path are each respectively the same, when the friction between the path and the body is negligible.**—The following figure illustrates an arrangement to prove, that the velocity acquired by a



APPARATUS TO PROVE THAT THE VELOCITY, THE MOMENTUM AND THE KINETIC ENERGY IMPARTED TO A FALLING BODY IS THE SAME WHICHEVER PATH IT FOLLOWS, IF FRICTION BE NEGLECTIBLE OR NEGLECTED.

body falling freely down any curved path is the same whatever be the shape of the path. It shows one straight and two curved paths. Each path is made of a smooth glass plate, or a pair of parallel polished metal rods, along which a truly spherical billiard ball may be allowed to roll down from a platform. The ball when it reaches the lowest position, is allowed to travel up an adjustable and very smooth, plate glass, inclined plane.

It will be noticed that in *each* instance, the ball ascends to the same height on this plane. Consequently we conclude from the results of the experiment that the *velocity*, the *momentum* and the *kinetic energy* (or energy of motion) of the ball when commencing to ascend this inclined plane must be respectively the same. Further, if the ball experienced no frictional resistance along its path, then it would run up the inclined plane until it reached exactly the same level from which it started with no



**The Path of a Projected Body which then falls under the Action of Gravity is a Parabola.**—The accompanying illustration shows a suitable form of apparatus for observing the path, which a body, projected horizontally, describes under the combined actions of the projecting force and of gravity. The whole apparatus should be firmly secured to the wall, and the ball should always be allowed to roll down from the top of the concave quadrant "race," in order that the path traced on the blackboard may be definitely followed.

When a body has a certain velocity imparted to it, such as a stone thrown at an object, or water issuing under pressure from a pipe, or a bullet fired from a rifle in any direction, other than that of a truly vertical one, it has two different motions imparted to it. The one direction and motion is the result of the initial velocity, while the other is due to the earth's attractive force, termed the force of gravity. In the same way, the ball, when it just leaves the lower end of the quadrant "race," has both a horizontal and a vertical motion. The resultant motion is determined from the relative values of these component motions, by the principle of the parallelogram of velocities. The ball will not move along a straight line, but will describe a parabolic curve, as shown by the dotted curved line in the blackboard of the figure. If the ball had left the quadrant with a velocity in any direction (other than that of a vertical one) it would still have described a parabolic path.

Now referring to the two upper left and right figures of a cone, we see that the parabolic curve BVC is the outline of the section of the cone GAL, by a plane passing through VO and parallel to the opposite generating line AL of the cone.

Generally speaking, a parabola is a curve traced by a point, which moves in such a manner, that the distance from the point at any instant to the *focus* of the curve is always equal to the normal distance of the same point from the *directrix* of the curve.

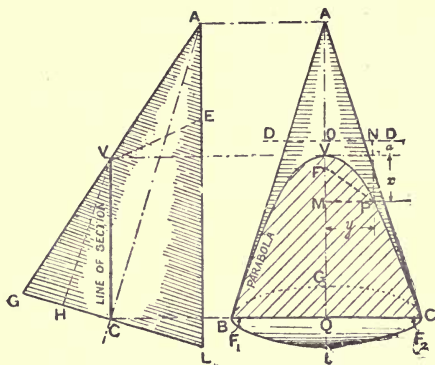
*Directrix and Focus.*—If a point so move that the ratio of its distance from a given fixed point, to its perpendicular distance from a fixed straight line be a constant, it describes a conic section, of which the fixed straight line is termed the *directrix* and the fixed point the *focus*. The constant ratio referred to is termed the *eccentricity*, and its magnitude determines the nature of the conic. Thus, if in the figure DOD be the *directrix* and F the *focus*, and if the point P moves so that the ratio of its distance from F, is to its distance PN from DOD, be a constant, then P will trace out a conic section which will be a parabola, an ellipse or a hyperbola, according as the ratio in question is equal to, less or greater than unity. That is as PF is equal to, less than or greater than PN or as  $FV = , < \text{or} > VO$ .

*Proof of Equation to the Parabola.*—In the right-hand upper figure, let P be a point on the curve BVC, then the distance of P from the *focus* F is equal to the length of the perpendicular line PN, let fall from the same point P upon the *directrix* line DOD.

Any conic section made by a plane, such as VE, which cuts the two extreme generating lines such as AG and AL of the cone GAL is called an ellipse. An example of an ellipse is shown by the curve BLCGB, of which  $F_1$  and  $F_2$  are the *foci*.

Any conic section made by a plane such as VH, parallel to the axis AC is called a hyperbola.

Referring to the right hand top figure, the equation of a parabola, viz.,  $y^2 = 4ax$  may be proved as follows:—

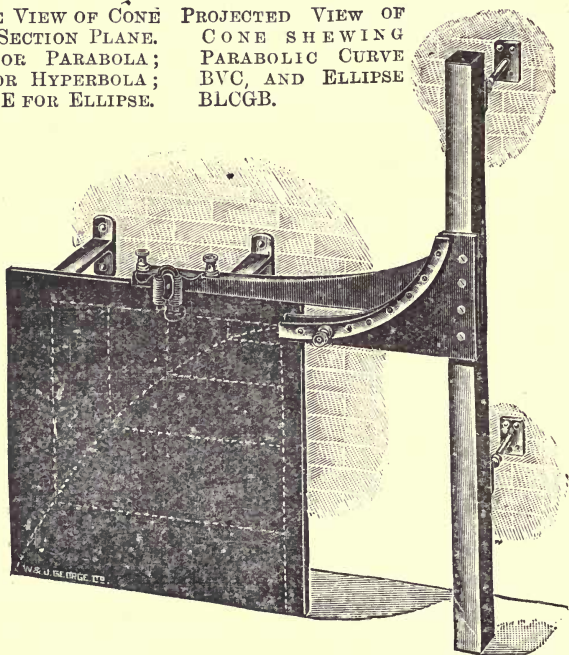


## INDEX OF PARTS.

- A for apex of cone.  
 AQ „ axis „  
 VC „ section „  
 BVC „ parabola at VC.  
 DOD „ directrix of parabola.  
 V „ vertex.  
 F „ focus.

OUTLINE VIEW OF CONE  
 WITH SECTION PLANE.  
 VC FOR PARABOLA;  
 VH FOR HYPERBOLA;  
 AND VE FOR ELLIPSE.

PROJECTED VIEW OF  
 CONE SHEWING  
 PARABOLIC CURVE  
 BVC, AND ELLIPSE  
 BLCGB.



APPARATUS FOR DETERMINING THE PATH OF A BODY PROJECTED  
 HORIZONTALLY AND FALLING FREELY.

By Euclid I.-47.

$$FP^2 = FM^2 + MP^2$$

$$\text{Or, } MP^2 = FP^2 - FM^2$$

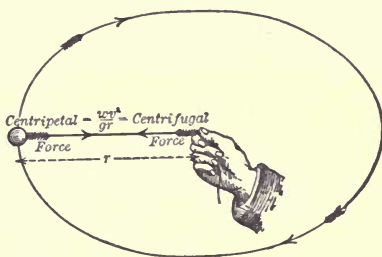
$$\text{That is, } MP^2 = PN^2 - FM^2$$

$$\text{Or, } MP^2 = (PT + TN)^2 - (VM - VF)^2$$

$$\text{That is, } MP^2 = 2TN \times 2PT \quad \left\{ \begin{array}{l} \text{But } MP = y; \\ \text{Al-o } PT = x \\ \text{And } TN = a. \end{array} \right.$$

$$\therefore y^2 = 4ax$$

**Centrifugal Force due to Motion in a Circle.**—EXPERIMENT I.—When a body—such as a stone—is attached to a cord and whirled round and round in a circle, the hand experiences a pull in the direction of the string, which is in tension under the action of a force, and the faster the body is moved the greater becomes the stress in the string, just as David of old must have felt it before he let go that pebble from his sling which went so straight for Goliath's brow. The stone is constantly tending to fly off at a tangent, and is only kept moving in the circular path by the reaction pulling it towards the centre of motion. The pull *from* the centre of motion is called the *centrifugal* or *centre-flying force*, and the exactly equal and opposite reaction is termed the *centripetal* or *centre-seeking force*. It may be proved by geometry that each of these forces is equal to



#### EXPERIENCING THE EFFECT OF CENTRIFUGAL FORCE.

the weight of the body  $\times$  the square of the velocity  $\div$  the acceleration due to gravity  $\times$  the radius of the circle described by the body.\*

\* At present the student must accept the above formula as correct. We shall have occasion to deduce the formula by aid of geometry in the Advanced Course. (See Text-Book on "Applied Mechanics and Mechanical Engineering," Vol. II., Lectures xxii. and xxiii.)

Or, . . . 
$$P = \frac{Wv^2}{gr} \text{ lbs.}$$

Where  $P$  = Pull on the cord, or the centrifugal force in lbs.

„  $W$  = Weight of the body in lbs.

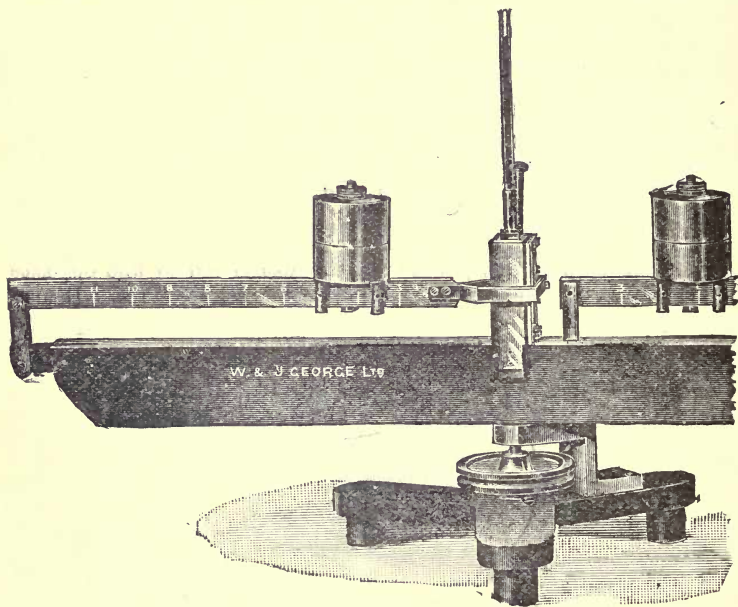
„  $v$  = velocity of the body in feet per second,

„  $g$  = gravity's acceleration = 32' per second *in one second*,

„  $r$  = radius from centre of motion to *c.g.* of body in feet.

**Centrifugal Force Machine.**—*General Description.*—This machine enables the student to systematically experiment and prove the laws connected with centrifugal force.

A polished wooden beam about 2 ft. in length is rotated by means of



CENTRIFUGAL FORCE EXPERIMENTAL MACHINE.

the pulley at any speed. Attached to the centre of this beam is an accurately machined box with one side flexible. The box is filled with mercury or coloured alcohol. An upright glass tube is fitted into the centre of this box, and exactly in the axis of rotation, so that the height of the mercury can be measured whatever be the speed of the machine. The height of the mercury in the tube can be adjusted by means of a screw at the side of the box. Fastened to the centre of the flexible side of the box is a long brass rod, which in turn is attached to one end of a flat brass rod, whilst the other end of the rod is connected to the end of the beam. This arrangement of supporting the long brass rod allows it to

move backward and forward with less friction than if it were made to slide on a bearing. This long rod is graduated, and a sliding weight can be clamped at any position from the box. Hence, we see that if the centre of the flexible side of the box be pulled or pushed, the mercury will rise or fall in the vertical glass tube; although, we cannot observe the yielding of the flexible side. With a fixed vertical scale alongside the glass tube it is easy to measure the rise and fall of the mercury in the latter. The weight shown on the left-hand side can be used for balancing the weight on the movable rod, and in this case it should be clamped at the same distance from the axis of the instrument.

*Prior Adjustments.*—Before commencing any experiments it is necessary to pull the end of the long movable brass rod with a force of 1, 2, or 3 lbs., &c., by means of a spring balance, to note the respective heights of the column of mercury in the glass tube. Hereby we can afterwards tell the value of our scale measurements. We can also make a number of experiments with the sliding weights removed from the rods, to find the centrifugal force of the remaining parts of the machine when run at different speeds, in order that these readings may be subtracted from subsequent observations to get true results.

*Working of the Centrifugal Machine.*—One experimenter turns the handle of a separate pulley, which drives the pulley shown in the figure by means of a rope. At the same time he keeps his eye upon the height of the mercury column in order to keep the speed constant, and he counts the number of revolutions per minute which his hand makes.

It is easy with this instrument to test the law which is usually given, that if a body be compelled to move in a curved path it exerts a force directed outwards from the centre of motion, and the amount in pounds is found by multiplying the mass of the body by the square of the velocity in feet per second, and dividing by the radius of the curved path we get the centrifugal force  $= P = \frac{Wv^2}{gr}$ . Or, if we multiply the mass by the radius of the circle and by the square of the angular velocity in radians per second, we get  $mr\omega^2 = P$ , the centrifugal force urging the weights from the centre of motion in a radial direction.

**EXPERIMENT II.**—Take a pail and half fill it with water. Attach a rope to the centre of the handle, and swing it round and round your head. The water does not fall out, even if you swing it in a vertical plane, if the velocity be sufficient to cause the centrifugal force to be greater than the force of gravity.

**EXAMPLE I.**—A small tin pail, containing 1 lb. of water, with a rope attached to its handle, is to be whirled in a vertical circle. If the distance from the hand or centre of motion, to the surface of the water be 2 feet, what is the least number of revolutions per minute that you can give it in order not to spill any of the water?

**ANSWER.**—Here  $P$  must be at least equal to 1 lb., for  $W = 1$  lb. and  $r = 2$  feet, whilst  $g = 32$ .

By the formula—

$$P = \frac{Wv^2}{gr}$$

$$\therefore v = \sqrt{\frac{P \times g \times r}{W}} = \sqrt{\frac{1 \times 32 \times 2}{1}} = \sqrt{64} = 8 \text{ ft. per second}$$

Or,  $v = 8 \times 60 = 480$  ft. per minute.

Now a circle of 2 feet radius = 12.56 feet circumference.

$$\therefore \frac{480}{12.56} = 38.2 \text{ revolutions per minute.}$$

Consequently, if you whirl the pail at 40 revolutions per minute, there will be no fear of any water coming out of it even when it is upside down at the highest part of the circle.

**EXPERIMENT III.**—Turn a disc of wood with a small barrel on one side of the centre. Fit the wheel and the barrel so truly with a turned axle that when the axle is supported by eye hooks at each end for bearings, a cord wound round the barrel and then pulled sharply, will cause the wheel to revolve freely at a high speed without vibration or oscillation. Now bore a hole through the disc near its circumference, and run in molten lead into this hole. Again spin the wheel rapidly, when it will be found to hobble to such an extent as to shake itself almost out of the bearings.

The centrifugal force due to the unbalanced piece of lead asserts itself so thoroughly that when it reaches the highest position of its revolution round the axis, it overcomes gravity, and lifts the whole wheel and barrel clean out of the bearings. It thereby creates such a disturbance as to leave a distinct impression on the mind of the student.

Next bore another hole through the disc of the same size as the former one, and at the same distance from the axle, but diametrically opposite to the front hole, and run in the same weight of

lead into it. Again spin the wheel, and it will be found to run smoothly.

This experiment conveys to the young engineer a most useful lesson, for it not only shows him the effect of centrifugal force due to want of balance, but it also gives him an idea how to rectify the evil.

**Balancing High-speed Machinery.**—All high-speed machinery, whether revolving or reciprocating, should as far as possible be most carefully balanced, in order to prevent centrifugal force coming into play and creating that horrid vibration and noise with which it is always more or less accompanied. There is nothing tends so much to the heating of bearings, and to the quick wearing out of brasses and other bearing surfaces as unbalanced moving parts; besides which, at very high velocities they become actually dangerous, and have frequently been known to cause destruction to life and property.\*

**Centrifugal Stress in the Arms of a Fly-Wheel.**—If the arms of a fly-wheel or pulley are not properly proportioned to resist the centrifugal force due to the mass of the revolving rim; or, if the casting has been carelessly cooled, so as to set up internal stresses between the arms and the boss or the rim, the wheel may give way. In fact, there is no fly-wheel or pulley made that would not burst, under the very great stress of centrifugal force, if you only ran it fast enough. The student will observe from the formula that the centrifugal force or stress in the arms of a fly-wheel is directly proportional to the square of the velocity, so that by merely doubling the number of revolutions per minute you quadruple the stress in the arms, and if the speed be increased three times, the stress becomes nine times as great.

**EXAMPLE II.**—Each segment of a fly-wheel, with its corresponding arm to which it is attached, weighs 1000 lbs., and the mass may be taken as collected at a distance of 4 ft. from the axis of the wheel. If each arm has a breaking stress of 100,000 lbs., what is the maximum number of revolutions per minute that the fly-wheel could be run at without breaking the arms, neglecting the binding strength of the rim of the wheel?

**ANSWER.**—By the previous formula for centrifugal force—

$$P = \frac{Wv^2}{gr}$$

$$100,000 = \frac{1000 \times v^2}{32 \times 4} \quad \therefore v^2 = 12,800$$

\* See Mr. C. A. Matthey's paper and the discussion on "The Mechanics of the Centrifugal Machine," in the Transactions of the Institution of Engineers and Shipbuilders of Scotland for Session 1898-99.

$\therefore v = 113$  ft. per second, fully.

Or,  $v = 113 \times 60 = 6780$  ft. per minute.

Now, the circumference of a circle of 4' radius = 25 ft.

$$\therefore \frac{6780}{25} = 271 \text{ revolutions per minute.}$$

**Energy.**—In applied mechanics energy means the *capability* of doing work.\*

**Potential Energy** is that form of energy which a body possesses in virtue of its position or its condition. For example, when a body of 10 lbs. is lifted 10 ft. high, it has a potential energy of 100 ft.-lbs.; for it takes that amount of work to lift the 10 lbs. through the 10 ft.; and if then allowed to fall, it would naturally give out the same quantity of work, either in overcoming friction, or, if it fell freely, it could be usefully employed to that amount and no more.

*Potential energy* may also be due to a condition of a body, such as the potential energy in the coiled spring of a watch or clock, which when wound up does work in moving the mechanism. We have also the case of potential energy in a lump of coal which when burned gives out heat, that will raise steam to be used in a steam engine for doing work. Or, in the case of an electric battery, where plates of copper and zinc are respectively placed in solutions of sulphate of copper and zinc, and on being suitably connected by wires to an electric motor, will give out electrical energy, which may be converted into mechanical work by the motor, and thereby effect some useful purpose.

**Kinetic Energy ( $E_k$ )** is energy due to motion. For example, in the first instance of potential energy the weight of 10 lbs., in falling freely down through 10 ft., had stored up in it, due to its motion, an amount of accumulated work equivalent to 100 ft.-lbs.

**Accumulated Work ( $E_p$ ).**—If a body of weight  $W$  lbs. be raised to a height  $h$  feet above the earth

$$\text{The potential energy stored up } E_p = Wh \text{ (ft.-lbs.)}$$

Now, if the body be allowed to fall freely, under the action of gravity, through  $h$  feet, it would have a velocity at the end of time  $t$  seconds of  $v$  feet per second.

Referring back to the formulæ for falling bodies previously given in this lecture we see that—

$$h = \frac{v^2}{2g} \therefore Wh = \frac{Wv^2}{2g} \text{ ft.-lbs.}$$

\* We have specially avoided using this term hitherto, as students are liable to confuse it with force, work, and power.

Therefore the *kinetic energy* or *accumulated work* stored up in a moving body is expressed by the formula—

$$E_k = \frac{Wv^2}{2g}$$

If a body of weight  $W$  lbs. were impressed forward along a perfectly smooth plane for a distance of  $l$  feet, by a force  $F$  lbs., causing an acceleration of,  $a$ , feet per second; then the previous set of formulæ for linear velocity would apply when the reaction from the plane cancelled the force of gravity.

Here, . . .  $F = \frac{W}{g}a$ ; and  $l = \frac{v^2}{2a}$

But the *Work Done* through distance  $l = F \times l = E_p$ .

And . . .  $F \times l = \frac{W}{g}a \times \frac{v^2}{2a} = \frac{Wv^2}{2g}$  ft.-lbs.

Therefore in this case the accumulated work stored up in the moving body would be expressed by the formula—

$$E_k = \frac{Wv^2}{2g}$$

**Accumulated work in a Rotating Body.**—If a body of  $W$  lbs. be concentrated at a distance of  $r$  feet from the centre of motion, and be rotated so that it has a velocity of  $v$  feet per second, then

$$\text{The Accumulated Work} = \frac{Wv^2}{2g} \text{ ft.-lbs.}$$

**The Energy of a Rotating Fly-wheel** is a good example of accumulated work. If the pressure of steam in the cylinder and the point of cut-off be kept constant, and if one or other of the machines which are being driven by the engine be thrown out of circuit—or, in other words, if the belt be moved to the loose pulley—the load on the engine will be lessened, and the engine will have a tendency to increase in speed. If, however, it be provided with a very heavy fly-wheel, the surplus power of the engine will be stored up in the fly-wheel, so that the increase of speed will not be so great as if it had a light one, or none at all. If a machine should be suddenly brought into circuit again after a short time, then the load on the engine will be as quickly increased; but the stored-up energy in the fly-wheel will enable it to overcome this sudden demand for power, so that the speed of the engine will not be greatly altered. The fly-wheel, therefore, acts as a regulator of speed, not only for alterations of load, but also for the variable pressures which exist in the cylinder of an engine. This is

particularly noticeable in the case of gas engines, where the almost instantaneous explosion of gas in the cylinder at the beginning of a stroke creates an immense force, which would urge the piston forward at lightning speed, if it were not for the very heavy fly-wheel with which the engine is provided. The fly-wheel stores up some of this sudden force and gives it out again during the intervening strokes when there is no explosion, thus tending to a uniformity of speed which would be conspicuous by its absence if the gas engine had only a light fly-wheel, or none at all. In fact, the motion of gas engines would be so erratic without fly-wheels as to prevent their application to many purposes for which they are admirably adapted when aided by *very* heavy ones.

**Radius of Gyration.\***—It will be evident, almost without explanation, that in the case of a fly-wheel or a rotating disc, those parts which are furthest from the centre of motion must accumulate more energy than those of the same weight which are nearer to that centre, because they move at a greater velocity. There is, however, for every body a *mean radius of rotation*, termed "*radius of gyration*," ( $k$ ) which is at such a distance from the centre of motion, that if the whole mass of the body were concentrated there, the same kinetic energy or accumulated work would be developed at the same speed or number of revolutions per minute. The length of this mean radius varies with the shape of the rotating body, and requires a knowledge of higher mathematics for its computation; so we will assume that in the case of a fly-wheel it is at the *c.g.* of the rim, or that the distance is given in any question requiring solution.

**EXAMPLE III.**—A fly-wheel weighing 10,000 lbs. has a mean radius of rotation,  $k=r=5$  feet, and turns normally at 100 revolutions per minute. Owing to the load being diminished, the speed increases to 110 revolutions per minute; what reserve energy is stored up in the fly-wheel, which is fit to overcome any sudden increase of load?

**ANSWER.**—Let  $v_1$  = the velocity in feet per second, at the normal speed  $n_1$  revolutions per minute,

And  $v_2$  = the velocity at the increased speed  $n_2$  revolutions per minute;

$$v_1 = 2\pi r n_1 = \frac{2 \times 22 \times 5 \times 100}{7 \times 60} = 52.4 \text{ ft. per sec.}$$

$$v_2 = 2\pi r n_2 = \frac{2 \times 22 \times 5 \times 110}{7 \times 60} = 57.6 \text{ ft. per sec.}$$

\* The *radius of gyration* is called the *swing radius* by some engineers.

$$\text{Stored energy at speed } n_1 = \frac{Wv_1^2}{2g}$$

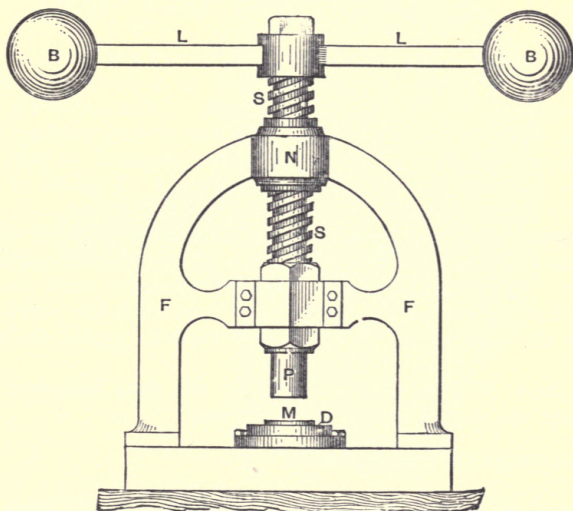
$$\text{,, ,, } n_2 = \frac{Wv_2^2}{2g}$$

$$\text{Reserved stored energy} = \frac{Wv_2^2}{2g} - \frac{Wv_1^2}{2g} = \frac{W}{2g}(v_2^2 - v_1^2)$$

$$\text{,, ,, } = \frac{10000}{2 \times 32}, (57 \cdot 6^2 - 52 \cdot 4^2)$$

$$\text{,, ,, } = 89,375 \text{ ft-lbs.}$$

**The Fly press.**—This machine is used, in the form shown by the figure, either for embossing or stamping pieces of metal with



THE FLY-PRESS.

#### INDEX TO PARTS.

D represents	Disc supporting M.	S represents	Screw.
M	„ Metal to be stamped.	N	„ Nut for S.
P	„ Punch or die.	L	„ Lever arms.
F	„ Frame of machine.	B	„ Balls or weights.

some design, or for punching thin metal plates. The piece of metal M, to be embossed or punched, is laid on a disc D, and the die or punch P is caused to come down on M with a large amount of stored-up energy, due to the operator taking hold of

one or other of the heavy balls B, and giving them a very rapid turn round. The result of this movement is to send the quickly pitched square-double-threaded screw rapidly through its nut N, thereby forcing the guided square carrying the punch straight downwards, and causing the latter to overcome the resistance of the hard metal. Neglecting friction at the screw and the guide, and considering the combined weight of the two balls as =  $W$  lbs., and  $v$  = their velocity in feet per second at the instant the punch meets the metal M, then—

The stored energy, or energy of the blow, =  $\frac{Wv^2}{2g}$  ft.-lbs.

If  $l$  = Length the punch or die goes into the metal in feet,  
And  $R$  = Resistance overcome (mean) in lbs.,

Then  $Rl = \frac{Wv^2}{2g}$  ft.-lbs.

**EXAMPLE IV.**—Distinguish between energy and power. What is the unit of power in this country? In a fly-press two balls, each weighing 60 lbs., are moving with a linear velocity of 15 feet per second, what is the measure of the energy existing in the balls (take  $g = 32$ )? What is the power required to raise 6600 gallons of water up 150 feet in 30 minutes? A gallon of water weighs 10 lbs. (S. and A. Exam. 1893.)

**ANSWER.**—(1) Energy is the capability of doing work which a body may possess on account of its position, or condition, or motion. Power is the *rate* of doing work, or the work done in a given time. The unit of power in this country is the horse-power and is the rate of doing work equivalent to 33,000 ft.-lbs. per minute.

(2) Here  $W$  = combined weight of the two balls = 120 lbs.  
 $v$  = linear velocity of balls = 15 ft. per sec.

Then energy existing in balls =  $\frac{Wv^2}{2g}$   
 " " " " =  $\frac{120 \times 15 \times 15}{2 \times 32} = 421.87$  ft.-lbs.

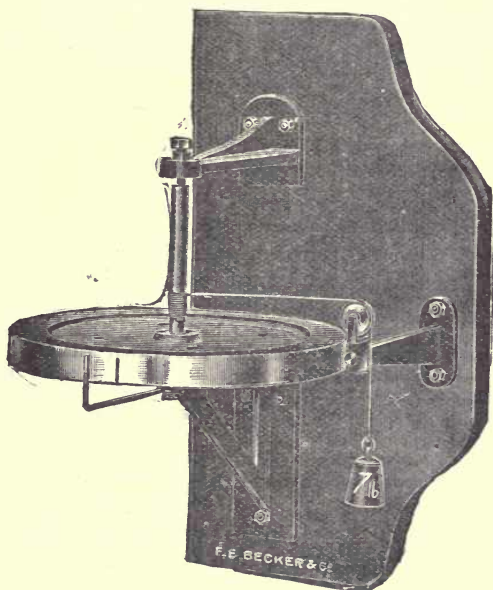
(3) Weight of water raised per minute =  $\frac{6600 \times 10}{30} = 2200$  lbs.

∴ Work done per minute =  $2200 \times 150 = 330,000$  ft.-lbs.

∴ Power required =  $\frac{330,000}{33,000} = 10$  H.P.

**To find Experimentally the Energy Stored in the Rotating Mass of a Fly-wheel.**—*Description of the Experimental Machine.*—The accompanying figure shows a fly-wheel mounted on an axle. This axle is supported on ball bearings, thereby reducing the friction to a minimum. The diameter of the fly-wheel is 18 inches and weighs about 100 lbs. If the centre of gravity of the fly-wheel is not exactly in the axis, then it is better to place the wheel as in the figure. One end of a cord is looped over a pin on the axle, and after being wound several times round the axle, the other end is led over an aluminium pivoted pulley, and attached to the 7-lb. weight.

*Object of Experiments.*—The object of this experiment is to illustrate



EXPERIMENTAL APPARATUS FOR DETERMINING THE ENERGY STORED UP IN A ROTATING FLY-WHEEL.

the "Principle of Work" or the law of the "Conservation of Energy." Since energy cannot be generated or destroyed, the quantity given to a machine can be traced in its transmission through the machine, as clearly pointed out in Lecture V. Hence the amount of energy given to the fly-wheel and the attached weight in this experiment is measured by the pull of the earth on the 7-lb. weight multiplied by the distance through which the latter falls.

*Conservation of Energy.*—Part of this energy is stored up in the fly-wheel as kinetic energy, part of it is used to turn the wheel against the friction of the bearings, and part is stored as kinetic energy in the falling weight. The last two items of the total energy are converted into heat energy, and

the several sub-divisions of the total energy may be connected under one general formula.

$$\left\{ \begin{array}{l} \text{Energy given to} \\ \text{the apparatus} \\ \text{during the} \\ \text{falling of the} \\ \text{weight.} \end{array} \right\} = \left\{ \begin{array}{l} \text{Kinetic} \\ \text{energy } E_k \\ \text{stored} \\ \text{in the} \\ \text{fly-wheel.} \end{array} \right\} + \left\{ \begin{array}{l} \text{Energy} \\ \text{converted} \\ \text{into heat} \\ \text{by} \\ \text{friction.} \end{array} \right\} + \left\{ \begin{array}{l} \text{Kinetic energy in} \\ \text{falling weight at} \\ \text{the instant when} \\ \text{the cord is released} \\ \text{from the axle.} \end{array} \right\}$$

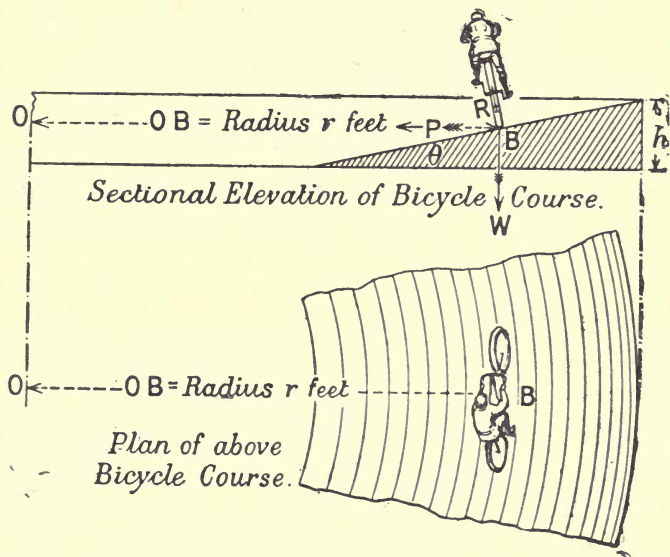
*Determination of Speed of Machine by the Fly-wheel.*—After setting the mark upon the rim of the fly-wheel opposite the fixed mark, and with the 7-lb. weight on the cord to keep it taut, turn the wheel by hand to see as near as possible how many turns the wheel makes in falling a certain number of feet. Let this distance through which the weight is to fall in the experiment be  $h$  feet, and let  $n$  be the number of turns made by the fly-wheel during this fall. Now determine by aid of a stop-watch how long it takes the 7-lb. weight to fall through  $h$  feet whilst turning the fly-wheel, and tabulate your results as follows:

Weight on cord in lbs.	Height of fall in feet	Turns of fly- wheel ( $n$ )	Time taken for weight to fall ( $h$ ) feet	Velocity of weight at the instant when cord leaves the axle in feet per second	Revolutions per second of fly-wheel at the instant when cord leaves the axle.
------------------------------	------------------------------	-----------------------------------	---	---	--

*Final Calculations.*—As we have found the total number of revolutions made by the wheel until the cord drops off its pin, and also observed the time taken in *seconds*. Then, as we know that the speed increases uniformly during this interval of time, the *mean* speed is just half the speed at the end of the time interval; consequently, if we divide the number of revolutions by the number of seconds in which they were performed, and multiply this quotient by 2, we will get the number of revolutions per second made by the wheel when the weight just ceases to act. You can test the accuracy of your result by counting the number of revolutions of the fly-wheel from the time that the cord drops off until the wheel comes to rest due to its own friction and dividing by the time which has elapsed, then multiply this quotient by 2 and this will give you the required speed of the fly-wheel.

**Motion on a Curved, Inclined, or "Banked" Track.**—Take as an example a cycling track, and let us suppose that the bicycle is moving with uniform velocity of  $v$  feet per second round a smooth circular course of radius  $OB$  equal to  $r$  feet. Then it is necessary to find at what angle to the horizontal plane the track should be inclined or "banked" in order that the bicycle may keep in its circular path.

We see from the diagram that two forces are acting on the bicycle—



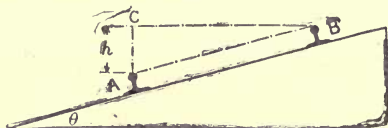
SECTIONAL ELEVATION AND PLAN OF A BICYCLE TRACK.

(i) Its own weight  $W$ ; and (ii) the reaction  $R$ , which is perpendicular to the smooth track. These two forces have a resultant horizontal force  $P = \frac{Wv^2}{gr}$ , which is acting towards the centre  $O$  of the horizontal circle in which the bicycle moves.

Hence, if we take any vector line to represent the weight  $W$  and from the upper end thereof draw a horizontal line, and from the lower end draw another vector line inclined at an angle  $\theta$  to the vertical until it meets the horizontal line in a point. The sides of the triangle will represent in direction and magnitude the forces  $W$ ,  $R$ , and  $P$ , and  $\tan \theta = \frac{P}{W} = \left( \frac{Wv^2}{gr} \div W \right) = \frac{v^2}{gr}$ , from which we can quite easily obtain the required angle of the inclination  $\theta$  of the track and the height  $h$  for the desired width of the same.

**Railway Curves.**—If the lines of a railway curve be laid at the same level, then the centripetal thrust of the rails on the wheel of trains passing round curves act on the flanges of the wheels, and the centrifugal thrust of the wheel on the track would tend to push it sideways out

of its place. But, in order to have this action and reaction normal to the track, the outer rail is raised and the track thereby inclined to the horizontal. The amount of this super-elevation which is suitable for a particular speed is quite easily calculated.



SECTIONAL ELEVATION OF A RAILWAY CURVE

Let  $G$  = gauge in inches between the rails.

„  $v$  = velocity in feet per second of the train.

„  $r$  = radius of curve in feet.

„  $h$  = height of super-elevation of one rail above the other in inches.

„  $\theta$  = the angle of “banking” or inclined plane.

Now draw the triangle  $ABC$  to represent the three forces,  $W$ ,  $R$ , and  $P$  as indicated in the previous case.

Then  $AC = AB \sin \theta = AB \tan \theta$ , since  $\theta$  is always very small.

$$\therefore h = G \frac{v^2}{gr} \text{ inches.}$$

**EXAMPLE V.**—A motor-car moves in a horizontal circle of 300 ft. radius at 30 miles per hour, what is the ratio of its centrifugal force to its weight? This is the tangent of the angle at which the track ought to be inclined sideways to the horizontal if there is to be absolutely no tendency to side-slip; find this angle. (B. of E. 1904.)

*Answer.*—

Let  $v$  = velocity of the car in feet per second.

„  $P$  = centrifugal force in lbs.

„  $W$  = weight of the car in lbs.

„  $r$  = radius of horizontal circle of car's motion in feet.

„  $g$  = acceleration due to gravity = 32 ft. per sec. per sec.

Then the forces which are acting at the centre of gravity of the car are as follows:

(i) The weight of the car,  $W$  lbs. acting vertically downwards.

(ii) The centrifugal force  $P$ , equal to  $\frac{Wv^2}{gr}$  lbs. acting in a horizontal direction.

(a) To Find the Ratio of the Centrifugal Force to the Weight of the Car.—

$$\begin{aligned} \frac{\text{Magnitude of centrifugal force}}{\text{Weight of the motor-car}} &= \frac{P}{W} = \frac{Wv^2}{gr} \div W = \frac{v^2}{gr} \\ &= \frac{P}{W} = \frac{30 \times 5280 \times 30 \times 5280}{60 \times 60 \times 60 \times 60 \times 32 \times 300} \\ &= \frac{P}{W} = \frac{121}{600}; \text{ or about } \frac{1}{5}. \end{aligned}$$

(b) To find the inclination of the angle  $\theta$  at which the track ought to be inclined to the horizontal if there is to be absolutely no tendency to side-slip.

$$\tan \theta = \frac{P}{W} = \frac{v^2}{gr} = \frac{121}{600} = .2016. \text{ Or } \theta = 11^\circ 24'.$$

**Momentum** is the quantity of motion possessed by a body. It is measured by the quantity of moving matter (i.e., its mass) multiplied by its velocity.\*

Or, 
$$\begin{array}{rcl} \text{Momentum} & = & \text{Mass} \times \text{Velocity} \\ M & = & m \times v \end{array}$$

The momentum of a body is, therefore, that constant force which, acting for unit time, would stop the body.

A force of 2 lbs. acting for unit time, or one second, on a body, will produce a certain amount of momentum; it is therefore obvious that twice the force acting for half the time, i.e., 4 lbs. acting for half a second, would produce the same momentum.

**EXAMPLE VI.**—A hammer head of  $2\frac{1}{2}$  lbs. moving with a velocity of 50 feet per second is stopped in .001 second. Find the average force of the blow.

**ANSWER.**—Momentum of Hammer Head = mass  $\times$  velocity.

$$,, \quad ,, = (2\frac{1}{2}/32) \times 50 = 3.906 \text{ lbs.-sec.}$$

$$\text{Average Force (F)} = ma = \frac{W}{g} \left( \frac{v_2 - v_1}{t} \right) = \frac{2\frac{1}{2}}{32} \left( \frac{0 - 50}{0.001} \right) = 3906 \text{ lbs.}$$

The negative signs (before  $v_1$  and 50) show that the acceleration ( $a$ ) is in this case a retardation. The momentum 3.906 represents the force in lbs. which, acting for one second, would stop the hammer head. The average force of the blow is therefore  $(3.906 \div .001) = 3906 \text{ lbs.}$

**EXAMPLE VII.**—A ship of 2000 tons, moving at 3 knots, is stopped in one minute; what is the average retarding force? Neglect the motion of the water. One knot is 6080 feet per hour.

**ANSWER.**—Here we may obtain the retarding force in tons and reduce the speed in knots to feet per second.

$$\text{Now, } 3 \text{ knots} = 3 \times 6080 \text{ feet per hour; or, } \frac{3 \times 6080}{60 \times 60} \text{ ft. per sec.}$$

$$\text{Hence the momentum} = \text{mass} \times \text{velocity} = \frac{2000}{32} \times \frac{3 \times 6080}{60 \times 60} \text{ units.}$$

\* The mass of a body is its weight in lbs. divided by the acceleration due to gravity. Hence, if a body is  $W$  lbs. in weight, and if  $g$  represents the acceleration due to gravity, or 32 ft. per sec. per sec. then the mass  $m = \frac{W}{g}$ . For example, if a body weighed 3.2 lbs., then its mass is  $3.2 \div 32$  or .1 unit of mass. If this body is moving with a velocity of 10 feet per second its momentum is  $.1 \times 10$  or 1. If this momentum be created or destroyed by a force acting for one second only on the body, the force must have a value of 1 lb. If it be created or destroyed in ten seconds, then the force is .1 of a lb.; if in  $\frac{1}{10}$  second, its value would be 10 lbs.

Since the retarding force acts for one minute, or sixty seconds.

The average retarding force = change of momentum  $\div$  60

$$\text{i.e.,} \quad \text{,,} \quad \text{,,} \quad \text{,,} \quad = \frac{2000}{32} \times \frac{3 \times 6080}{60 \times 60} \div 60$$

$$\text{Or,} \quad \text{,,} \quad \text{,,} \quad \text{,,} \quad = 5.27 \text{ tons.}$$

**EXAMPLE VIII.**—A railway train starting from rest along a level line acquires a speed of 30 miles per hour in five minutes. What has been the mean pull between the engine and the train; the resistances to motion being taken at 10 lbs. per ton, and weight of train exclusive of engine 150 tons?

**ANSWER.**—Constant pull due to resistances =  $10 \times 150 = 1500$  lbs.

$$\text{Mass of train} = m = \frac{150 \times 2240}{32} = 150 \times 70$$

$$\text{Velocity of train} = v = 30 \text{ miles per hour} = \frac{30 \times 5280}{60 \times 60} \text{ feet per second.}$$

$$\text{,,} \quad \text{,,} \quad = v = 44 \text{ feet per second.}$$

$$\text{Momentum of train} = m \times v = 150 \times 70 \times 44 = 462,000.$$

And the time taken to produce this momentum =  $5 \times 60$  seconds.

$$\therefore \left. \begin{array}{l} \text{Average pull for change} \\ \text{of the momentum} \end{array} \right\} = \left( \frac{mv_2 - mv_1}{t} \right) = \frac{462,000}{5 \times 60} = 1540 \text{ lbs.}$$

$$\therefore \left. \begin{array}{l} \text{Total mean pull between the} \\ \text{engine and carriages} \end{array} \right\} = 1500 + 1540 = 3040 \text{ lbs.}$$

**Mass** is defined as *the quantity of matter in a body*. It is measured by the weight of the body in pounds at London divided by the acceleration of gravity at the same place—i.e.,  $\text{Mass} = w \div g$ ; or,  $w$  in lbs.  $\div 32.2$ .

**Inertia** is defined as *that property of matter whereby it tends to remain in a condition of rest or of uniform motion*. Hence, we have Newton's first law of motion as stated in the second page of this Lecture, and which is sometimes termed the "Law of Inertia."

**NOTE.**—The force in lbs. is the *distance-rate* at which work is done in foot-pounds, and it is also the *time-rate* at which momentum is produced or destroyed.

**Moment of Momentum.**—Unit moment of momentum or angular momentum is unit momentum at unit perpendicular distance.

**EXAMPLE IX.**—A body weighing 3220 lbs. was lifted vertically by rope, there being a damped spring balance to indicate the pulling force of  $F$  lb. on the rope. When the body had been lifted  $x$  ft. from its position of rest, the pulling force was automatically recorded as follows:

$x$	0	18	43	60	74	95	111	130
$F$	7700	7680	7430	7130	6770	5960	5160	3970

Find approximately the work done on the body when it had risen 115 ft. How much of this is stored as potential and how much as kinetic energy? What is, then, the velocity of the body? (S. E. B. 1900.)

*Answer.*—Use squared paper, and plot the values of  $x$  along the

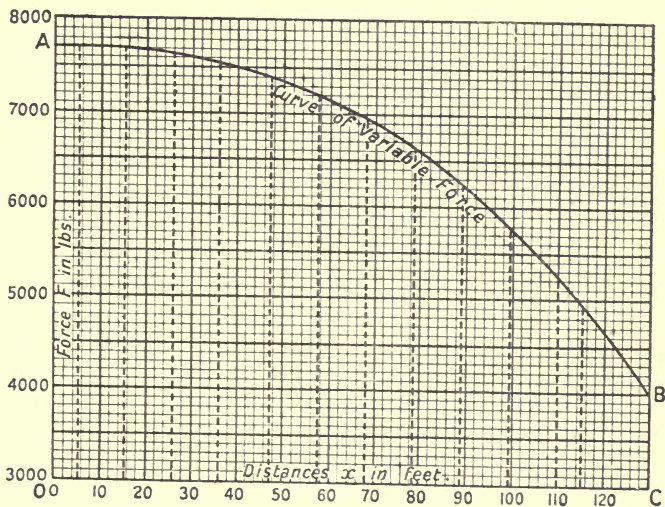


DIAGRAM OF WORK DONE BY A VARIABLE FORCE.

abscissæ OC, with the corresponding values of  $F$  as ordinates. Then draw a curve AB through the several points. Divide OC into, say, 15 equal parts, and from the middle point of each division read off upon the vertical scale the corresponding values of  $F$ . The sum of these ordinates divided by 15 will give the mean force  $F$  in lbs. This force multiplied by the distance through which it acts, viz., 130 ft. gives the *total* work done by the force or the area of the diagram OABC in ft.-lbs.

Total work done by force  $F$  = area of diagram OABC in ft. lbs.

$$\text{" " " } F = \left\{ \begin{array}{l} \text{average value.} \\ \text{of force in lbs} \end{array} \right\} \times \text{distance } x, 130 \text{ ft.}$$

Work done by force  $F$  in } = " "  $\times 115 \text{ ft.}$   
 passing through 115 ft.

$$= 6951 \times 115 = 799,360 \text{ ft.-lbs.}$$

But, work done in lifting the } =  $Wh = 3220 \times 115$  = 370,300 ft.-lbs.  
 weight  $W$  through  $h$  feet

Hence, work done by force } = \left\{ \begin{array}{l} \text{Work done in} \\ \text{lifting weight} \end{array} \right\} + \left\{ \begin{array}{l} \text{Kinetic energy,} \\ E\_k, \text{ in the body at} \\ \text{end of the lift.} \end{array} \right\}

115 ft.

$$\text{Or, } 799,360 = 370,300 + \frac{Wv^2}{2g},$$

$$\frac{Wv^2}{2g} = 799,360 - 370,300 = 429,060 \text{ ft.-lbs.}$$

$$v^2 = \frac{429,060 \times 2g}{W} = \frac{429,060 \times 64}{3220}.$$

$$v^2 = 8528.$$

$$v = \sqrt{8528} = 92.3 \text{ feet per second.}$$

Poten'ial Energy  $E_p$  in body at height of 115 ft. = 370,300 ft.-lbs.

Kinetic Energy  $E_k$  in body at height of 115 ft. = 429,060 ft.-lbs.

## LECTURE XXI.—QUESTIONS.

1. A body moves in a circle with a uniform velocity; show that it must be acted on by a constant force tending towards the centre, and find the magnitude of the force in terms of the radius of the circle, and of the mass and velocity of the body.

2. A body weighing  $2\frac{1}{2}$  lbs., fastened to one end of a thread 4 feet long, is swung round in a circle, of which the thread is the radius; what will be its velocity when the tension of the thread is a force of 20 lbs. ( $g=32$ )? *Ans.* 32 feet per second.

3. When an unbalanced wheel is set in rapid rotation, a considerable amount of shake and vibration is experienced. You are required to explain this result from first principles, and to state the mechanical laws which appear to be at work. How would you calculate the amount of pull that this unbalanced weight exerts?

4. What primary law in mechanics asserts itself when some revolving piece of machinery moves at a high velocity, and is unbalanced? A weight of 1 lb. is placed on the rim of a wheel 2 feet in diameter, which revolves upon its axis and is otherwise balanced. The linear velocity of the rim being 30 feet per second, what is the pull on the axis as caused by the weight of 1 lb.? *Ans.* 28.1 lbs.

5. A segment of a fly-wheel, with the arm to which it is attached, weighs 3500 lbs., and the mass of the portion may be taken as collected at a distance of 8 feet from the axis of the wheel, which makes 40 revolutions per minute. What is the force tending to pull away the segment and arm from the boss of the wheel? *Ans.* 15,365 lbs.

6. Define kinetic energy. How does it differ from potential energy? If a velocity of 300 ft. per second is impressed on a weight of 10 lbs., what is the measure of the energy now imparted to the weight. *Ans.* 14,062.5 ft.-lbs.

7. State the rule for finding the amount of work stored up in a given weight when moving with a given velocity. A weight of 6 cwt. moves with a velocity of 20 feet per second; how many units of work are stored up in it? *Ans.* 4200 ft.-lbs.

8. Write down the formula for the amount of energy stored up in a given weight when moving with a given velocity. Describe, with a sketch, the action of a fly-press. If each ball of the press weighs 50 lbs., and the work stored up in the balls is 400 ft.-lbs., find the velocity with which they are moving. Take the number 32 to represent  $g$ . *Ans.* 16 feet per second.

9. Account for the storing up of energy in a rotating fly-wheel. If the weight of the rim be doubled while the rate of rotation remains unchanged, how much is the energy increased? *Ans.* Twice.

10. State the formula for the energy stored up in a fly-wheel, on the supposition that the whole of the material is collected in a heavy rim of given mean radius. Apply the formula to show (1) the effect of doubling the number of revolutions per minute; (2) the effect of doubling the weight; (3) the effect of increasing the mean radius in the proportion of 3 to 2.

11. A fly-wheel weighs  $2\frac{1}{2}$  tons, and its mean rim has a velocity of 40 feet per second. If the wheel gives out 10,000 ft.-lbs. of energy, how much is its velocity diminished? *Ans.* 1.455 feet per second.

12. Explain the use of the fly-wheel in any machine with which you are acquainted. To what class of machines is such a wheel usually applied? What is the kinetic energy in a wheel revolving at 150 revolutions per minute, if the wheel loses 5000 ft.-lbs. of energy when its speed is reduced to 147 revolutions per minute? *Ans.* 126,263 ft.-lbs.

13. A fly-wheel of a shearing machine has 150,000 foot-pounds of kinetic energy stored in it when its speed is 250 revolutions per minute; what energy does it part with during a reduction of speed to 200 revolutions per minute? *Ans.* 54,000 ft.-lbs.

If 82 per cent. of this energy given out is imparted to the shears during a stroke of 2 inches, what is the average force due to this on the blade of the shears? (S. E. B. 1902.) *Ans.* 265,680 lbs.

14. A fly-wheel is required to store 12,000 ft.-lbs. of energy as its speed increases from 98 to 102 revolutions per minute; what is its kinetic energy at 100 revolutions per minute? (S. E. B. 1900.) *Ans.* 150,000 ft.-lbs.

15. A machine is found to have 300,000 foot-pounds stored in it as kinetic energy when its main shaft makes 100 revolutions per minute; if the speed changes to 98 revolutions per minute, how much kinetic energy has it lost? (S. E. B. 1901.) *Ans.* 11,880 ft.-lbs.

16. What do you understand by work, potential and kinetic energy? A bullet weighing 1 oz. leaves the muzzle of a rifle with a velocity of 1350 feet per second; what is the kinetic energy of the bullet in ft.-lbs.? *Ans.* 1780 ft.-lbs.

17. If a gun delivers 400 bullets per minute, each weighing 0.5 oz., with 2000 feet per second horizontal velocity; neglecting the momentum of the gases, what is the average force exerted upon the gun? (S. E. B. 1900.) *Ans.* 12.94 lbs.

18. A bullet of 0.1 lb., with a speed of 2200 feet per second, is fired into the middle of a block of wood of 30 lbs., which is at rest but free to move; find the speed of the block and bullet afterwards. What is the loss of kinetic energy in foot-pounds? (S. E. B. 1902.) *Ans.* 7.3 ft. per sec.; 7537 ft.-lbs.

19. A man and his bicycle weigh 170 lbs.; he has a speed indicator (not a mere counter). When going at 10 miles an hour on a level road he suddenly ceases to pedal, and in 15 seconds finds that his speed is 8 miles an hour. What is the force-resisting motion? (S. E. B. 1901.) *Ans.* 1 lb.

20. A car weighing  $2\frac{1}{2}$  tons and carrying 40 passengers, the average weight of each of them being 145 pounds, is travelling on a level rail at the rate of 6 miles an hour. What is its momentum in engineer's units? If the propelling force be withdrawn, what average force in pounds must be exerted to bring the car to rest in two seconds? and supposing the force to be constant, what distance would the car travel before it came to rest? *Ans.* 3135 lbs.-ft.-secs.; 1567.5 lbs.; 8.8 ft.

21. A car is drawn by a pull of  $P$  lbs., varying in the following way  $t$  being seconds from the time of starting:

$P$	1020	980	882	720	702	650	713	722	805
$t$	0	2	5	8	10	13	16	19	22

The retarding force of friction is constant and equal to 410 lb. Plot  $P-410$ , and the time  $t$ , and find the *time average* of this excess force. What does this represent when it is multiplied by 22 seconds? (S. E. B. 1902.) *Ans.* 366 lbs.

22. A body weighing 1610 lbs. was lifted vertically by a rope, there being a damped spring balance to indicate the pulling force  $F$  lb. of the rope. When the body had been lifted  $x$  feet from its position of rest, the pulling force was automatically recorded as follows:

$x$	0	11	20	34	45	55	66	76
$F$	4010	3915	3763	3532	3366	3208	3100	3007

Find approximately the work done on the body when it has risen 70 feet. How much of this is stored as potential energy, and how much as kinetic energy? *Ans.* 247,000 ft.-lbs.; 112,700 ft.-lbs.; 134,300 ft.-lbs. (S. E. B. 1901.)

23. A tramcar, weighing 15 tons, suddenly had the electric current cut off. At that instant its velocity was 16 miles per hour. Reckoning time from that instant, the following velocities,  $V$ , and times,  $t$ , were noted:—

$V$ .—Miles per hour	-	16	14	12	10
$t$ .—Seconds	-	0	9.3	21	35

Calculate the average value of the retarding force and find the average value of the velocity from  $t = 0$  to  $t = 35$ . (B. of E. 1903.)

*Ans.* Retarding Force = 264 lbs.; Average Velocity = 12.74 miles per hour.

24. A projectile has kinetic energy = 1,670,000 foot-pounds at a velocity of 3000 feet per second. Later on its velocity is only 2000 feet per second, how much kinetic energy has it lost? What is the cause of this loss of energy? *Ans.* 927,778 ft.-pounds. (B. of E. 1903.)

25. A man weighing 160 lb. is in a lift which starts to descend with an acceleration of 2 feet per second per second. What force is exerted by the man upon the floor of the lift? What would the force be if the lift were descending at a uniform speed? *Ans.* 150 lbs.; 160 lbs. (B. of E. 1903.)

26. In a gun, of which the internal diameter is 6 inches, a projectile weighing 100 lb. has imparted to it in a distance of 12 feet a velocity of 2500 feet per second. Find the average pressure of the gases on the base of the projectile up to the time it leaves the gun. (Neglect friction and the energy of rotation of the projectile.)

*Ans.* Average Pressure = 7200 lbs. per sq. inch. (B. of E. 1903.)

27. A weight of 120 lb. falls to the ground from a height of 18 feet and just rebounds. If the time of contact between weight and ground be the energy of rotation of the projectile.)

*Ans.* Average Pressure = 28,800 lbs. per sq. inch. (B. of E. 1903.)

28. A train weighing 250 tons is moving at 40 miles per hour, what is its momentum in engineers' units? If this momentum is destroyed in ten seconds, what is the average force acting on the train during these ten seconds. Define what is meant by *force* by people who have to make exact calculations. *Ans.* Average Force = 456 tons; Momentum = 14,670 ton-feet seconds? (B. of E. 1904.)

29. A fly wheel weighs 24,000 lb., its mean radius (or rather radius of gyration) is 10 feet, it revolves at 75 revolutions per minute, what is its kinetic energy?

If suddenly disconnected from its engine, in how many revolutions will it come to rest, if we know that in each revolution the energy wasted in overcoming friction is 3000 foot-pounds?

*Ans.*  $E_k = 2,315,051$  ft.-lbs.; and 772 revs. (B. of E. 1904.)

30. An ordinary steam engine has a stroke of 18 ins., and the connecting rod is 36 ins. long. The crank shaft makes 400 turns per minute. Find the velocity of the piston, in feet per minute, when it has moved through one-quarter of the stroke, reckoned from the back end.

*Ans.*  $V_p = 1760$  feet per minute. (C. & G., 1905, O., Sec. A.)

31. A cricket ball, weighing 0.28 lb., reaches the batsman when it is travelling horizontally at 96 feet per second; what is its momentum in engineers' units? The batsman drives the ball straight back to the bowler with the same speed; what has been its change of momentum? If the time of the blow is one-thirtieth of a second, find the average magnitude of the force exerted by the bat upon the ball. (B. of E. 1905.).

*Ans.* Change of momentum = 1.68 units; Average Force = 50.4 lbs.

32. A casting is bolted to an angle plate on the face plate of a lathe. The casting, angle plate, and bolts are equivalent to 75 lbs. at a radius of  $4\frac{1}{2}$  inches. In what position must a weight of 20 lbs. be fixed to the face plate to effect a balance? *Ans.*  $16\frac{2}{3}$  inches from centre of plate.

(B. of E. 1905.)

33. The angular position  $D$  of a rocking shaft at any time  $t$  is measured from a fixed position. Successive positions at intervals of  $1/50$  second have been determined as follows:—

Time $t$ , seconds }	0.00	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18
Position $D$ , radians }	0.106	0.208	0.337	0.487	0.651	0.819	0.978	1.111	1.201	1.222

Find the change of angular position during the first interval from  $t = 0.0$  to  $t = 0.02$ ; calculate the mean angular velocity during this interval in radians per second, and set this up on a time base as an ordinate at the middle of the interval. Repeat this for the other intervals, tabulating the results, and drawing the curve showing approximately angular velocity and time. Read off the angular velocity when  $t = 0.075$  second.

(B. of E. 1905.)

## LECTURE XXII.

CONTENTS.—Some Properties of Materials employed by Mechanics—Essential Properties—Extension—Impenetrability—Contingent Properties—Divisibility—Porosity—Density—Cohesion—Compressibility and Dilatability—Rigidity—Tenacity—Malleability—Ductility—Elasticity—Fusibility—Load, Stress, and Strain—Total Stress and Intensity of Stress—Tensile Stress and Strain—Example I.—Compressive Stress and Strain—Example II.—Limiting Stress or Ultimate Strength—Safe Loads and Elasticity—Limit of Elasticity—Hooke's Law—Factors of Safety—Modulus of Elasticity—Ratio of Stress to Strain—Examples III.—V.—Resilience or Work Done in Extending or Compressing a Bar within the Elastic Limit—Examples VI.—IX.—Single Riveted Lap Joints—Example X.—Questions.

**Some Properties of Materials employed by Mechanics.**—The properties of matter are almost innumerable, but they may be divided into two classes: (1) *Essential* properties; (2) *Contingent* properties. The *essential* properties are those without which matter cannot possibly exist. The *contingent* properties are those which we find matter possessing, but without which we could conceive it to exist.

**Essential Properties—1. Extension** means that property by which every body must occupy a certain bulk or volume. When we say that one body has the same volume as another, we do not mean that it has the same quantity of matter, but only that it occupies the same space.\*

**2. Impenetrability** means that every body occupies space to the exclusion of every other body, or that two bodies cannot exist in the same space at the same time.

**Contingent Properties.—1. Divisibility** means that matter may be divided into a great but not an infinite number of parts. The ultimate particles of matter are termed *atoms*, derived from a Greek word signifying indivisible.

**2. Porosity** signifies that every body contains throughout its mass minute spaces or interstices to a greater or less extent. This has been proved to be the case with every known substance. These spaces are supposed to be filled with a highly elastic fluid called ether.

For example, when the steel or cast-iron cylinder of a hydraulic

\* For Simple Rules of Mensuration see the Author's *Elementary Manual* on "Steam and the Steam Engine," Lectures I., II., III.

press is subjected to enormous pressure, water will ooze through the metal from the interior to the outside.

3. Density is that property by which one body differs from another in respect of the quantity of matter which it contains.\*

Let  $M_1, M_2$  = Masses of two bodies

Let  $V_1, V_2$  = Volumes of two bodies.

Let  $D_1, D_2$  = Densities of two bodies.

If  $V_1 = V_2$ , then  $\frac{M_1}{M_2} = \frac{D_1}{D_2}$ ; if  $D_1 = D_2$ , then  $\frac{M_1}{M_2} = \frac{V_1}{V_2}$

If both vary, then  $\frac{M_1}{M_2} = \frac{V_1 \times D_1}{V_2 \times D_2}$

4. Cohesion is that property by which particles of matter mutually attract each other at *insensible* or indefinitely small distances. It is therefore different from *gravitation*, since the latter acts at all distances. It is evident that without this property we could not have a solid, for if a solid body be lifted by one part, the remainder sticks to it, and the whole is kept together by cohesion.

5. Compressibility and Dilatability are properties common to all bodies, by which they are capable of being compressed like a sponge or extended like a piece of india-rubber in a greater or less degree.

6. Rigidity signifies the stiffness to resist change of shape when acted on by external forces. Unpliable materials which possess this property in a large degree are termed *hard*, whilst those which readily yield to pressure, without disconnection, are called *soft*. Substances which cannot resist a change of shape without breaking are termed *brittle*, whilst those that do resist and at the same time change their form are said to be *tough*.

7. Tenacity is the resistance (due to cohesion) which a body offers to being pulled asunder, and is measured by the tensile strength in lbs. per square inch of the cross section of the body. We will consider this property in the case of metals, &c., when dealing with stress and strain.

8. Malleability is that property by which certain solids may be pressed, rolled, or beaten out from one shape to another without fracture. It is therefore a property depending upon the softness, toughness, and tenacity of the material. Gold possesses this property in a higher degree than any other metal, and con-

\* The Density of a substance is either the number of units of mass in a unit of volume, in which case it is equal to the *heaviness* (i.e., weight of unit volume of substance in unit weight); or it is the ratio of the mass of a given volume of the substance to the mass of an equal volume of water, in which case it is equal to the *specific gravity*.

sequently sheets of gold are procurable of less than one-thousandth of an inch in thickness. Copper is one of the most useful of the malleable metals, and it may be beaten out into most elaborate shapes from the solid ingot. The Swedish iron of which horse-shoe nails are made is also very malleable, and is therefore highly prized by the blacksmith. Lead, although possessing softness, is not sufficiently tenacious to be considered a *very* malleable metal, but still it finds one of its most useful applications in the form of rolled lead sheathing for roofs of houses and interiors of water tanks, &c.

9. Ductility\* is that property by which some metals may be drawn down through a die-plate into wire or tubes. This property depends chiefly on toughness and tenacity. For example, we find that the very fine pianoforte wire used with Lord Kelvin's deep-sea sounding machine is both hard and rigid, but possesses great toughness and tenacity. The copper wire used for electrical conductors becomes harder and harder as it gets drawn down to smaller and smaller sizes, and it has therefore to be annealed in order to comply with the many bendings and unbendings which it has afterwards to undergo in winding and unwinding it upon bobbins whilst twisting it into a stranded conductor or in covering it with a dielectric of cotton, silk, gutta-percha, or india-rubber, &c. Solid-drawn copper pipes are frequently used for conveying steam and liquids where a sound light job is required to resist great pressures. This flowing property of metals is now taken great advantage of by the engineer in a variety of ways. For example, lead and tin, when subjected to great hydraulic pressure, and properly guided through a die, can be squirted into long continuous rods or pipes, or squeezed on to insulated electric light conductors, so as to form a water-tight protecting sheathing thereto, just as if these metals were composed of so much plastic dough. In fact, all you have to do in order to cause many harder and stronger metals, such as copper, iron, and steel, to *flow cold* into almost any shape of mould, is to *apply sufficient pressure and to give sufficient time* for them to retain their natural homogeneous and isotropic structure, or to adopt means for restoring the structure should they have departed therefrom during any part of the process. A metal is said to be *homogeneous* when it is of the same density and composition throughout its mass. It is *isotropic* when it has the same elastic properties in all directions.

\* Refer to the description of the Lever Testing Machine, illustrated in Lecture IV., and to Lord Kelvin's Hydrostatic Wire Testing Machine, illustrated in Lecture XVII., as examples of machines whereby the comparative ductility of certain materials may be ascertained by their percentage elongation.

**10. Elasticity** is that property, possessed by different solids in a greater or less degree, of regaining their original size and shape after the removal of the force which caused a change of form. We shall see later on that there are limits of elasticity beyond which the bodies will not regain their exact normal size or shape.

**12. Fusibility** is that property whereby metals and many other substances, such as resins, tallows, &c., become liquid on being raised to a certain temperature. The following table shows in *round numbers* the melting-points of a few of the commoner metals:—

MELTING POINTS OF METALS IN DEGREES FAHRENHEIT.

Mercury . . . . .	- 38	Copper . . . . .	2000
Tin . . . . .	+ 440	German silver . . . . .	2000
Bismuth . . . . .	500	Gold . . . . .	2000
Lead . . . . .	600	Cast iron . . . . .	2200
Zinc . . . . .	700	Steel . . . . .	2500
Antimony . . . . .	800	Nickel, also Aluminium . . . . .	2800
Brass . . . . .	1800	Wrought iron . . . . .	3300
Silver . . . . .	1850	Platinum . . . . .	3500

**Load, Stress, and Strain.**—When force is applied to a body so as to produce either elongation or compression, bending, torsion, shearing, or a tendency to any of these, the force applied is termed the *load*, the corresponding resistance or reaction in the material is termed the *stress* due to the load. Any alteration produced in the length or shape of the body is termed the *strain*.

**DEFINITIONS.**—*Load* is the force or forces applied to the body.

*Stress* is the reaction in the body due to the load.

*Strain* is the alteration in shape as the result of the load.

The load is called a *dead load* when it produces a steady or a gradually increasing or diminishing stress. For example, the weight of a roof on the walls of a building is a steady or dead load. The gradually increasing pull produced on the specimen in the lever-testing machine, illustrated by the fourth figure in Lecture IV., is also a dead load.

The load is termed a *live load* when it varies from instant to instant. For example, a regiment of soldiers, or a series of vehicles, or a train passing over a bridge creates a *live load* on the bridge.

**Total Stress and Intensity of Stress.**—The *total stress* is the total reaction due to the total load. The *intensity of stress*, or simply the word *stress*, expresses the reaction per unit area of the cross section. Thus, if *P* be the total force applied in lbs., and *A* be the total cross section in square inches, then the

Mean *Intensity of Stress* on the section =  $\frac{P}{A}$  lbs. per square inch.

**Tensile Stress and Strain.**—If the line of action of a load be along the axis of a bar, tie-rod, or beam, so as to tend to elongate the same, the reaction per square inch of cross section is termed the *tensile stress*, and the elongation per unit of length is called the *tensile strain*.

**EXAMPLE I.**—A wire  $\frac{1}{16}$  square inch in cross section, and 10 feet long, is fixed at its upper end. A load of 1000 lbs. is hung from the lower end, and then the wire is found to stretch 1 inch. (1) What is the stress? (2) What is the strain?

**ANSWER.**—(1) Here  $P = 1000$  lbs., and  $A = \frac{1}{16}$  sq. in.

Let  $p$  = stress or pull per square inch in lbs.

$\therefore$  The stress, or  $p = \frac{P}{A} = 1000 \div \frac{1}{16} = 10,000$  lbs. per sq. inch.

(2) Original length =  $L = 10' = 120''$ , and the increase of length =  $l = 1''$ .

Let  $e$  = strain or extension per unit of length, i.e., per inch in this case,

$\therefore$  The Strain, or  $e = \frac{\text{increase of length}}{\text{original length}} = \frac{l}{L} = \frac{1''}{120''} = .0083$

**Compressive Stress and Strain.**—If the line of action of a load be along the axis of a bar, shore, strut, or pillar, so as to tend to compress or shorten the same, the reaction per square inch of cross section is termed the *compressive stress*, and the diminution per unit of length is called the *compressive strain*.

**EXAMPLE II.**—A vertical support in the form of a hollow pillar, having 2 square inches cross section of metal, is 10 feet long. With a load of 10,000 lbs. resting on the top, it is found to be compressed  $\frac{1}{16}$  of an inch in length. (1) What is the stress? (2) What is the strain?

**ANSWER.**—(1) Here  $P = 10,000$  lbs., and  $A = 2$  sq. inches. Let  $p$  = stress or compression per sq. in. of cross section in lbs.

$\therefore$  The stress, or  $p = \frac{P}{A} = \frac{10,000}{2} = 5000$  per square inch.

(2) Original length =  $L = 10' = 120''$ , and the diminution of length =  $l = \frac{1}{16}''$

Let  $e$  = strain or compression per unit of length, i.e., per inch in this case,

$\therefore$  The strain, or  $e = \frac{\text{diminution in length}}{\text{original length}} = \frac{.1''}{120''} = .00083$

**Limiting Stress or Ultimate Strength.**—For every kind of material and every way in which a load is applied, there must be a value, which, if exceeded, causes rupture or fracture of the

body. The greatest stress which the material is capable of withstanding is called the *limiting stress* or *ultimate strength per square inch of cross section* of the substance, for the particular way in which the load is applied.

**Factors of Safety.**—The ratio of the *ultimate strength* or *limiting stress* to the *safe working load* is called the *factor of safety*. This factor of necessity varies greatly with different materials, and even with the same material, according to circumstances. For materials which are subjected to oxidation or to internal changes of any kind, the factor of safety must of necessity be larger than in those which are always kept dry or are well painted and carefully handled. There is no condition in engineering structures which requires a more careful calculation, or estimate of the necessary factors of safety, than that of railway bridges, which are exposed to all sorts of weathers and to extremely variable live loads. The skill of the engineer is therefore brought out, when he designs structures so as to include all possible circumstances to which they may be subjected, and so proportions the material at his disposal, that there shall be a minimum of internal stress and strain, with a maximum resistance to dead or live loads for a minimum cost of material and workmanship.\*

**TABLE OF ULTIMATE STRENGTH AND WORKING STRESS OF MATERIALS WHEN IN TENSION, COMPRESSION, AND SHEARING.**

Materials.	Ultimate Strength. Tons per sq. inch.			Working Stress. Tons per sq. inch.		
	Tension.	Compression.	Shearing.	Tension.	Compression.	Shearing.
Cast iron . . .	7.5	45	14	1.5	9	3
Wrought-iron bars .	25	20	20	5	3.5	4
Steel bars . . .	45	70	30	9	9	5
Copper bolts . . .	15	25	—	3	5	—
Brass sheet . . .	14	—	—	3	—	—

**Safe Loads and Elasticity.**—As a rule, however, the object of the engineer is not to put such a stress on his materials of construction as will cause rupture or destruction, but rather to

\* For other tables relating to the Strength of Materials in Engineering Constructions, Factors of Safety, &c., refer to Rankine's "Rules and Tables," Molesworth's "Pocket Book of Engineering Formulæ," D. K. Clarke's "Rules and Tables," "The Practical Engineer's Pocket Book," and for Electrical Engineering Materials to Munro and Jamieson's "Pocket Book of Electrical Rules and Tables."

make machines and raise structures that will withstand all reasonable forces likely to be brought to bear upon them. Consequently, he is quite as much interested in what may be termed *safe loads* as in ultimate or destructive ones. He therefore requires to know what loads can be safely applied to materials under different circumstances, so as to comply with that most useful property termed *elasticity*, which we again define as *the capability of regaining their original size, shape, and even strength, after the removal of the forces which caused a change of form in them.*

**Limit of Elasticity—Hooke's Law.**—So long as the stress or reaction per square inch of cross section does not exceed a certain limit, called the *limit of elasticity*, then the material will return to its original shape, size, and strength, after the removal of the load. This limit has been ascertained for most materials of construction by elaborate experiments, which are to be found tabulated in the Proceedings of the Institutions of the Civil and Mechanical Engineers, and in such books as Rankine's "Rules and Tables," Molesworth's "Pocket Book of Engineering Formulæ," and D. K. Clark's "Rules and Tables." For example, with a bar of good wrought iron the elastic limit is only reached after a stress of 24,000 lbs. per square inch has been brought to bear upon it, and in a similar degree every other material has a corresponding limit, beyond which it is not safe to stress it, for fear that it should be overstrained, and thus lose, to a certain extent, its property of recuperation or restitution, or take a permanent set.

*Within this limit, Hooke's Law* holds good for metal bars under the action of forces tending to elongate or compress them. This law states that:

(1) The amount of extension or compression for the same bar is in direct proportion to the stress.

(2) The extension or compression is directly proportional to the length.

(3) The extension or compression is inversely proportional to the cross sectional area; consequently, if the area be doubled the extension or compression will be halved, or the resistance to the load will be doubled.

Let  $P$  = Pull, push, or load in lbs. on the bar.

„  $A$  = Area of cross section of the bar.

„  $L$  = Length of the bar before the load was applied.

„  $l$  = Length by which the bar is extended or compressed.

„  $p$  = Stress or load per square inch of cross section =  $P/A$ .

Then, so long as  $\frac{P}{A}$  does not exceed the elastic limit,  $l$  varies directly

as  $P$  for the same bar; or  $\frac{l}{L}$  varies directly as  $\frac{P}{A}$ , for different bars of the same material and subjected to the same conditions.

In other words, so long as the stress does not exceed the elastic limit, the strain will be proportional to the stress.

**Modulus of Elasticity, or Ratio of Stress to Strain.**—As we have just indicated, by HOOKE'S LAW, if a metal under test be gradually subjected to a stress, and if the load does not exceed the limits of elasticity of the material, the strain will be in proportion to the load.

Consequently, the ratio of the *stress* to the *strain* is a constant quantity for each particular substance within the limits of Hooke's Law, and is termed the *Modulus of Elasticity* of the substance.

$$\begin{array}{lll} \text{But} & . & \text{Stress} \propto \text{Strain} \\ \therefore & . & \text{Stress} = E \times \text{Strain}.^* \end{array}$$

Where  $E$  represents a constant number or modulus depending on the natural elasticity of each material—

$$\therefore E = \frac{\text{Stress}}{\text{Strain}} = \frac{\frac{P}{A}}{\frac{l}{L}} = \frac{PL}{Al}$$

$$\text{i.e.,} \quad \therefore \quad . \quad . \quad . \quad . \quad PL = AE$$

Or, imagine—it is *pure imagination*—that a substance could be elongated to double its length or compressed to zero by subjecting it to a certain load, we should then have an index value, or constant number, or modulus, by which we could compare it with every other substance which behaved likewise under similar circumstances. This imaginary value is termed the *Modulus of Elasticity*.

For example, take a bar of wrought iron of 1 square inch cross section, which is found to stretch  $\frac{1}{24000000}$  part of its length under a stress of 1 lb., and consequently by Hooke's Law twice that amount under a stress of 2 lbs., and so on; then this number (24,000,000) is called the *Modulus of Elasticity* of the iron bar. For, if the elasticity of the bar were perfect, it is evident that a stress of 24,000,000 lbs. would produce a strain or elongation equal to the length of the bar, or,  $\frac{24000000}{24000000} = 1$ . In other words, the length of the bar would be doubled under this stress. Consequently, we have the following definition,

\* Since *stress* is reckoned by so many lbs. per square inch of cross section of a material, and *strain* is simply an abstract number, it follows that the Modulus of Elasticity ( $E$ ) must also be reckoned by so many lbs. per square inch.

**DEFINITION.**—*The Modulus of Elasticity of any substance is that load which would double its length on the supposition that the elongation was proportional to the stress, and that the cross section of the bar was of unit area, or one square inch, and supposing the bar to remain perfect during the operation.*

From this we again see that—

$$\text{Modulus of Elasticity} = \frac{\text{stress}}{\text{strain}} = E = \frac{P}{A} \bigg/ \frac{l}{L}$$

Or, . . . .  $PL = AE$

**MODULI OF ELASTICITY TO STRETCHING.**

(See Rankine's Rules and Tables for complete Data.)

Material.	Modulus of Elasticity in lbs. per sq. in. in round numbers.	Material.	Modulus of Elasticity in lbs. per sq. in. in round numbers.
	(Mean values.)		(Mean values.)
Wood, Elm . . .	1,000,000	Lead (sheet) . . .	700,000
„ Larch . . .	1,100,000	„ (wire) . . .	1,000,000
„ Beech . . .	1,300,000	Brass (cast) . . .	9,000,000
„ Birch . . .	1,400,000	„ (wire) . . .	14,000,000
„ Mahogany . .	1,400,000	Copper (cast) . . .	15,000,000
„ Oak . . .	1,500,000	„ (wire) . . .	17,000,000
„ Pine (yellow) .	1,600,000	Cast Iron . . . .	18,000,000
„ Ash . . .	1,600,000	Wrought Iron . . .	25,000,000
„ Teak . . .	2,000,000	Steel . . . . .	35,000,000

**EXAMPLE III.**—A steel bar 5' long and  $2\frac{1}{4}$  sq. in. in cross section is suspended by one end; what weight hung on the other end will lengthen it by .016 inch, if the modulus of elasticity of steel is 30,000,000 lbs. per square inch? (S. and A. Exam. 1877.)

**ANSWER.**—First ask what is wanted? Viz., *stress*.

Now the universal rule is  $\text{Modulus of Elasticity} = \frac{\text{stress}}{\text{strain}}$

Or,  $\text{stress} = \text{modulus} \times \text{strain}$ .

For, *The strain is the elongation per unit of the length.*

Consequently,  $e = \frac{.016''}{5' \times 12''} = \frac{.016}{60} = .00026.$

∴ The Stress = Modulus  $\times$  strain

$$= 30,000,000 \times .00026 = 8000 \text{ lbs. per sq. in.}$$

And, The *Total Stress* = 8000 lbs.  $\times$  2.25 sq. in. = 18,000 lbs.

Or, we might have applied the formula previously deduced—viz.,

$$PL = AE,$$

where P is the total pull required in lbs.

$$\therefore P = \frac{AE}{L} = \frac{2.25 \text{ sq. in.} \times .016'' \times (30 \times 10^6)}{5 \times 12} = 18,000 \text{ lbs.}$$

EXAMPLE IV.—What do you understand by stress and strain respectively? If an iron rod, 50 feet long, is lengthened by  $\frac{1}{2}$  inch under the influence of a stress, what is the strain? (S. and A. Exam. 1892.)

ANSWER.—*Stress* is the reaction per unit area of cross section due to the load. Let P = the total tension acting on area A;

$$\text{Then stress} = p = \frac{P}{A}$$

*Strain* is the ratio of the increase or diminution of length or volume to the original length or volume. Let L = original length of a bar of the material,  $l$  = amount by which the length is increased or diminished; then, when the bar is subjected to stress,

$$\text{The strain} = e = \frac{l}{L}$$

In the example given,  $L = 50' \times 12'' = 600$  inches; and  $l = \frac{1}{2}$  inch.

$$\therefore \text{Strain, } e = \frac{l}{L} = \frac{\frac{1}{2}}{600} = \frac{1}{1200} = .0008\bar{3}$$

EXAMPLE V.—From the above question and answer determine the modulus of elasticity of the iron of which the rod is composed, if the load was 4366 lbs., and the cross section of the rod 2 square inches.

$$\text{ANSWER.—(1) Stress} = \frac{\text{Total load}}{\text{Cross area}}$$

$$\text{Or, } p = \frac{P}{A} = \frac{4366 \text{ lbs}}{2} = 2183 \text{ lbs. per sq. in.}$$

$$(2) \text{ Modulus of Elasticity} = \frac{\text{Stress}}{\text{Strain}}$$

$$\text{Or, } E = \frac{p}{e} = \frac{2183}{.0008\bar{3}} = 25,000,000$$

∴ A load of 25,000,000 lbs. would elongate a rod of the iron to double its length by tensile stress.

**Resilience or Work done in Extending or Compressing a Bar within the Elastic Limit.**—*Definitions of Resilience.*—(1) When a bar is strained within the elastic limits either by a compressive or a tensile force, then the work done in extending or compressing it is equal to the amount of compression or extension multiplied by the mean stress which produces the strain. The amount of work thus done when the stress just reaches the elastic limit is termed *resilience*.

(2) *Resilience* may also be defined as half the product of the stress into the strain, where the stress and the strain are those produced when the elastic limit is reached.

Let  $P$  = push or pull in lbs. applied gradually.

"  $A$  = area of bar in square inches of cross-section.

"  $L$  = length of bar in inches.

"  $l$  = elongation of the bar in inches due to the force  $P$ .

"  $p$  = stress per square inch when the elastic limit is reached,  $= P/A$ .

"  $R$  = resilience of the bar.

**I.—Where the Load is Gradually Applied.**—If the load be gradually increased from 0 up to  $P$  lbs., its mean value will be  $\frac{P}{2}$ , and the work

done or *resilience*,  $R = \frac{P}{2} l$  inch-lbs.

But  $P = pA = EA \frac{l}{L}$ .

∴ work done or *resilience*  $R = \frac{EAl^2}{2L} = \frac{1}{E} \left( \frac{P}{A} \right)^2 \frac{AL}{2} = \frac{1}{2} p^2 \frac{AL}{E}$ ,

" " "  $R = \frac{p^2}{E} \times \frac{\text{volume of bar}}{2}$  inch-lbs.

Work done per unit  
of volume  $\left. \vphantom{\begin{matrix} \text{Work done per unit} \\ \text{of volume} \end{matrix}} \right\} = \frac{p^2}{2E}$

This latter equation gives the *strain energy stored in the bar*, since the material is still elastic.

**II.—When the Load is Suddenly Applied without Initial Velocity.**—Let  $l$  be the elongation or shortening of the bar when the load  $P$  is *suddenly applied*, but without initial velocity.

Then, the work done by the external load must be equal to the energy stored up in the bar.

Let  $P_{max}$  be the maximum stress per square inch which is produced in the bar.

The *mean* stress will be  $\frac{P_{max}}{2}$  as its initial value was 0.

Hence, the work done on the bar  $= \frac{P_{max} l}{2} = \frac{EAl^2}{2L}$ ,

and the work done by external load  $= Pl$ .

Therefore,  $\frac{P_{max} l}{2} = \frac{EAl^2}{2L} = Pl$ ,

or,  $P_{max} = 2P$ ; and  $E \frac{l}{L} = 2 \frac{P}{A}$ .

Consequently, the maximum intensity of stress induced in a bar by the sudden application of a constant load, without initial velocity, is double the intensity of the stress produced by the load itself.

**III.—When the Load is Suddenly Applied and with an Initial Velocity.**—Let a weight of  $W$  lbs. be dropped from a height of  $h$  inches upon a bar, and that the *maximum* stress produced in the bar was  $P_{max}$  lbs. per square inch, whilst the elongation or compression was  $l$  inches.

Work done by the falling weight =  $W(h + l)$  = energy stored up in the bar.

Therefore,  $\frac{P_{max}l}{2} = \frac{EAl^2}{2L} = W(h + l) = Wh$  if the value of  $l$  is small.

If  $v$  = velocity of load  $P$  at moment of impact in ft. per sec.,

then kinetic energy,  $E^k = \frac{Wv^2}{2g} = \frac{P_{max}l}{2A} = \frac{W(h + l)}{12}$  ft.-lbs.

EXAMPLE VI.—Calculate the resilience of a steel tie-bar, 1 inch in diameter and 4 feet long if the elastic limit is reached under a load of 20 tons, and modulus of elasticity = 13,000 tons per square inch.

Answer.—

$$\text{Work done or resilience, } R = \frac{1}{E} \left( \frac{P}{A} \right)^2 \frac{AL}{2}.$$

$$" " " " = \frac{1}{13,000} \left( \frac{20}{.7854} \right)^2 \left( \frac{.7854 \times 4 \times 12}{2} \right)$$

$$" " " " = \frac{48}{51} \text{ inch-tons.}$$

$$" " " " = \frac{48 \times 2240}{51} = 2108 \text{ inch-lbs.}$$

EXAMPLE VII.—A round bar of steel is 20 feet long and 1 inch in diameter. Find the tensile load which, if suddenly applied, would cause an instantaneous elongation of the bar of .1 inch.

Taking  $E = 13,000$  tons per square inch.

Answer.—

$$P = \frac{EAl}{2L}, \text{ where symbol letters represent the values stated in the text.}$$

$$P = \frac{13,000 \times .7854 \times .1}{2 \times 20 \times 12}$$

$$P = \frac{102.1}{48} = 2.12 \text{ tons.}$$

EXAMPLE VIII.—Determine the greatest weight that can be dropped from a height of 1 foot on a bar of steel which is 1 inch in diameter and 10 feet long. The modulus of elasticity,  $E = 13,000$  tons per square inch, and the elastic limit 18 tons per square inch. Also, find the alteration in the length of the bar.

Answer.—

$$p = \frac{P}{A} = \frac{El}{L}$$

$$18 = \frac{13,000 \times l}{10 \times 12}$$

$$l = \frac{18 \times 10 \times 12}{13,000} = .16 \text{ inch.}$$

$$\frac{P_{max}l}{2} = \frac{1}{2} pAl = W(h + l).$$

$$W = \frac{pAl}{2(h + l)}$$

$$W = \frac{18 \times .7854 \times .16}{2(12 + .16)} = \frac{2.262}{24.32} \text{ ton} = 209 \text{ lbs.}$$

**EXAMPLE IX.**—In a tensile test of a piece of flat wrought iron bar the following results were obtained:

- (i) Original dimensions of cross-section, 2·02 inches by 0·51 inch.
- (ii) Final dimensions of cross-section at point of fracture. 1·49 inches by 0·39 inch.
- (iii) Gross load at limit of elasticity, 36,000 lbs.
- (iv) Gross load at fracture, 59,000 lbs.
- (v) Total final extension on a length of 10 inches, 1·63 inches, extension when load was 22,000 lbs., ·0075 inch.

Find from the above data, (a) the modulus of elasticity, (b) limit of elasticity, and tenacity in lbs. per square inch; also (c) the reduction of area per cent., and (d) the approximate work done in fracturing this specimen. (L.U.B.Sc. Eng. 1903.)

*Answer.*—

$$(a) \left. \begin{array}{l} \text{Young's} \\ \text{modulus of} \\ \text{elasticity} \end{array} \right\} E = \frac{PL}{Al}$$

$$E = \frac{22,000 \times 10}{2 \cdot 02 \times \cdot 51 \times \cdot 0075} = \frac{220,000}{\cdot 007726}$$

$$E = 28,473,440 \text{ lbs. per square inch.}$$

$$(b) \left. \begin{array}{l} \text{Stress at} \\ \text{elastic limit} \\ \text{in tons per} \\ \text{square inch} \end{array} \right\} = \frac{\text{Load at elastic limit}}{\text{Original area}} = \frac{36,000}{1 \cdot 0302}$$

$$= 34,944 \text{ lbs. per square inch.}$$

$$(c) \left. \begin{array}{l} \text{Percentage} \\ \text{contraction of} \\ \text{area} \end{array} \right\} = \frac{\text{Original area} - \text{final area}}{\text{Original area}} \times 100$$

$$= \frac{(2 \cdot 02 \times \cdot 51) - (1 \cdot 49 \times \cdot 39)}{2 \cdot 02 \times \cdot 51} \times 100$$

$$= \frac{(1 \cdot 032 - \cdot 5811) 100}{1 \cdot 0302} = 43 \cdot 6 \text{ per cent.}$$

$$(d) \left. \begin{array}{l} \text{Work done} \\ \text{per cubic} \\ \text{inch in frac-} \\ \text{turing the} \\ \text{specimen} \end{array} \right\} = \frac{\text{Work done in stretching the test-piece}}{\text{Volume of the test-piece}}$$

$$= \frac{\left\{ \begin{array}{l} \text{Mean load between starting-point and point of} \\ \text{fracture} \times \text{distance moved through in inches} \end{array} \right\}}{\left\{ \begin{array}{l} \text{Area in square inches} \times \text{length of test-piece in} \\ \text{inches} \end{array} \right\}}$$

$$= \frac{32,750 \times 1 \cdot 63}{10} = 5338 \cdot 25 \text{ inch-lbs.}$$

$$(e) \left. \begin{array}{l} \text{Percentage} \\ \text{elongation} \\ \text{on 10 inches} \end{array} \right\} = \frac{\text{Final length} - \text{original length}}{\text{Original length}} \times 100$$

$$= \frac{1 \cdot 63 \times 100}{10} = 16 \cdot 3 \text{ per cent.}$$

**Single-Riveted Lap Joints.** (See the author's text-book on Steam, &c., re Riveted Joints.)

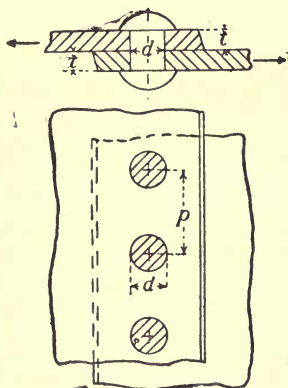
Let  $p$  = pitch of rivets in inches.

"  $d$  = diameter of rivet hole in inches.

"  $t$  = thickness of plate in inches.

"  $f_t$  = tearing resistance or tensile stress of the plate in lbs. per square inch.

"  $f_s$  = shearing resistance or shear stress of the rivet in lbs. per square inch.



SINGLE-RIVETED LAP JOINT.

Then for a single-riveted lap joint,

the area of plate under tensile stress  $= (p-d)t$ ,

and the area of rivet under shear stress  $= \frac{\pi d^2}{4}$

Hence, for *equal strength*, the following equation must be true:

$$f_t (p-d)t = f_s \frac{\pi d^2}{4}$$

In practice,  $d = 1.2 \sqrt{t}$ , or  $t = \frac{d^2}{1.44}$  is usually taken for boilers.

$$\therefore f_t (p-d) \frac{d^2}{1.44} = f_s \frac{\pi d^2}{4}$$

$$f_t (p-d) = f_s \frac{\pi}{4} \times 1.44 = .36 \times \pi \times f_s$$

$$\text{Or, } (p-d) \frac{f_t}{f_s} = 1.131.$$

The values to be taken for  $f_t$  and  $f_s$  in any given case depends upon the number of rows of rivets; upon the material which is used (iron or steel), and on whether the holes have been punched or drilled. But, in practice  $f_t$  ranges from 35,000 to 67,000 lbs. per square inch, and  $f_s$  from 43,000 to 53,000 lbs. per square inch. Whilst for iron plates and iron rivets, with drilled holes, the ratio  $\frac{f_t}{f_s}$  may be taken as equal to .94; which would give for the single-riveted lap joint  $(p-d)0.94 = 1.131$ .

EXAMPLE X.—In a single-riveted lap joint the thickness of the plate is  $\frac{3}{4}$  inch, and the diameter of the rivet is 1 inch. If the tearing resistance of the plate is 60,000 lbs. per square inch, and the shearing resistance of the rivet is 50,000 lbs. per square inch, find the proper pitch of the rivets.

(C. & G., 1905, O., Sec. B.)

Answer.—Substituting the numerical values given by the question in the above formula, we get :

$$(p - d) \frac{f_t}{f_s} = 1.131$$

$$(p - d) \frac{60,000}{50,000} = 1.131$$

$$p - 1 = \frac{1.131 \times 50,000}{60,000} = .95$$

$$\therefore p = .95 + 1 = 1.95 \text{ inches.}$$

## LECTURE XXII.—QUESTIONS.

1. State and define the essential and contingent properties of matter, and give the names of those engineering materials with which you happen to be practically acquainted, that best exemplify each property.

2. What is the meaning of the term *ductility* as applied to wrought iron? Describe, with sketches, some apparatus for testing a piece of metal as to ductility. If a uniform bar of iron 10 inches long is found to stretch  $1\frac{1}{2}$  inches at the time of fracture, what is the measure of the ductility of the material of the bar? *Ans.* 15 per cent.

3. Give the approximate breaking tensile stress for a bar of cast iron of one square inch sectional area, and the same for a bar of wrought iron? What is the meaning of the term ductility as applied to wrought iron, and how is the ductility of iron measured?

4. What must be the diameter in inches of a round rod of wrought iron in order to sustain a load of 50 tons? It is given that a bar of iron 1 square

inch in section will just support a load of 25 tons. *Ans.*  $d = \sqrt{\frac{2}{.7854}} = 1.6''$ .

5. What is the modulus of elasticity of a substance? A round bar of iron, 12 feet long and  $1\frac{1}{2}$  square inches in sectional area, is held at one end and pulled by a force till it stretches  $\frac{1}{2}$  inch; find the force, the modulus of elasticity being 30,000,000. *Ans.* 39,063 lbs.

6. A round bar of steel 1" in diameter and 10 feet long, is fixed at its upper end, and a load is applied to the bottom end and stretches it .05". Find the load if the modulus of elasticity is 30,000,000. *Ans.* 9817.5 lbs.

7. Find the dimensions of a transverse section of a square rod of fir to sustain a suspended load of 10 tons, the rod being held vertically. The breaking load of a rod of fir one square inch in section is 6 tons. *Ans.* 1.29 inches.

8. Find the extension produced in a bar of wrought iron 4 feet long and 2 square inches in section by a suspended weight of  $4\frac{5}{8}$  tons, the modulus of elasticity of the material being 29,000,000 pounds per square inch. *Ans.* .009 inch.

9. What do you understand by the terms stress, strain, and modulus of elasticity? A tie-rod, 100' long and 2 square inches cross area, is stretched .75" under a tension load of 32,000 lbs. What is the intensity of the stress, the strain, and the modulus of elasticity under these circumstances? *Ans.* 16,000 lbs. per square inch; .000625; 25,600,000.

10. Define what is meant by "dead load," "live load," "limiting stress," "limit of elasticity," and "factors of safety."

11. What do you understand by stress and strain respectively? If an iron rod, 50 ft. long, is lengthened by  $\frac{1}{4}$  in. under the influence of a stress, what is the strain? If the rod is 2 sq. in. in section, and the load 11,000 lbs., what is the modulus of elasticity? *Ans.* .000417; 13,200,000.

12. Find the stress produced in a pump-rod 4" diameter, lifting a bucket 28" diameter if the pressure on the top of the bucket be 6 lbs. per square inch in addition to the atmosphere, and the vacuum below the bucket be 26" by gauge. Reckon each 2" of vacuum = 1 lb. *Ans.* 931 lbs. per sq. inch.

13. If the rod in question is 5' long, find its extension if the modulus of elasticity = 9,000,000. *Ans.* .006 inch.

14. What do you understand by the terms *tensile*, *compressive* and *shearing strength* respectively of any material? Define "modulus of elasticity." If a wrought-iron bar of 1 square inch sectional area just breaks under a tensile stress of 60,000 lbs., what would be the area of the section of a tie-rod which would just support a load of 20 tons?

Ans. 75 sq. inch.

15. A wrought-iron tie bar,  $\frac{3}{4}$  inch in diameter, has a modulus of elasticity of 28,000,000 lbs. per square inch. Its length is 23 inches; find the load under which the bar will extend .015 of an inch. Find also the stress per square inch.

Ans. 8067.6 lbs. and 18,261 lbs. per sq. inch.

16. How would you find out for yourself the behaviour of steel wire loaded in tension till it breaks? What occurs in the material? Use the words "stress" and "strain" in their exact senses.

17. An iron rod, of 1 inch diameter and 12 feet in length, stretches  $\frac{3}{32}$ -inch under a load of 6 tons suspended at its extremity. Determine the stress, strain, and modulus of elasticity of the bar. Ans. 17,112.3 lbs. per sq. in.; 0.00065;  $E = 26,326,615$  lbs. per sq. in.

18. What do you mean by Stress, Strain, and Modulus of Elasticity?

A wire 10' long and  $\frac{1}{8}$  sq. inch in sectional area is hung vertically, and a load of 450 lbs. is attached to its extremity, when the wire stretches .015" in length. What are the stress and strain respectively? And also the modulus of elasticity? Ans. 3600 lbs. per sq. inch; .000125;  $E = 28,800,000$  lbs. per sq. inch.

19. An iron wire is loaded with gradually increasing tensile loads till it breaks. We want to know its modulus of elasticity, its elastic limit stress, and its breaking stress. What measurements and calculations do we make? (S. E. B. 1900.)

20. Sketch apparatus and describe a laboratory experiment by which you could find  $E$ , Young's modulus of elasticity, for an iron wire 10 feet long and 0.05 inch diameter. How would you secure the upper end of the wire? How apply the load? And how measure the elongation? How would you plot your results and how deduce the value of  $E$ ?

(S. E. B. 1902.)

21. Describe an experiment by which you could determine  $E$ , Young's modulus of elasticity, by stretching an iron wire. (B. of E. 1903.)

22. A bolt  $2\frac{1}{2}$  inches diameter has a tensile load of 30 tons, what is the stress? What is the strain if Young's modulus of elasticity is  $3 \times 10^7$  pounds per square inch? What is the elongation of a part which when unloaded was 102 inches long?

(B. of E. 1904.)

Ans. Stress = 13,689.8 lbs. per sq. inch. Strain = .000456.

Elongation = .0465 inch.

23. Two bars of equal length, both of rectangular section but of different materials, are firmly riveted together at their ends and subjected to a pull so that they are compelled to stretch the same amount. If  $A_1$ ,  $A_2$  represent their sectional areas, and  $E_1$ ,  $E_2$  the values of Young's modulus for the two materials, show that when the pull is  $P$  lbs. the intensities of stress induced in the two bars are

$$\frac{PE_1}{A_1 E_1 + A_2 E_2} \text{ and } \frac{PE_2}{A_1 E_1 + A_2 E_2}$$

respectively—the limits of elasticity being not exceeded.

(C. & G., 1904, O., Sec. B.)

24. A bar of mild steel, of rectangular section, is 2 inches wide and  $\frac{1}{2}$  inch thick, and is 10 inches long. If Young's modulus is 12,500 tons per square inch, find the amount the bar stretches when the load on it is 10 tons. How much work is then stored up in the bar?

Ans.  $l = .008$  inch; work stored up in bar = 89.6 in.-lbs.

(C. & G., 1905, O., Sec. B.)

25. An iron column is 12 inches in external diameter, and the metal is  $1\frac{1}{4}$  inch thick. The load on the column is 125 tons. What is the compressive stress in the metal? By what amount will the column be shortened, if its length is 15 feet, and if Young's modulus of elasticity is 12,500,000 pounds per square inch?

Ans.  $f_c = 6630$  lbs. per square inch;  $l = .095$  inch. (B. of E. 1905.)

26. The following results were obtained during a tensile test of a mild steel bar  $\frac{3}{4}$  inch in diameter :

Total load on the bar in tons . . }	0.88	1.76	2.64	3.52	4.40	5.28	6.16	7.04
Elongation on a length of 8 inches stated in inches . }	0.0012	0.0024	0.0035	0.0047	0.0061	0.0075	0.0088	0.0102

(a) Plot a curve on squared paper, going evenly through the points, to show the relation between the load and the elongation.

(b) Find the load necessary to cause an elongation of 0.0040 inch.

(c) Find the total work done in inch-tons upon this 8" length of the bar during the test.

(B. of E. 1905.)

Ans. (b) Load = 3.05 tons.

(c) Total work done = 0.036 inch-ton.

NOTES AND QUESTIONS.

## LECTURE XXIII.

CONTENTS.—Stresses on Chains—Shearing Stress and Strain—Example I. — Torque or Twisting Moment—Torsion of wires—Table giving the strength, moduli of Elasticity and Rigidity of various materials—Strength of Solid Round Shafts—Example II.—Table giving the Horse-Power which steel shafting will transmit at various speeds—Strength of Hollow Round Shafts—Relation between the Twisting Moment and Horse-Power transmitted by shafting, as well as the diameter necessary to transmit a given Horse-Power—Examples, III. IV.—Questions.

In this Lecture we will continue the subject of “strength of materials,” and finish the course with reasons for the shapes generally given to sections of cast iron, wrought iron, and steel girders.

**Stresses on Chains.**—The only stress to which the sides of the links of chains are subjected under ordinary circumstances, is that of tension. This stress tends to bring the sides of the links closer together, and consequently we find that large chain cables for mooring ships (where very sudden and severe stresses are encountered) have a cast-iron stud or wedge fitted between the inner sides of the links. These studs most effectually keep the sides of the links apart, and prevent any link jamming a neighbouring one. They add materially to the strength of the chain, for they are in compression whilst the sides of the links are in tension. Being composed of cast iron, which offers the immense resistance to compression of fully 45 tons per square inch,\* there is not much fear of their giving way before the sides of the links.

The strength of a stud-link may be taken as equal to double the strength of a rod of wrought iron, of the same diameter and quality of material as that of which the chain is composed, whereas the strength of an open-link chain is only about 70 per cent. of this amount, even with perfect welding.†

In Molesworth’s “Pocket-Book of Engineering Formulæ,” the student will find at page 54 a formula for the safe load on chains, viz.—

$$W = 7 \cdot 1 d^2$$

Where  $W$  = Safe load in tons.

„  $d$  = Diameter of iron in inches.

\* See Table of the Ultimate Strengths and Safe Working Loads given in Lecture XXII.

† Some well-known authorities give less than 70 per cent.

Now, such a formula is very easy of application, but the student should never rest content until he finds out how the constants have been arrived at, and what relation the various symbols have towards each other. If he refers back to the short table of "Ultimate Strengths and Working Loads" given in the previous Lecture, he will find opposite wrought-iron bars and under tension, the value 5 tons per square inch as the safe working load. Consequently, applying what was said above about perfect stud-link chains, he will see that—

$$W = \begin{cases} \text{twice the load of a rod of the same diameter and} \\ \text{quality as that of which the chain is composed.} \end{cases}$$

$$\therefore W = 2 \times 5 \times \text{cross area of the chain iron.}$$

$$W = 2 \times 5 \times \frac{\pi d^2}{4} = 2 \times 5 \times \frac{1}{4} \times \frac{22}{7} d^2 = 7.8 d^2$$

This is near enough to the constant given by the above empirical formula to enable him to see how it has been obtained.

Chains which are subjected to many sudden jerks (such as lifting chains for cranes and slings) become in time crystalline, or short in the grain, and consequently brittle and unsafe. The best precaution to adopt in order to periodically remove this enforced internal condition, is to draw them once a year very slowly through a fire, thus allowing them to become heated to a dull red, and then to cool them slowly in a heap of ashes. This method is followed at Woolwich Arsenal and some other Government works.

**Shearing Stress and Strain.**—The action which is produced by shearing and punching machines on iron, steel, or copper plates, &c., is to force one portion of the metal across an adjacent portion. The shearing stress is the reaction per square inch opposing the load or pressure applied to the shears or punch, and the shearing strain is the deformation per unit length or volume. Rivets holding boiler plates together, fulcrum of levers, the pins of the links of the chain of a suspension bridge, the cotter keys of a pump rod, are all subjected to shearing stresses and strains. The ultimate and the working shearing stresses for a few engineering materials were given in a table in Lecture XXII.

In the case of loaded beams (which we will consider shortly in connection with bending moments) the *shearing force* at any point or any transverse section thereof is equal to the algebraical sum of all the forces on *either* side of the point or section.

**EXAMPLE I.**—A steel punch 1" diameter is used in a large shipyard punching machine to make holes in steel plates 1" thick. What will be the total shearing stress or least pressure required?

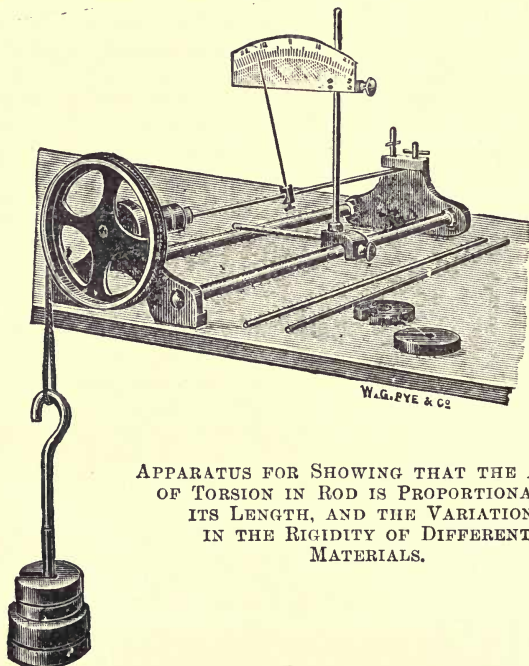
ANSWER.—Referring to the table in last Lecture, we see that the ultimate shearing strength or shearing stress for steel bars (which we will assume to be the same as for plates) is 30 tons per square inch. Now a hole 1" diameter has a circumference  $= \pi d = 3.14"$ , and since the plate is 1" thick, the area of the resisting section must be the circumference of the hole  $\times$  its depth, or  $= 3.14" \times 1" = 3.14$  sq. in., and the total pressure required  $= 30$  tons  $\times 3.14 = 94.2$  tons.

**Torque, or Twisting Moment.\***—In the case of a shaft having a lever, pulley, or wheel fixed to it with a force  $P$  lbs., applied at radius  $R$  feet from the centre of the shaft, then

*The twisting moment T.M. is  $= P \times R$  lbs.-feet.*

Or, *The torque  $= P \times R \times 12$  lbs.-inches.*

\* The term *torque* was devised by the late Professor James Thomson, of Glasgow University, to signify the twisting or torsional moment. The lbs.-feet of torque must not be confused with ft.-lbs. of work; or with *resilience* (which is the work done in *straining* a body, as measured by the elongation or compression in feet  $\times$  the mean load causing the strain). It will therefore save confusion, if we take the force applied at the end of the arm in lbs., and the leverage or arm in inches, and *then* multiply them, so as to get the *torque* in *lbs.-inches*.



APPARATUS FOR SHOWING THAT THE ANGLE OF TORSION IN ROD IS PROPORTIONAL TO ITS LENGTH, AND THE VARIATIONS IN THE RIGIDITY OF DIFFERENT MATERIALS.

**Torsion in Rods.**—It will be seen from the previous figure, that the front pulley carries a cord with known weights suspended from it. The spindle of this pulley moves in ball bearings, and carries at its inner end a three-jaw chuck. This chuck is for holding one end of the rod under test, whilst the other end is for tightly clamping it to the back bracket of the machine. The torsional couple is applied by means of weights hung from the afore-mentioned cord, and the torsion in degrees is read off on the dial. To eliminate errors the weight should be hung first on one side of the pulley, then on the other side, and the mean of the two readings taken. It will be noticed, that both the pointer and the scale can be readily moved to any distance from the fixed end of the rod.

It can thus be shown that the angle of torsion in a rod is proportional to its length and that different materials have different rigidities.

**Torsion of Wires.**—In the figure, AB represents a wire held firmly at the top end of a supporting rod, which is 8 feet long. A pulley is fixed firmly to the wire at B, and this pulley is acted upon by two cords which tend to turn it without moving its centre sideways, i.e., they act on the pulley with a turning moment only. But, the pulley can only turn by giving a twist to the wire. Hence, if a light pointer be fastened to the wire at C, the former moves over a dial, and the angle turned through by the wire and the pointer is called the total angle of twist at C. Similar pointers fixed to the wire at D and E give the angle of torsion at these points. The dials at D and E are supported upon adjustable sliders, so that they may be moved up or down the vertical rod in order that the torsion of any length of rod may be measured.

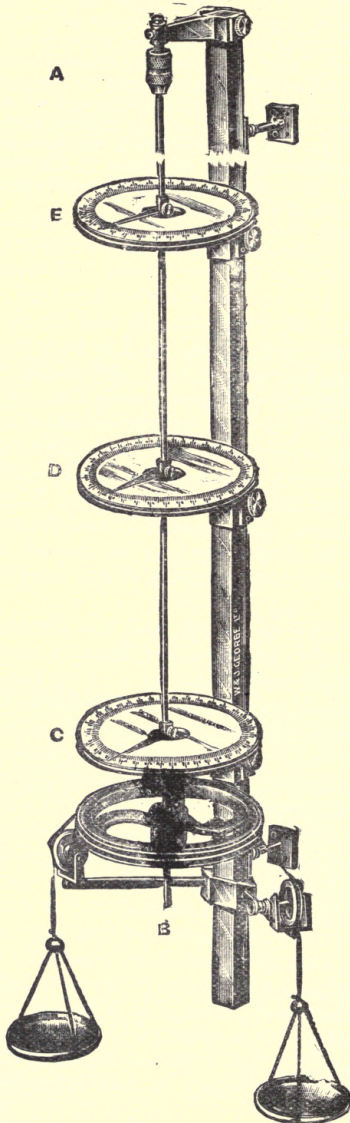
(i) If the length AE be 1 foot, and the distance between the dials E and D, D and C, be also 1 foot, then we should find that the angles of twist at E, D and C are as 1 : 2 : 3 respectively. *That is, the angle of twist is proportional to the length of wire twisted.*

(ii) By varying the twisting moment and noting the angle of twist which is produced at each variation, you can prove that *the angle of torsion or twist is proportional to the twisting moment.*

(iii) If you try *different sizes of wires of the same material and length*, and apply to each of them the same twisting moment, then you will find that *the amount of twist produced in them, will be inversely as the fourth power of the diameter of the wires.*

(iv) If you take wires of the *same diameter and length, but of different materials*, and apply the same twisting moment to them, then you will find that *the amount of twist will be inversely proportional to the modulus of rigidity of the material.*

*Note, the Modulus of Rigidity of a Material* in lbs. per square inch may be defined as the ratio of shearing stress to shearing strain. This constant or coefficient of shearing elasticity for each particular material is indicated in the following table by the letter C.



EXPERIMENTAL APPARATUS FOR MEASURING THE  
TORSION OF WIRES.

## STRENGTH, MODULI OF ELASTICITY, AND RIGIDITY OF VARIOUS MATERIALS.

Materials.	Breaking strength to resist tension, in tons per square inch.	Modulus of elasticity, E, in tons per square inch.	Modulus of rigidity, C, in tons per square inch.
Aluminium - bronze (90% copper and 10% aluminium) . . .	40	6500	2500
Brass wire . . . .	20 to 25	5000 to 6500	2000 to 2300
Cast-iron . . . .	5 to 15	4500 to 7000	1700 to 2700
Charcoal-iron (hard- drawn) . . . .	35 to 40	12,500 to 13,500	5000 to 5500
Charcoal-iron (annealed) . . . .	30		
Copper (cast) . . . .	8 to 12	5000 to 6000	1900 to 2300
" (rolled) . . . .	13 to 16	5500 to 7500	2100 to 2900
" wire (annealed) . . . .	18 to 20	—	—
" " (hard-drn.) . . . .	26 to 30	—	—
Delta Metal (forged) . . . .	22 to 24	6350	2340
German-silver wire . . . .	30	4800	1800
Gun-metal (90% cop- per and 10% tin) . . . .	12 to 17	5000	1900
Muntz metal (rolled or forged) . . . .	22	6350	2340
Phosphor bronze . . . .	16 to 18	6000 to 7000	2300 to 2700
" " wire (hard-drawn) . . . .	45 to 70		
Platinum . . . .	—	10,500	4000
Steel (ordinary) . . . .	70	13,000 to 14,000	5200 to 5700
" (annealed) . . . .	100		

**Strength of Solid Round Shafts.**—It is evident from the above, that a shaft subjected to a twisting moment must offer a sufficient *resistance* thereto, otherwise it would be twisted, or sheared, or ruptured through by the torque. It may be proved that in the case of solid round shafts their resistance to torsion is *directly proportional to the cubes of their diameters* when made of the same material and quality.\*

\* This is evident from the fact that the shaft must offer a *moment of resistance*, or *shearing moment*, equal to the *twisting moment* at the instant of rupture. Now, the area to be sheared, is the cross area of the shaft

$= \frac{\pi}{4} D^2$ , where D is the diameter of the shaft. The mean arm or leverage at

which this resistance acts is equal to *half the radius* of the shaft, for at the centre the arm is = 0, and at the circumference it is = *r*, the radius of the shaft.

The mean arm is therefore =  $\frac{r}{2}$ . And, if the shearing resistance per

square inch of cross section of the material be = *f*, the product of these three quantities will be the *total shearing moment*, and must equal the *twisting moment*—viz. = *P* × *R*, where *P* is the force applied at the end of the

Let  $D_1, D_2, D_3$  = Diameters of three shafts, 1", 2", and 3" diameter respectively.

$T_1, T_2, T_3$  = Torques which they will respectively resist when stressed to the same extent.

Then, . . .  $T_1 : T_2 : T_3 :: D_1^3 : D_2^3 : D_3^3$

Or, . . .  $T_1 : T_2 : T_3 :: 1^3 : 2^3 : 3^3$   
 $:: 1 : 8 : 27.$

In other words, the strengths of the three solid shafts will be as 1 : 8 : 27.

A good wrought-iron shaft of 1" diameter has been found to withstand a torque of 800 lbs.-ft., or 9600 lbs.-inches, which means that they will resist 800 lbs. force at 1 foot, or 12" leverage, or 400 lbs. at 2 feet, or 24", and so on.

Or, . . .  $P \times R' = 800$  lbs.-feet of torque

i.e., . . .  $P \times R'' = 9600$  lbs.-inch torque.

EXAMPLE II.—On the above basis, what force acting at the circumference of a pulley 20" diameter will break a wrought-iron shaft 2" diameter?

ANSWER.—By the above rule we have the proportion :

$$T_1 : T_2 :: D_1^3 : D_2^3$$

$$\text{But } T_1 = P_1 \times R_1'' = 800 \text{ lbs} \times 12''$$

$$\text{And } T_2 = P_2 \times R_2'' = P_2 \times 10''$$

$$\therefore P_1 R_1 : P_2 R_2 :: D_1^3 : D_2^3$$

$$\text{i.e., } P_2 R_2 \times D_1^3 = P_1 R_1 \times D_2^3$$

$$\text{Or, . . . } P_2 = \frac{P_1 R_1 \times D_2^3}{D_1^3 \times R_2} = \frac{800 \times 12'' \times 8}{1 \times 10''} = 7680 \text{ lbs.}$$

lever or circumference of the pulley, and R the length of the arm or radius of the wheel or pulley.

$$\text{Consequently, } P \times R = \sqrt[3]{\left( \frac{\pi}{4} D^3 \times \frac{D}{4} \right)} = \sqrt[3]{\frac{\pi}{16} D^3}$$

But S is a constant quantity for any particular material. Also,  $\pi$  and 16 are constants.

$$\therefore P \times R \text{ vary as } D^3.$$

At the instant of rupture the *strength* of the shaft just balances or is equal to the *twisting moment*  $P \times R$ .

$\therefore$  The strength of shaft *varies* as  $D^3$ .

This is the same as the general statement in the text above. Without some such algebraical explanation, students are sorely puzzled how the cube of the diameter crops up; or still more so when they see the following which appears in some text-books.

$$\left. \begin{array}{l} \text{The moment of resistance of} \\ \text{a round shaft to torsion} \end{array} \right\} = \frac{3 \cdot 1416}{16} \times \text{diameter}^3 \times \text{shearing stress.}$$

Such a statement is, however, quite evident after the above analysis. (We must leave the consideration of hollow shafts, tubes, &c., to our Advanced Course.)

POWER THAT STEEL SHAFTING WILL TRANSMIT AT VARIOUS SPEEDS.

From *The Practical Engineer*, September 2, 1892. By A. G. town, M.E.

DIAMETERS OF SHAFTS IN INCHES.														
Revs. per Minute.	1 1/2	2	2 1/2	3	3 1/2	4	5	6	7	8	9	10		
HORSE-POWERS THEY WILL TRANSMIT.														
50	3.3	5.3	8.0	10.9	15.6	27	43	64	125	216	343	512	729	1000
60	4.0	6.4	9.6	13.1	18.8	32	51	77	150	259	412	614	875	1200
70	4.7	7.5	11.2	15.2	21.9	38	60	89	175	302	480	717	1021	1400
80	5.4	8.5	12.8	17.4	25.0	43	69	102	200	346	549	819	1166	1600
90	6.0	9.6	14.4	19.6	28.1	49	77	115	225	389	617	922	1312	1800
100	6.7	10.7	16.0	21.8	31.2	54	86	128	250	432	686	1024	1458	2000
110	7.4	11.8	17.6	23.9	34.4	59	94	141	275	475	755	1126	1604	2200
120	8.1	12.9	19.2	26.1	37.5	65	103	154	300	518	823	1229	1750	2400
130	8.7	13.9	20.8	28.3	40.6	70	111	166	325	562	892	1331	1895	2600
140	9.4	15.0	22.4	30.5	43.8	76	120	179	350	605	960	1434	2041	2800
150	10.1	16.1	24.0	32.6	46.9	81	129	192	375	648	1029	1536	2187	3000
160	10.8	17.1	25.6	34.8	50.0	86	137	205	400	691	1097	1638	2333	3200
170	11.5	18.2	27.2	37.0	53.1	92	146	218	425	734	1166	1741	2479	3400
180	12.2	19.3	28.8	39.2	56.3	97	154	230	450	778	1235	1843	2624	3600
190	12.8	20.4	30.4	41.3	59.4	103	163	243	475	821	1303	1945	2770	3800
200	13.5	21.4	32.0	43.5	62.5	108	172	256	500	864	1372	2048	2916	4000
225	15.2	24.1	36.6	49.0	78.1	122	193	288	593	972	1543	2304	3280	4500
250	16.9	26.8	40.0	54.4	85.9	135	214	320	625	1080	1715	2560	3645	5000
275	18.6	29.5	44.0	59.8	93.7	149	236	352	688	1188	1886	2816	4009	5500
300	20.3	32.2	48.0	65.3	101.6	162	257	384	750	1296	2058	3072	4374	6000
325	21.9	34.8	52.0	70.7	109.4	176	279	416	813	1404	2229	3328	4739	6500
350	23.6	37.5	56.0	76.2	109.4	189	300	448	875	1512	2401	3584	5103	7000
375	25.3	40.2	60.0	81.6	117.2	203	322	480	938	1620	2572	3840	5468	7500
400	27.0	42.9	64.0	87.0	125.0	216	343	512	1000	1728	2744	4096	5832	8000
425	28.7	45.6	68.0	92.5	132.8	230	364	544	1063	1836	2915	4352	6197	8500
450	30.4	48.2	72.0	97.9	140.6	243	386	576	1125	1944	3087	4608	6562	9000
475	32.1	50.9	71.0	103.4	148.4	257	407	603	1188	2052	3258	4864	6926	9500
500	33.7	53.6	80.0	108.8	156.2	270	429	640	1250	2160	3430	5120	7290	10000

**For power of wrought-iron shafts take 70 per cent. of steel shafts of same size.**

**Strength of Hollow Round Shafts.**—The shafts of large land and marine engines are sometimes made hollow. The ratio between the torsional strength of a solid round shaft and a hollow one is as follows:—

Let  $D$  = outer diameter of either shaft.

$d$  = inner diameter of the hollow shaft.

Then, their strengths are as  $D^3 : \frac{D^4 - d^4}{D}$ ,

and their weights are as  $D^3 : D^3 - d^3$ .

**Relation between the Twisting Moment and Horse Power Transmitted by Shafting, as well as the Diameter Necessary to Transmit a given Horse Power.**

Let  $P$  = the force of the twisting couple, either constant or the mean value if variable, in pounds.

„  $R$  = the length of lever arm of the twisting couple in feet.

„  $N$  = revolutions of shaft per minute.

„ T.M. = mean twisting moment =  $P \times R$  lb.-feet.

„  $f_s$  = shearing stress per square inch on cross-section of shaft.

Then, T.M. = resisting moment =  $\frac{\pi}{16} d^3 f_s = 196 d^3 f_s$ .

$$\text{Or, } d = \sqrt[3]{\frac{\text{T.M.}}{196 f_s}} = \sqrt[3]{5 \cdot 1 \times \frac{\text{T.M.}}{f_s}}$$

But, work done per minute =  $P \times 2\pi RN = \text{T.M.} \times 2\pi N$  ft.-lbs.

Also, work done per minute = H.P.  $\times 33,000$  ft.-lbs.

$\therefore \text{T.M.} \times 2\pi N = \text{H.P.} \times 33,000$

$$\text{Or, } \text{T.M.} = \frac{\text{H.P.} \times 33,000}{2\pi N} \text{ lbs.-ft.} = \frac{12 \times 33,000 \times \text{H.P.}}{2\pi N} \text{ lbs.-inches.}$$

$$\text{Hence, } \text{T.M.} = 63,030 \frac{\text{H.P.}}{N} \text{ lbs.-inches} = \frac{63030 \times \text{H.P.}}{2240 \times N} = \frac{28 \text{ H.P.}}{N} \text{ ton-inches.}$$

$$\text{But, } \text{T.M.} = \frac{\pi d^3 f_s}{16}$$

$$\text{Hence, } \frac{\pi d^3 f_s}{16} = 63,030 \frac{\text{H.P.}}{N}, \text{ or } d = 68 \cdot 5 \sqrt[3]{\frac{\text{H.P.}}{N f_s}}$$

If we assume the safe values of the stress  $f_s$  to be as follows: cast-iron = 3600 lbs. per square inch; wrought-iron = 9000 lbs. per square inch; and steel = 13,500 lbs. per square inch, then the diameter in inches for a round shaft in terms of the horse-power to be transmitted is, for Cast-iron

$$d = 4 \cdot 5 \sqrt[3]{\frac{\text{H.P.}}{N}}; \text{ Wrought-iron, } d = 3 \cdot 3 \sqrt[3]{\frac{\text{H.P.}}{N}}; \text{ and Steel, } d = 2 \cdot 9 \sqrt[3]{\frac{\text{H.P.}}{N}}.$$

Of course, the twisting moment is here assumed to remain constant at its mean value. In practice the twisting moment varies in many cases, and to allow for this it is usual to take the maximum twisting moment from 1·3 to 1·5 times the mean twisting moment, thus the values of diameter  $d$  as found above are slightly increased.

It should also be borne in mind that shafts in practice are subjected to bending as well as twisting, owing to the loads due to the weights of

pulleys and the pulls of the belts. Hence, if the usual rule for the diameter of a wrought-iron shaft is  $d = 3 \cdot 3 \sqrt[3]{\frac{\text{H.P.}}{N}}$  when torsion only is considered, then it will be  $d = c \times 3 \cdot 3 \sqrt[3]{\frac{\text{H.P.}}{N}}$  when bending is taken into account. Some values of the coefficient  $c$  are given in the following table:

Kind of Shaft.	Value of $c$ .
Propeller shafts of steamships, and shafts with similar load	1.13
Line shafting in mills, etc.	1.3
Crank-shafts and shafting subjected to shocks, such as shafts in some machine tools, &c.	1.42

**EXAMPLE III.**—(a) Find the diameter of a solid steel propeller shaft to transmit 12,000 H.P. at 80 revolutions per minute. (b) If the shaft is to be hollow, find its external diameter, from strength considerations, when its internal diameter is two-thirds of its external diameter.

**Answer.**—Let  $D_1$  = diameter of the *solid* steel propeller shaft.  
 „  $D_2$  = outer diameter of the *hollow* steel propeller shaft.  
 „  $d$  = inner diameter of the hollow steel propeller shaft.  
 „  $N$  = number of revolutions per minute of shaft.

(a) Referring to the previous article on the relation between the diameter of shaft necessary to transmit a given horse-power, we deduced the following formula for a solid round *steel* shaft, when the stress  $f_s$  was assumed as 13,500 lbs. per square inch.

$$d = 2 \cdot 9 \sqrt[3]{\frac{\text{H.P.}}{N}}.$$

Substituting the numerical values given by the question, we get—

$$d = 2 \cdot 9 \sqrt[3]{\frac{\text{H.P.}}{N}} = 2 \cdot 9 \sqrt[3]{\frac{12,000}{80}} = 2 \cdot 9 \sqrt[3]{150} = 15 \cdot 4 \text{ inches.}$$

But, to allow for variations in stress, bending moments, and shocks to which the propeller shaft may be submitted, we find from the table that the value of the coefficient  $c$  is 1.13.

Hence  $D_1 = cd = 1 \cdot 13 \times 15 \cdot 4 = 17 \cdot 4$  inches for the solid round shaft.

(b) From the above, we see that  $d^3 = (2 \cdot 9)^3 \frac{\text{H.P.}}{N}$ . Also, from the relation of the strength of solid to hollow round shafts, we get,  $d^3 : \frac{D^4 - d^4}{D}.$

$$\therefore \frac{D^4 - d^4}{D} = (2 \cdot 9)^3 \frac{\text{H.P.}}{N} \text{ for hollow steel shafts subjected to torsion only—}$$

$$\frac{D^4 - d^4}{D} = 14 \cdot 39 \frac{\text{H.P.}}{N}.$$

Now, substituting the values given by the question, we get—

$$\frac{D_2^4 - (2/3 D_2)^4}{D_2} = \frac{14.39 \times 12000}{80} = 14.39 \times 150 = 2158.5.$$

$$\text{Or, } \frac{1}{3} D_2^3 = 2158.5.$$

$$\text{i.e. } D_2^3 = 3 \times 2158.5 = 6475.5.$$

$$\therefore D_2 = \sqrt[3]{6475.5} = 18.64 \text{ inches.}$$

If bending is taken into account, then

$$D_2 = 1.13 \times 18.64 = 21 \text{ inches for the hollow round shaft.}$$

EXAMPLE IV.—The screw shaft of a marine engine is 10 ins. diameter, and the revolutions 100 per minute. It is replaced by twin screw shafts rotating 500 times a minute. If the total horse-power developed in the two cases be the same, and the working stress is also the same in the twin screw shafts as in the single screw shaft, find the proper diameter of shafts in the second case and compare their weights.

(C. & G., 1905, Q., 23. 3.)

Answer.—

Let  $D_1$  = diameter of *single* screw shaft.

"  $D_2$  = diameter of *one* of the *twin* screw shafts.

"  $N_1$  = number of revs. per minute of *single* shaft.

"  $N_2$  = " " " " twin screw shafts.

H.P. = total horse-power to be developed in each case.

$f$  = working stress in lbs. per square inch.

$$\text{Now, } D_1 = \sqrt[3]{5.1 \times \frac{12 \times 33,000 \times \text{H.P.}}{2\pi N_1 f}}$$

$$\text{and, } D_2 = \sqrt[3]{5.1 \times \frac{12 \times 33,000 \times \text{H.P.}}{4\pi N_2 f}}$$

$$\therefore \frac{D_1^3}{D_2^3} = \frac{5.1 \times 12 \times 33,000 \times \text{H.P.}}{2\pi N_1 f} \div \frac{5.1 \times 12 \times 33,000 \times \text{H.P.}}{4\pi N_2 f}$$

$$\text{Or } \frac{D_1^3}{D_2^3} = \frac{2 N_2}{N_1} = \frac{2 \times 150}{100} = \frac{3}{1}$$

$$\therefore \frac{D_1}{D_2} = \sqrt[3]{\frac{3}{1}} = \frac{1.442}{1}$$

$$\text{i.e., } D_1 = 1.442 D_2. \therefore D_2 = \frac{10''}{1.442} = 7''$$

But, the weights of shafts are proportional to their cross sectional areas if their lengths are equal.

$$\text{Hence, } \frac{D_1^2}{2 D_2^2} = \frac{(1.442 D_2)^2}{2 D_2^2} = \frac{2.08}{2} = \frac{1.04}{1}.$$

Therefore, the weights of the shafts in the two cases are approximately the same, but the diameter of the single screw shaft is nearly  $1\frac{1}{2}$  times the diameter of one of the twin screw shafts.

## LECTURE XXIII.—QUESTIONS.

1. An open link chain is constructed of round wrought-iron rod,  $\frac{5}{8}$  inch in diameter; calculate what is the probable breaking load of the chain. Wrought-iron chains are liable to deterioration by constant use; what change do they undergo, and what precaution is taken to prevent their breaking? *Ans.* 10½ tons.

2. A steel punch  $\frac{3}{4}$  inch in diameter is employed to punch a hole in a plate  $\frac{3}{8}$  inch in thickness. What will be the least pressure necessary in order to drive the punch through the plate when the shearing strength of the material is 35 tons per square inch? *Ans.* 51.56 tons.

3. Define what is meant by "shearing stress and strain," "torque," "twisting moment." Show by an example that a shaft subjected to torque bears a shearing stress tending to sever it at right angles to its axis.

4. What is meant by the "twisting moment" of a shaft? If a wrought-iron shaft 1 inch in diameter breaks in torsion by a force of 800 lbs. at the end of a lever 1 foot long, what force at the end of a lever 2 feet long will break a shaft of the same material, but 2 inches in diameter? Find also the diameter of a wrought-iron shaft to resist a force of 2 tons at a distance of 18 inches from its centre. *Ans.* 3200 lbs. 2 inches full.

5. If a shaft, 2 inches in diameter, is found equal to the transmission of 4 horse-power, what amount of power can be transmitted by a shaft 4 inches in diameter, all other questions remaining the same? *Ans.* 32 horse-power.

6. If a revolving shaft, which is 2 inches diameter, is found sufficiently strong to transmit 4 horse-power, how much power may be transmitted by a shaft which is 3 inches in diameter, supposing all the other conditions to be the same, and that the iron of both shafts is subjected to the same stress? *Ans.* 13.5 H.P.

7. If 800 lbs. at the end of a 12-inch lever be a safe stress to apply to a wrought-iron bar one square inch in section, find the effort which a shaft 2 inches in diameter can transmit at the circumference of a pulley one foot in diameter, and making 300 revolutions per minute. Find also the horse-power transmitted. *Ans.* 8893 lbs.; 254 H.P.

8. If a wrought-iron shaft of 1 inch diameter is broken by the torsion of a load of 800 lbs. acting at the end of a 12-inch lever, find the weight which, when applied to the end of the same lever, would break a shaft of the same material, but 3 inches in diameter. State, in general terms, the reasoning by which you arrive at the result. *Ans.* 21,600 lbs.

9. Suppose that a shaft of 1 inch diameter may be safely subjected to a torque of 2000 lb.-inches; what torque will a 2½ inch shaft safely resist? Calculate the horse-power which may be safely transmitted by the latter shaft if its speed is 150 revolutions per minute. (B. of E., 1902.)

*Ans.* 22,780 lb. inches; 54 horse-power.

10. A wire of Siemens' steel 0.1 inch diameter is to be twisted till it breaks. Sketch the arrangement and show how the angle of twist and the twisting moment are measured, how the results may be plotted on squared paper, and the sort of results that may be expected. In what way may a wire of twice this diameter be expected to behave?

(B. of E. 1901.)

11. If a shaft 4 inches in diameter will safely withstand a torque of 120,000 lb.-inches, what torque would a 9-inch shaft take? What H.P. would the former transmit at 200 revolutions per minute, and what would the latter transmit at 50 revolutions per minute? (B. of E., 1903).

Ans. Torque = 1,367,000 lb.-inches; H.P.<sub>1</sub> = 384.7 and H.P.<sub>2</sub> = 1110.

12. Compare the strengths and weights of a solid wrought-iron shaft and a hollow steel shaft of the same external diameter—assuming the internal diameter of the hollow shaft half the external, the working stress of steel  $1\frac{1}{2}$  times that of iron, and the densities of wrought-iron and steel to be the same. (C. & G., 1903, O., Sec. B.).

$$\begin{aligned} \text{Ans.}— \quad & \frac{\text{Strength of solid W.I. shaft}}{\text{Strength of hollow steel shaft}} = \frac{32}{45} = \frac{1}{1.4} \\ & \frac{\text{Weight of solid W.I. shaft}}{\text{Weight of hollow steel shaft}} = \frac{4}{3} = \frac{1.3}{1} \end{aligned}$$

13. The propeller shaft of a vessel, whose engines develop 1000 horsepower at 60 revolutions per minute, is 8" dia. Assuming the shaft subjected to pure torsion, and that the maximum twisting moment on the shaft is  $1\frac{1}{2}$  times the mean, estimate the maximum shear stress induced in the shaft. (C. & G., 1904, O., Sec. B.).

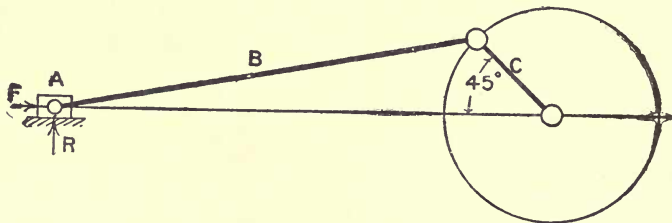
Ans. Max. shear stress = 13,050 lbs. per sq. inch.

14. The figure shows the skeleton mechanism of a direct-acting steam-engine. *A* is the cross-head, *B* is the connecting-rod, and *C* is the crank.

The connecting rod is 4 cranks long, and in the position shown in the figure, the crank has turned through an angle of  $45^\circ$  from the dead centre in a clock-wise direction.

A force *F*, due to the steam pressure on the piston of 12,000 pounds, acts upon the cross-head. Find graphically, or in any other way, the thrust in the connecting rod, and the magnitude of the force *R* between the cross-head and slide bar. All friction to be neglected.

(B. of E. 1905.)



Ans. *F* = 12,200 lbs.; *B* = 12,430 lbs.; and *R* = 2220 lbs.

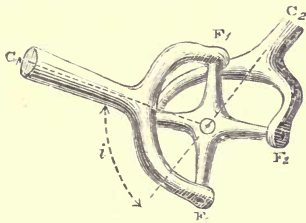
15. Describe, with a sketch of the apparatus, how you would experimentally determine the law connecting the twisting moment and the angle of twist for a piece of steel wire. (B. of E. 1905.)

## LECTURE XXIV.

CONTENTS.—Hooke's Coupling or Universal Joint—Double Hooke's Joint—Sun and Planet Wheels—Cams—Heart Wheel or Heart-shaped Cam—Cam for Intermittent Motion—Quick Return Cam—Example—Pawl and Ratchet Wheel—Reversible Pawl—Masked Ratchet—Silent Feed—Watt's Parallel Motion—Parallel Motion—Questions.

IN this and the following Lecture we shall examine a few of the many devices for transmitting circular motion and for converting it into rectilinear motion, or *vice versa*, together with other miscellaneous mechanisms.

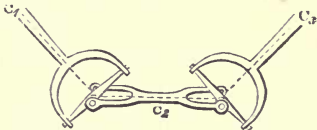
**Hooke's Coupling or Universal Joint.**—This is a contrivance sometimes used for connecting two intersecting shafts. Each of the shafts ends in a fork,  $F_1$ ,  $F_2$ , which embraces two arms of the crosspiece, O. The four arms of this cross are of equal length. As  $C_1$  rotates,  $F_1$  and  $F_2$  describe circles in planes perpendicular to their respective axes. Since these planes are inclined to each other the angular velocity of  $C_2$  at any instant is different from that of  $C_1$ , but the mean angular velocities are



HOOKE'S JOINT.

equal to one another, because at one instant  $C_2$  goes faster than  $C_1$ , and at another slower. This joint will not work when the two shafts are inclined at  $90^\circ$ , or any smaller angle, to each other.

**Double Hooke's Joint.**—The variable velocity ratio obtained with a Hooke's joint may be obviated by the use of two joints instead of one. The forks are connected by an intermediate link,  $C_2$ , which must be carried on corresponding arms of the two crosses, as shown in the next figure. If the intermediate shaft be equally inclined to the other two shafts, the irregularities caused in the motion by its transmission through the first coupling are exactly neutralised by the equal and opposite ones caused by the second joint. The first and third

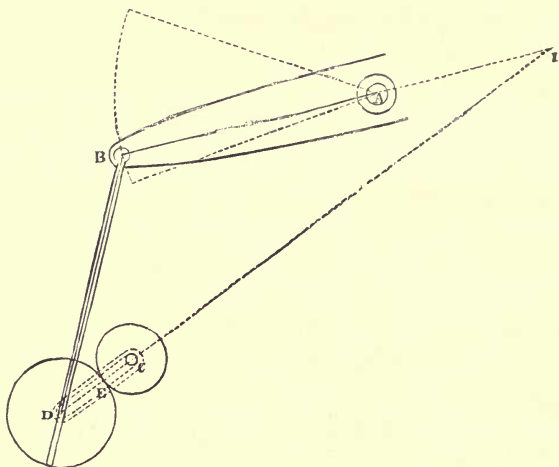


DOUBLE HOOKE'S JOINT.

shafts, therefore, revolve with the same velocity at every instant. The double joint works equally well whether the two extreme axes are inclined as shown in the figure, or are parallel to each other but not in line.

Both the single and double Hooke's joint are, as a rule, used only for light work, such as for astronomical instruments.

**Sun and Planet Wheels.**—This device was invented by Watt to convert the oscillatory motion of the beam in his engines into the circular motion of the flywheel. As will be seen from the



SUN AND PLANET WHEELS.

first figure, it consists of a wheel D, rigidly fixed to the connecting rod D B, and kept in gear with another wheel C, by the link D E C. The wheel C, is keyed to the flywheel shaft. As the beam oscillates up and down, the connecting-rod pulls D up one side of C, and pushes it down the other. It thereby causes C to rotate, and with it the shaft and flywheel.\*

**Cams.**—Cams are usually of the form of discs or cylinders. They rotate about an axis, and give a reciprocating motion to

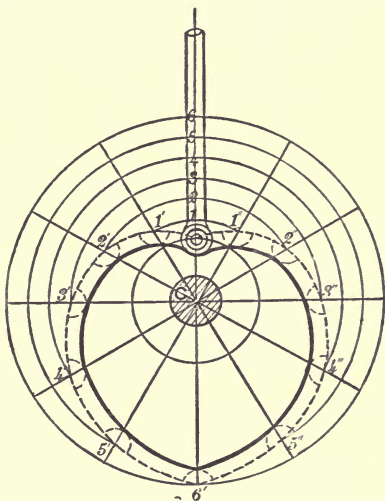
\* See Vol. I., Lecture XIX., of the author's text-book on "Applied Mechanics" for a description of epicyclic trains and the application of the formula to this case. Watt first applied this motion to his "Double Acting Steam Engine" in 1784. See Lecture XVIII. of the author's elementary manual on "Steam and the Steam Engine."

some point in a rod by means of the form of their periphery or surface, or by grooves in their surface.

The cam generally revolves uniformly round its axis, whilst the reciprocating motion may be of any nature, depending on the shape of the cam, and may be in a plane inclined at any angle to the axis of rotation. In the following examples, uniformity of rotation is assumed in the case of the cam, and the motion of the reciprocating piece takes place in a plane perpendicular to the axis.

**Heart Wheel or Heart-shaped Cam.**—Suppose that it is required to give a uniform reciprocating motion to a bar moving vertically between guides, and in a line passing through C, the centre of motion of the cam plate.

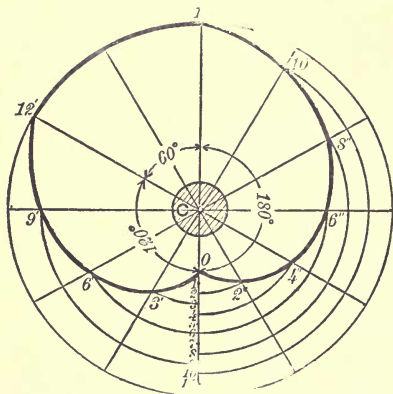
Let the sliding bar be at its lowest position, as shown, and when in its highest position let its extremity be at the point 6. The distance thus moved is called the *travel* and will be passed over during one-half revolution of the cam. The required curved outline may be obtained in the following manner:—With centre C, describe circles passing through the extreme positions of the end of the rod. Divide the travel into, say, six equal parts at the points 1, 2, 3, &c. Divide the semi-circumference into the same number of equal parts by radial lines C 1', C 2', &c. Then with centre C, draw the concentric arcs 1, 1'; 2, 2'; &c., intersecting these radii in the points 1', 2', 3', &c. The dotted line drawn through these points will represent the required curve.



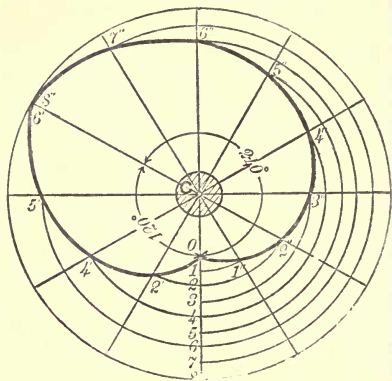
HEART-SHAPED CAM.

If the end of the sliding bar rests on this curve it is clear, that for equal angles turned through by the cam, the bar will move outwards through equal distances, and consequently, will have uniform linear motion imparted to it. The return motion will evidently be obtained by the similar and equal curve 1'', 2'', 3'', &c., on the opposite side of the cam.

A cam so formed would impart the required motion to a *point*. If the end of the sliding bar be provided with a roller in order to diminish the friction, then the shape of the cam must be altered so that the centre of the roller shall move over the outline of the cam as traced above. To accomplish this, we must draw a curve inside the original one by describing small arcs with centres on the original curve as at 1', 2', 3', &c., with a radius equal to that of the roller, and then by drawing a smooth curve touching these arcs, as shown by the heavy line in the figure.



CAM GIVING AN INTERVAL OF REST.



CAM GIVING A QUICK RETURN.

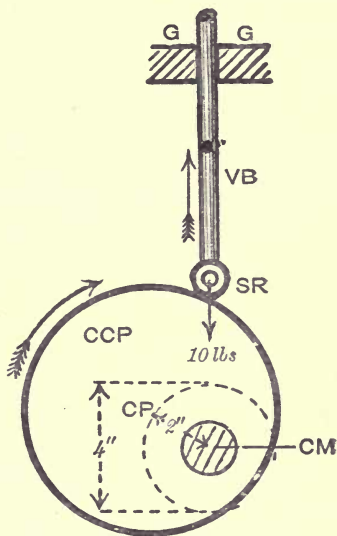
**Cam for Intermittent Motion.**—Sometimes the motion imparted by a cam is intermittent. For instance, a common form of lever punching machine is fitted with a cam which gives the punch an upward movement, then a period of rest, and finally a downward movement during each revolution. As an example of this, let us set out a cam to impart vertical motion to a bar, so that the latter shall be raised uniformly during the first half revolution, remain at rest during the next one-sixth, and descend uniformly during the remainder of the revolution.

As before, suppose the reciprocation to be in a line passing through C, the centre of motion of the cam plate. Then, with centre C, draw circles passing through the extreme positions of the end of the bar. Divide the circumference into three parts corresponding to the periods of one-half, one-sixth, and one-third revolution, by drawing radial lines making angles of 180°, 60°, and 120°. Since the motion is to be uniform, divide the travel

into a convenient number of equal parts, say twelve; and the circumference into the same number of equal parts by radial lines. Draw the concentric arcs 2, 2"; 4, 4"; &c., and 3, 3'; 6, 6'; &c., as shown. The curves through the points so determined will give the required motions. The interval of rest will evidently be given by the circular portion from 12" to 12'. The complete outline is represented by the heavy line in the diagram.

**Quick Return Cam.**—The student will readily understand from the right-hand figure, that if two-thirds of a revolution be occupied in raising the motion bar and the remainder in lowering the same, the return stroke will be performed in half the time of forward stroke. The curves of this cam are found in the same way as in the previous examples.

**EXAMPLE.**—A vertical bar, moving in guides, is driven by a circular cam plate having a centre of motion in the centre line of the bar. The distance from the centre of motion to the centre of the plate is 2 inches, and the bar exerts a pressure of 10 lbs. when rising, but falls by its own weight. Find the work done in 100 revolutions of the plate.



CIRCULAR CAM PLATE.

## INDEX TO PARTS.

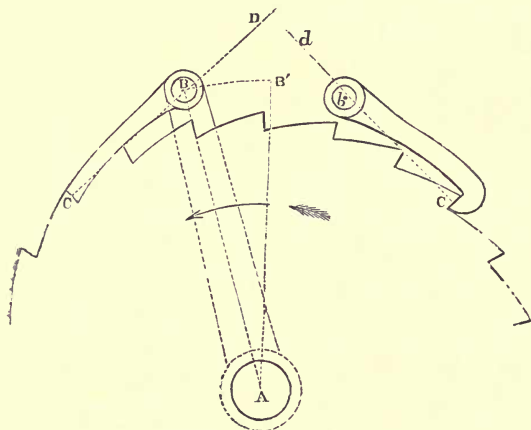
G represents Guides.		
VB	"	Vertical bar.
SR	"	Sliding roller.
CCP	"	Circular cam plate.
CP	"	Centre of plate.
CM	"	Centre of motion.

**ANSWER.**—Since the distance between the roller S R, and the centre of the plate C P, remains constant as the plate revolves, it is evident that the bar will move as if it were actuated by a crank of length equal to the distance between C M and C P, and a connecting-rod of length equal to the radius of the plate. Hence, the stroke of the bar will be 4 inches, or  $\frac{1}{3}$  foot—i.e., twice the length of the equivalent crank. Neglecting friction, the work done in raising the bar by one revolution of the plate, will be:—

$$\text{Pressure} \times \text{distance moved} = 10 \times \frac{1}{3} \text{ ft.-lbs.}$$

$$\therefore \text{Work done in 100 revolutions} = 100 \times 10 \times \frac{1}{3} = 333\cdot\dot{3} \text{ ft.-lbs.}$$

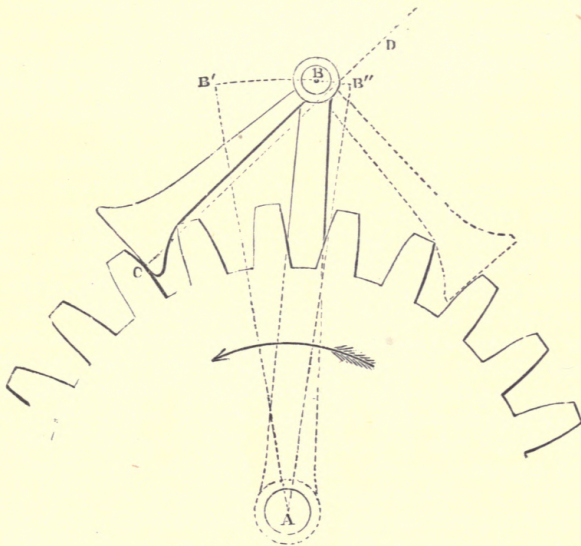
**Pawl and Ratchet Wheel.**—A toothed wheel which is acted upon by a vibrating piece, termed a *click* or *pawl*, is called a *ratchet wheel*. Ratchet wheels are made in many different forms, and are



PAWL AND RATCHET.

used for a variety of purposes. For instance, clocks and watches are usually provided with ratchet wheels to allow the spring or weight to be wound up, without disturbing the rest of the works, and they are used to drive the feeding arrangements of many machines. When, as in the latter case, the click or pawl drives the ratchet wheel, it is carried on a vibrating arm. In the first figure, A B is the vibrating bar which drives the ratchet wheel, by means of the click B C, and teeth C C, when moving in the direction shown by the arrow. When A B moves back to A B', the click slides over the top of the next tooth and drops behind it. It is then ready to drive the wheel through the space of another tooth when A B again moves forward. While the pawl is moving back from B to B', the wheel is prevented from moving with it by another *pawl* or *detent*, b C. In this case, the vibrating bar is on the same axis as the ratchet wheel; but this is not always so. The reactions between the teeth and the pawl keep them in

contact with each other. The resultant pressure of the teeth on the pawl must therefore be such, that its moment tends to turn the pawl towards A, the centre of the ratchet wheel. This condition evidently is satisfied if C D, the direction of the resultant pressure at C, passes between A and the axis B, about which the pawl turns. Similarly, the moment of the resultant pressure on the detent must tend to turn it towards A, but its direction,  $d C$  (not C  $d$ ), must lie outside A  $b$ , because this detent ends in a hook.



REVELSIBLE CLICK.

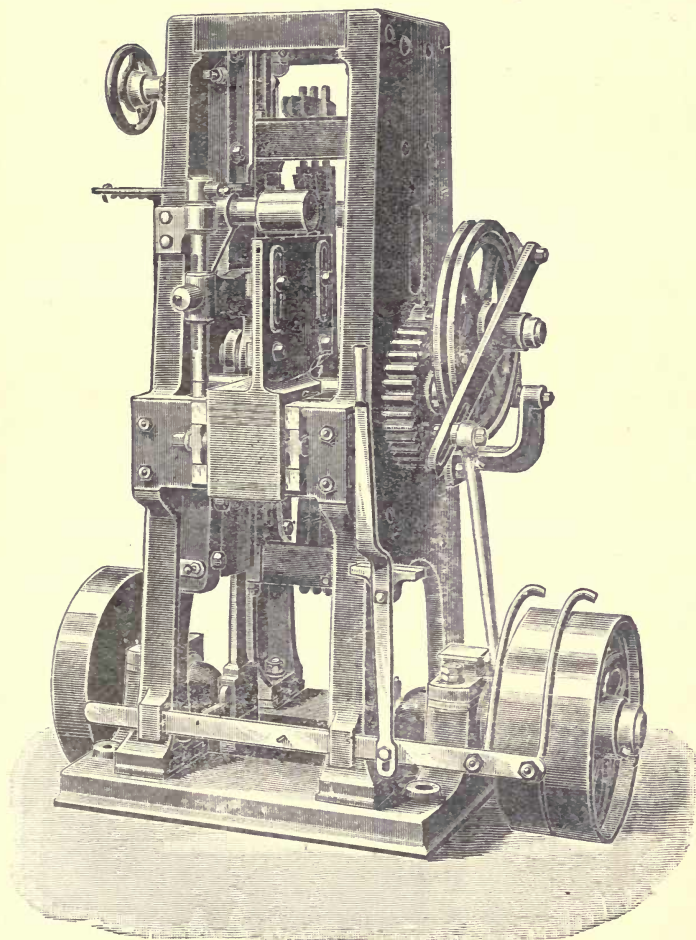
Both pawls might have been like B C, which acts by pushing, or both hooks, which act by pulling, like  $b C$ . The pawls are pressed against the ratchet by their own weight, or by springs, according to circumstances. When a ratchet wheel is used only to prevent the recoil of the axis on which it is fixed, the vibrating arm is, of course, not required, and only the detent is used.

**Reversible Pawl.**—The above figure shows a form of click used in the feed motion of shaping and other machines. The ratchet wheel is here an ordinary toothed wheel, and the click B C is so shaped as to be able to drive it either way. When the click is in

the position shown in full lines, it drives the ratchet wheel in the direction of the arrow. When the wheel is required to rotate the other way, the click is lifted over to the dotted position; and, if it be desired to stop the feed motion without stopping the machine, the click is put in an upright position. A portion of the pin at B, which turns with the click, is triangular in section. A spring presses on this part and so keeps the click in any one of its three positions. The ratchet wheel is keyed to A, the axis of the screw which moves the slide carrying the cutter, and the friction between this screw and its nut is sufficient, without any detent, to prevent the ratchet from moving back. The vibrating arm A B, which carries the click is driven by a small eccentric or crank. The pawl may, of course, be made to move the ratchet more than one tooth at a time by adjusting the angle through which A B vibrates.

**Masked Ratchet.**—In numbering machines it is often necessary to print the same number twice, as in cheques and their counter-foils. The ratchet which shifts the type wheels must therefore be moved at every alternate back-stroke of the printing machine. This may be accomplished by putting a second ratchet, running free on the shaft, alongside the driving one and making the pawl broad enough to move both. The second ratchet has the same number of teeth as the other, but its teeth are made alternately deep and shallow. It is also a little larger than the driving ratchet, so that the pawl passes over the top of the teeth of the latter, without moving it, when in a shallow tooth. Next stroke the pawl drops into a deep tooth. This allows it to catch the teeth of the main ratchet and so shift the type wheel. This arrangement is called a *masked ratchet*.

**Silent Feed.**—A ratchet wheel is always more or less noisy in action, and the wear caused by the sudden drop of the pawl is considerable. To avoid this, a friction catch is sometimes substituted for the pawl and a grooved wheel for the toothed one. The pawl and ratchet then becomes a *silent feed*. The action of this arrangement will be easily understood by a reference to the figures. E C is an eccentric cam tapered at its edge to fit the groove in the grooved wheel G W. When E C moves in the direction of the arrow the friction causes it to turn about its axis, and, since the axis is not concentric with the circular part of its rim, it gets wedged in the groove. Hence, for the rest of the stroke, the lever carries G W round with it. At the beginning of the return stroke, E C turns in the opposite direction, and so gets released from the groove. A detent E D, precisely similar to E C, but carried on a fixed arm, prevents the wheel from moving back-



VERTICAL SAWING MACHINE, BY JOHN M'DOWAL & SONS OF  
JOHNSTONE, SHOWING SILENT FEED.



by a set of links. This system of links has been called *Watt's Parallel Motion*. The first figure will serve to show the principle on which an approximate rectilinear motion is obtained. Part of the beam of the engine is shown in three different positions,  $C T_1$ ,  $C T_2$ , and  $C T_3$ . The point,  $T$ , in it is connected by the link,  $T t$ , to the end of a lever or radius rod,  $c t$ , pivoted at  $c$ . In their mid positions,  $C T_2$ ,  $c t_2$ , these two levers are usually parallel to each other, and perpendicular to the line  $P_1 P_2 P_3$ . The point,  $T$ , describes an arc of a circle round  $C$ , and  $t$  round  $c$ . As these arcs curve in opposite directions, we should expect some intermediate point on the link  $T t$ , to curve in neither direction, but to describe an approximate straight line. This point  $P$  may be found by making  $\frac{P t}{P T} = \frac{C T}{c t}$ . The actual path of  $P$  is like the figure 8, and the parts which cross are very nearly exact straight lines for a short distance on either side of the crossing.

Prof. Rankine gives the following construction for the lengths of the links in his *Machinery and Millwork*.—Let  $A$  be the centre of the beam,  $G D$  the centre line of the piston-rod's motion, and  $B$  the mid position of its end. Draw  $A D$  perpendicular to  $G D$ . Make  $D E$  equal to one-fourth of the stroke, and join  $A E$ . Draw  $E F$  perpendicular to  $A E$ , and meeting  $A D$  in  $F$ .  $A F$  is the length of the beam. If  $G$  be the point where the radius rod cuts  $G D$ , draw  $G K$  at right angles to  $G D$ , and make  $D H$  equal to  $G B$ . Join  $A$  to  $H$ , and  $F$  to  $B$ , and produce  $A H$  and  $F B$  to meet  $G K$  in  $K$  and  $L$ . Then,  $F L$  is the connecting link,  $K L$  is the radius rod, and  $B$  is the point on the link  $F L$ , to which the piston-rod must be attached.

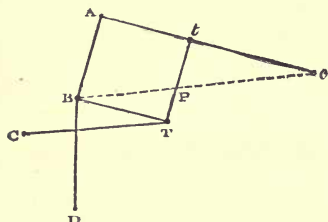
**Parallel Motion.**—In the accompanying figure  $A B T t$  is a parallelogram, and  $c$  is a point in  $A t$  produced. In the meantime we will leave the links  $C T$  and  $B D$  out of account and consider the parallelogram only. Join  $B c$  and we have two similar triangles,  $B A c$  and  $P t c$ .

$$\therefore \frac{P t}{B A} = \frac{t c}{A c} \quad \text{Or, } P t = B A \frac{t c}{A c} = \text{a constant.}$$

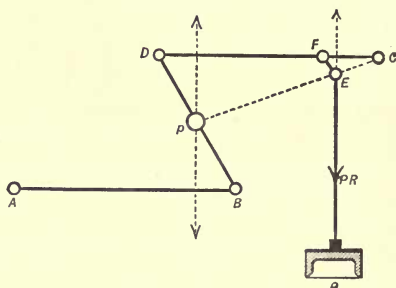
That is, in every position of the parallelogram, the point  $P$  remains in one fixed position in the link  $T t$ . Moreover the ratio  $\frac{B c}{P c}$  is constant, and, therefore, whatever path  $P$  traces out,  $B$  will trace out a similar one. This is the principle of the *pantograph*, which is used for enlarging or reducing drawings. Now, we have

just seen how we may make  $P$  move in an approximate straight line by the link  $C T$ .  $B$  will, therefore, also move in an approximate straight line. We might have guided  $B$  instead of  $P$  with a radius rod, but this would have necessitated longer and heavier links and would have occupied more space.

In applying this motion to his engine, Watt made  $A t c$  the beam, and attached the piston-rod to  $B$  and the air pump-rod to



PARALLEL MOTION.



PARALLEL MOTION FOR RICHARD'S INDICATOR.

P. The lengths  $A c$ ,  $t c$  were, therefore, proportional to the strokes of the piston and pump bucket respectively. Sometimes a third link was added so as to get a second parallelogram and a second point moving parallel to  $P$ , and this was used to drive the feed-pump.

The right-hand figure shows the parallel motion of Richard's steam engine indicator.\* The student will at once see that it is a modification of Watt's parallel motion. In this case the piston-rod  $P R$  is guided by a collar so as to move vertically, and is attached by the link  $E F$  to the bar  $C D$ , between  $D$  and the centre  $C$ . The motion of  $p$ , to which the pencil is attached is, therefore, a *magnified* copy of the piston's motion.

\* See Lecture XVI. of the author's "Elementary Manual of Steam and the Steam Engine" for a description of this indicator.

## LECTURE XXIV.—QUESTIONS.

1. Describe Hooke's joint for connecting two axes whose directions meet in a point.

2. Sketch and describe the double Hooke's joint, and explain why it is used in certain cases in preference to the single joint.

3. Explain the manner in which Watt used the so-called Sun and Planet Wheels as a substitute for a crank and connecting-rod, and account for the result which he obtained.

4. Sketch a cam for giving a bar a uniform reciprocating motion, and explain how you find the form of its periphery.

5. Set out a form of cam which, when acting on a bar by uniform rotation, will cause the backward and forward motion of the bar to have an interval of rest between each.

6. Describe, by the aid of the necessary sketches, how the circular motion of the driving pulley is converted into the reciprocating motion of the punch in an ordinary machine for punching holes in metal plates. Calculate the approximate maximum pressure in pounds at the end of a punch in cutting a hole 1 inch in diameter through a steel plate  $\frac{3}{8}$  inch thick, the resistance of the plate to shearing being taken as 50,000 lbs. per square inch of section.

*Ans.* 98,175 lbs.

7. Describe the nature of a cam, and give any examples you know of for which it is used.

8. What is a cam? Show, with a sketch, how to obtain by means of a cam a motion in a direction parallel to the axis of rotation.

9. What is a cam? For what purposes in mechanism are cams generally used? Sketch and describe the construction and actual form of a cam in use in any machine with which you are acquainted. Sketch a cam which would give a slow forward and quick return motion to a reciprocating piece, with an interval of rest between the two motions.

10. How is a cam in the form of a heart set out, so that when the cam rotates uniformly it may cause a sliding piece to move to and fro with a uniform velocity?

11. Determine a form of cam which, by rotating uniformly, will communicate a reciprocating movement to a sliding bar, but with an interval of rest at the beginning and end of each stroke.

12. Sketch a pawl and ratchet wheel as used for preventing the recoil of the gear.

13. Describe a ratchet wheel, and the manner in which it is held and driven. Show its application in a ratchet brace where it is combined with a lever and screw. Sketch the contrivance, and point out the manner in which the drill is advanced.

14. Sketch a ratchet feed motion, such as is suitable for a planing machine, and explain the manner in which the amount of feed is regulated.

15. What is a ratchet wheel, and how is it driven? In what way can it be arranged that a ratchet wheel shall advance half the space of a tooth at each stroke of the driver?

16. By what contrivance may a ratchet wheel with 60 teeth be made to act as if it had 120 teeth?

17. Sketch and describe some form of pawl which will drive a ratchet wheel during both the forward and backward strokes.

18. It is sometimes useful to advance a ratchet wheel at every *alternate* forward stroke of the driver, instead of at every stroke, as is commonly the case; describe and sketch a mechanical contrivance which will give such a movement.

19. Describe, with the necessary sketches, some form of silent-feed arrangement commonly used instead of a ratchet wheel, for advancing the timber in sawing machines. Explain the principle of the friction grip upon which such a contrivance depends.

20. A pinion with 20 teeth works in a straight rack, the distance from centre to centre of the teeth in the pinion being  $\frac{1}{2}$  inch. The pinion is driven by a ratchet wheel with 40 teeth fixed on the pinion shaft; find the advance of the rack in inches for each tooth moved through by the ratchet wheel.

*Ans.* .25 inch.

21. Sketch and describe a vertical sawmill, showing how the silent feed is applied.

22. Explain the principle of Watt's approximate straight-line motion, commonly called a "parallel motion" By what combination of linkwork is an exact straight-line motion obtained?

23. Sketch and describe the action of a ratchet wheel. For what purposes are such wheels used in machine tools? Show how a ratchet wheel of 30 teeth can be used to give a feed through  $\frac{1}{80}$ th of a revolution at each stroke of the arm of the pawl. (S. E. B. 1900.)

24. Design a cam to lift vertically a sliding piece at a uniform speed through 2 ins., the return motion being also at a uniform rate, but at half the speed; having given that the line of stroke, produced, of the slider passes through the axis of the cam shaft, that the nearest approach of the centre of the roller to the cam centre is 2 ins., and that the diameter of the roller is  $\frac{1}{2}$  in.

(C. & G., 1904, O., Sec. A.)

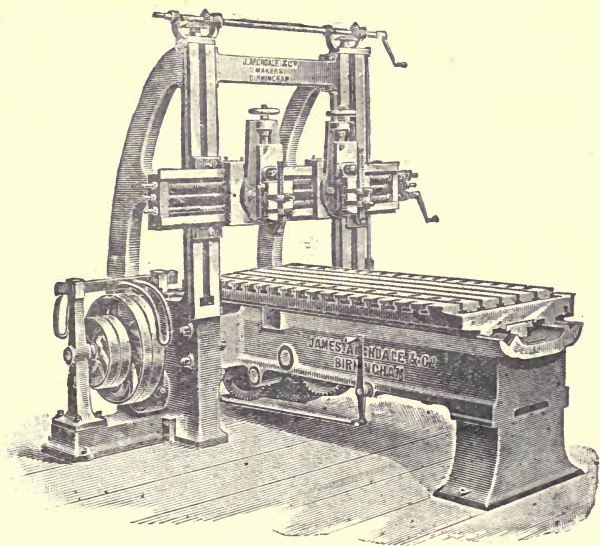
25. Describe, with the aid of a sketch, Watt's parallel motion, and show how to find the position of the point in the coupler which most nearly describes a straight line. Sketch the path which the tracing point describes for all possible positions.

(C. & G., 1905, O., Sec. A.)

## LECTURE XXV.

**CONTENTS.**—Reversing Motions—Planing Machine—Reversing by Friction Cones and Bevel Wheels—Whitworth's Reversing Gear—Quick Return Reversing Motion—Whitworth's Quick Return Motion—Whitworth's Slotting Machine—Common Quick Return—Horizontal Shaping Machine—Quick Return with Elliptic Wheels—Vertical Slotting Machine—Questions.

**Reversing Motions.—Planing Machine.**— In Lecture XI. we illustrated and described a belt reversing motion for uniform forward and return speeds and also for a quick return as applied



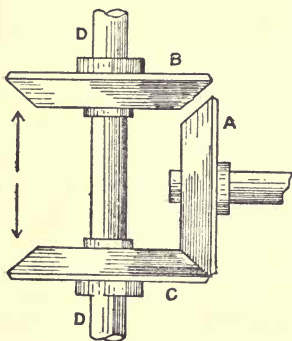
HORIZONTAL PLANING MACHINE WITH QUICK RETURN BELT  
GEARING BY MESSRS. J. ARCHDALE & CO., BIRMINGHAM.

to planing machines. The above figure will serve to show the manner in which the quick return and slower forward motions

are applied to a modern planing machine. Here, the smaller fixed and loose pulleys are situated to the extreme left, whilst the larger ones are placed alongside of them. The pulleys are connected by underneath toothed gearing with the rack of the moving table upon which the material to be planed is bolted. The under side of this table has two straight, truly planed and scraped V-shaped surfaces, which accurately fit corresponding V grooves on the upper surface of the strong heavy bed plate. Two vertical standards, with planed and scraped surfaces on their front faces, serve to guide the cross-piece which carries two tool-holders. An up and down motion is given to this cross-piece by means of the uppermost handle, spindle, two pairs of bevel pinions and vertical screws actuating nuts connected to the back of it. A horizontal motion is given to the tool-holders by the lower right-hand handle and horizontal screws. The tool-holders themselves are adjustable up and down by handles and screws as shown. They may also be so set as to cut at any angle, or a horizontal tool-holder can be fixed upon one or other of the upright standards to plane vertical surfaces. All the above-mentioned motions may be actuated from either side of the machine.

We shall now illustrate and describe several other forms which are frequently used in connection with machinery of different kinds.

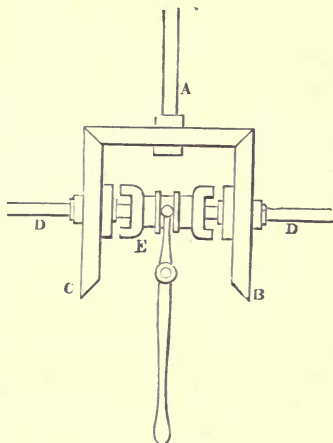
**Reversing by Friction Cones and Bevel Wheels.**—In the first of the two following figures, the two cones B and C are fixed to the shaft D D, which can be moved up or down so that the cones B or C may be alternately brought into contact with the other cone A, which is fixed to its shaft. Suppose A to be the driver; then, when it is in contact with B, the shaft D D will be turned in one direction, and when in contact with C it will be rotated in the opposite way. The spindle or shaft D D may, however, be driven by a belt or toothed gearing, and consequently when A is in contact with B, it will be turned in one direction and when in contact with C it will be rotated in the opposite way. This device is frequently used in connection with governors for engines, in order to lengthen or



FRICITION CONE REVERSING  
GEAR.

shorten the rod to the throttle valve or cut-off gear and thus enable the governor balls to regain their normal position, without altering the quantity of steam being admitted.

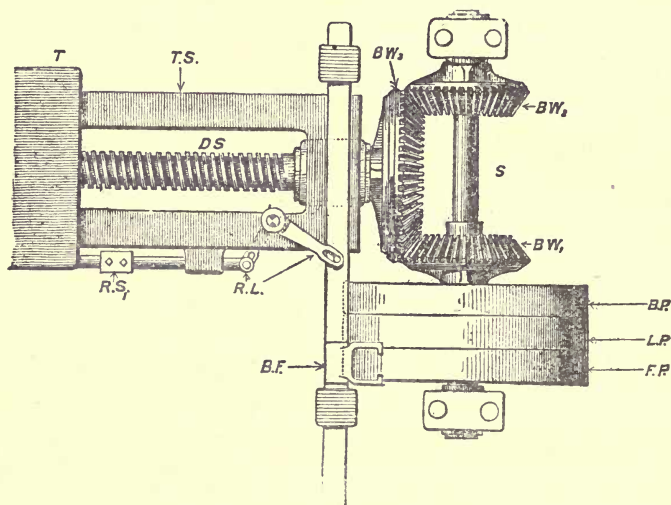
If we require to transmit more force than the friction between the cones will effect without slipping, then we must substitute toothed bevel wheels for plain cones. In such a case, if the speed and stress were considerable the sudden engagement of the wheels on one side or the other would be apt to damage the teeth; hence, it is usual with steam cranes, winches, windlasses, screwing gear, &c., to have B and C free to rotate upon their shaft D D, and always in gear with A, as shown by the second figure. The reversal in this case is effected by means of the clutch E, which can be slid along a feather on the shaft by the lever, so as to engage B or C and thus make it turn with the shaft. It will be easily seen that the direction in which A will rotate depends upon whether B or C is locked to the shaft D D.



BEVEL WHEEL AND CLUTCH  
REVERSING GEAR.

**Whitworth's Reversing Gear.**—Another modification of this gear is that made by Sir Joseph Whitworth & Co. for planing machines. In this case, the reversal is effected by shifting the driving belt by the fork B F, from the forward pulley F P to the backward one B P. The latter is cast in one piece along with or keyed to the boss of the bevel wheel B W<sub>1</sub>, which runs loose on the shaft S; whilst, the former and also B W<sub>2</sub>, are rigidly connected to this shaft. A loose pulley L P is placed between the forward and backward ones in order to facilitate the shifting of the belt from the one to the other and to carry the belt when the machine is not at work. The table T upon which is placed the casting or other material to be operated upon, has a strong nut fixed upon its under side, and is moved along the bed or slide T S by the driving screw D S. This screw is keyed to the bevel wheel B W<sub>3</sub>, and is consequently driven in a forward or backward direction according as the belt is on F P or B P. In order to save the time that would otherwise be wasted if the cutting tool

was so fixed as to cut in one direction only, Sir Joseph Whitworth designed a cylindrical revolving tool-holder, which automatically turns the tool half round at the end of each stroke of the table. The reversal of the motion of the table is automatically effected by its pushing the reversing stops  $RS_1$  (and  $RS_2$ , not shown in the figure) as it nears the end of a stroke. These stops are fixed in

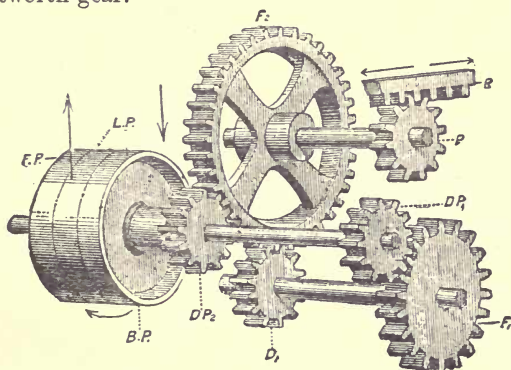


WHITWORTH REVERSING GEAR.

such positions upon a rod connected to the reversing levers  $RL$ , as to shift the belt fork at the proper time and thus give the required length of stroke.

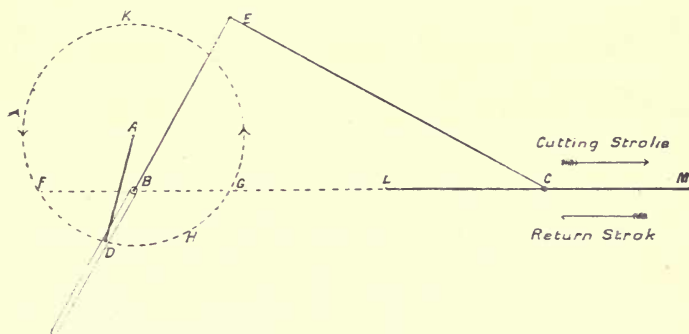
**Quick Return Reversing Motion.**—Another form of reversing motion, used for planing machines which only cut one way, is made up of a train of spur wheels and a rack  $R$ . As in the previous case, there are three belt pulleys, each of the outer ones being connected to a separate pinion. One of these pinions,  $DP_2$ , drives the rack on the planing table through a spur wheel  $F_2$  and pinion  $P$  fixed on an intermediate shaft; the other,  $DP_1$ , which is connected with  $FP$ , transmits its motion through another pair of wheels  $F_1$  and  $D_1$ . This will cause the rack and table to move in the opposite direction, and as these wheels are made of unequal sizes, the motion is also slower than when driving through  $DP_2$ . This slower motion is used for the cutting stroke

and the quicker one for the return stroke. The reversal is effected by shifting the belt from  $FP$  to  $BP$ , or *vice versa*, as in the Whitworth gear.



QUICK RETURN REVERSING GEAR FOR PLANING MACHINES.

**Whitworth's Quick Return Motion.**—In a shaping or slotting machine the table carrying the work is fixed and the tool moves over it, cutting in one direction only. In such a case, the tool usually obtains a reciprocating motion from a compound crank, so arranged as to give a quicker return stroke in order to

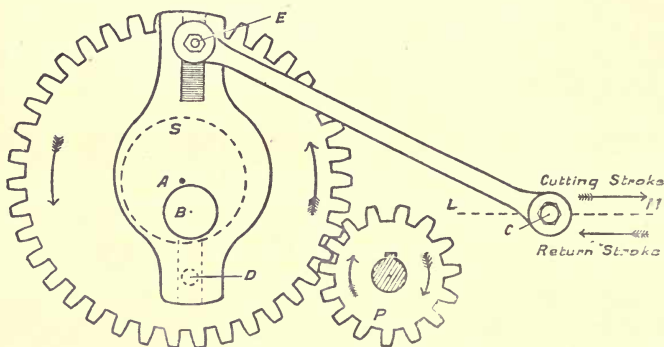


QUICK RETURN MOTION.

save time. Looking at the above diagrammatic figure,  $A$  and  $B$  are two fixed points and  $C$  is a point connected to the tool-holder, so guided as to move along  $LM$  at right angles to  $AB$ . A crank

D B E, centred at B, has its outer end joined to C, by the connecting-rod E C. A second crank A D rotates round A and drives the first by having a pin at D which moves in a slot B D. Now, when D is at the position G, C will be at the extreme left of its stroke; and when D is at F, C will be at the other end of its stroke. Hence, if A D rotates uniformly in the direction of the arrows; C will make its cutting stroke from left to right while D is moving round G K F; but during its return stroke D only requires to move round F H G. The return stroke will therefore occupy less time than the cutting stroke in the same ratio as the arc F H G is less than the arc G K F.

Sir Joseph Whitworth applied this principle to shaping machines in the manner shown by the next figure, which has the

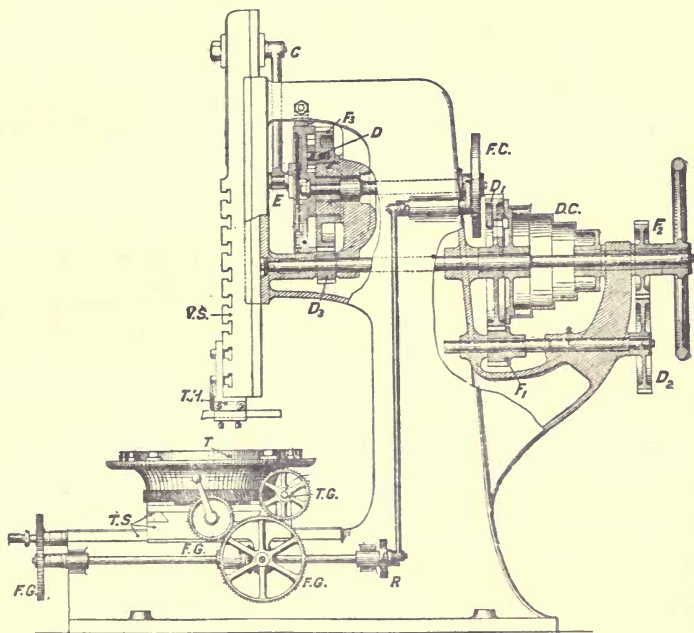


WHITWORTH'S QUICK RETURN MOTION FOR SHAPING MACHINES.

same lettering for corresponding parts as the previous one. Here, the crank A D is obtained by putting a pin D on a toothed wheel which rotates freely on a *fixed* shaft S, whose centre is at A; and the crank D B E is supported by a pin in a hole bored at B in the end of this shaft. In the back of this crank-piece there is a slot B D in which D can slide, and in the front another slot B E in which E can be clamped in any position so as to adjust the length of B E and thus give the required travel to the tool. The large wheel is driven uniformly by a pinion P connected with the belt pulley.\*

\* Students who desire further information on machine tools such as drilling and milling machines or on measuring appliances should refer to Professor Shelley's "Workshop Appliances," Professor Goodeve's "Elements of Mechanism," and Lineham's "Text-book of Mechanical Engineering," &c.

**Whitworth's Slotting Machine.\*** — The following side elevation of Sir Joseph Whitworth & Co.'s slotting machine will serve to show how their quick return motion is practically applied. The driving stepped cone DC receives its motion from a corresponding overhead cone fixed to the workshop

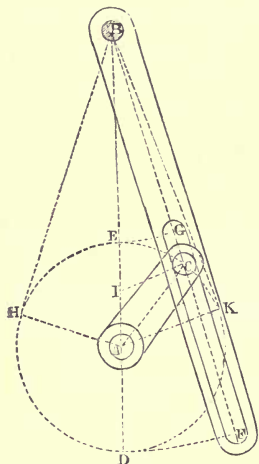


WHITWORTH'S SLOTTING MACHINE.

shafting. Its motion is in turn communicated to the compound crank E B D (just described and illustrated by the two preceding figures) through the toothed drivers and followers  $D_1, F_1$ ;  $D_2, F_2$ ;  $D_3, F_3$ . The quick up and slow down motion of the vertical slide bar V S, with its tool-holder T H, is obtained from the compound crank through the connecting-rod C E. The table T, upon which the metal to be slotted is bolted, may be shifted by hand levers or automatically moved to and fro, cross-

\* The above figure is reduced from a lithographed drawing which appears in Mr. Lineham's book on "Mechanical Engineering," to which students may refer for further views and details.

wise and turned at pleasure, by the feed gearing F G, actuated by a ratchet at R. This ratchet has a reversible click of the kind illustrated in the previous lecture and it is driven by a rod from

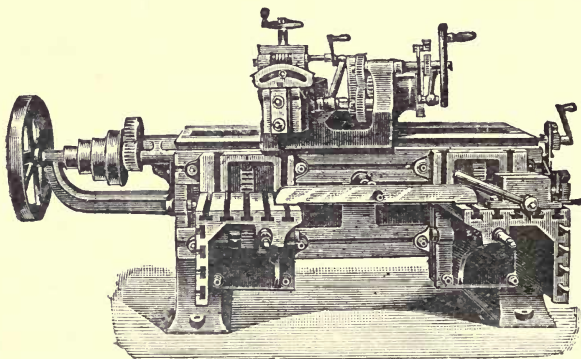


COMMON QUICK RETURN  
FOR SLOTTING MACHINE.

the feed cam F C. The whole of the moving parts are supported by a strong heavy bed plate, cast in one piece with the upright framing so as to prevent vibration in the material being operated upon or chattering of the cutting tool.

**Common Quick Return.**—Another form of quick return very often used for shaping and slotting machines is based on the mechanism of the oscillating steam-engine. A crank A C rotates uniformly and imparts motion to a slotted arm B F. This arm will have its extreme positions at B H and B K. It will therefore make its forward and back swings while C is moving round H D K and K E H respectively, and hence it has the quick return motion desired for the tool. The tool-holder is connected by a rod to such a point in B F as will give the required travel.

**Horizontal Shaping Machine.**—The following illustration shows a shaping machine with this gear for driving the cutting

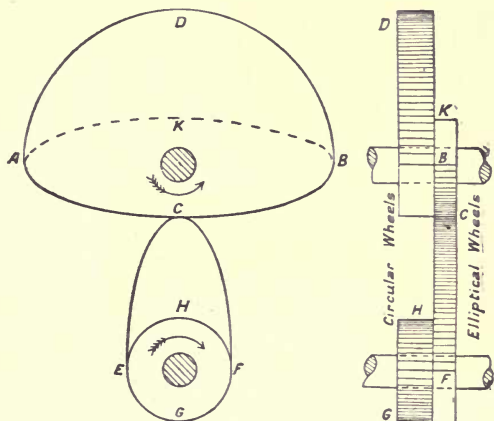


SHAPING MACHINE WITH QUICK RETURN BY MESSRS.  
SELIG SONNENTHAL & Co., LONDON.

tool. The work is fixed in a vice mounted on a table at the front of the machine, and this table can be moved up and down by a rack or screw, and traversed along the bed in either direction, backwards and forwards by a screw. The tool-holder moves inwards and outwards over this vice and is provided with a screw for moving the tool vertically, and a worm for adjusting its inclination. The gear for driving the tool is actuated by a sliding pinion on a shaft which lies along the whole length of the back of the machine and is driven by a stepped cone pulley. The feed of the tool-holder towards the left or right, is effected by a nut and screw underneath. This screw is worked by a ratchet seen to the right, in a similar way to that described for the previous machine.

**Quick Return with Elliptic Wheels.—Vertical Slotting Machine.**—We illustrate another slotting machine which has a different method

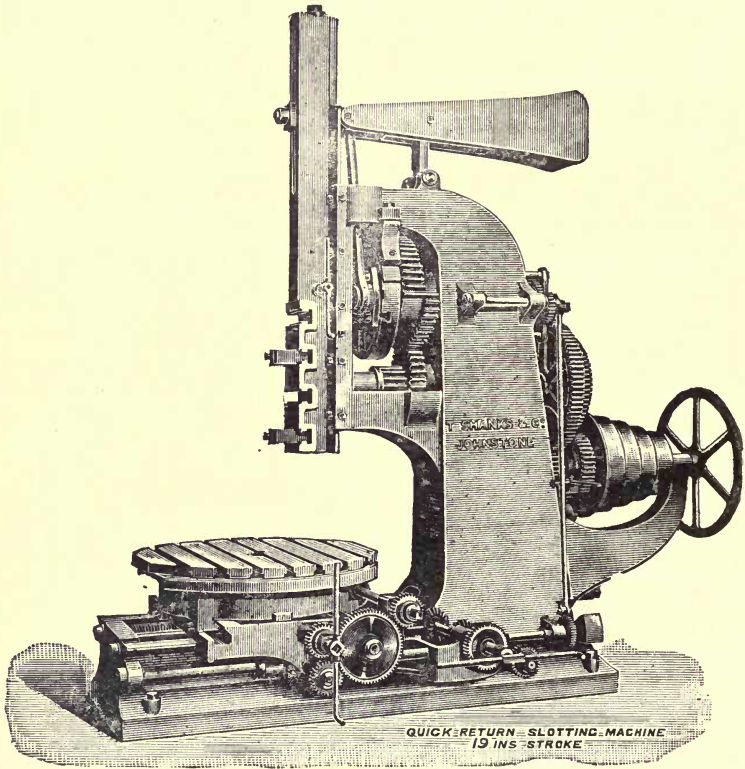
of obtaining a quick return. In this machine, the moving bar is actuated by a simple crank, but this crank is driven at different speeds for the cutting and return strokes by the following device. One portion of the circumference of the wheel on the crank-piece is part of an ellipse A C B, and the other, B D A, is



ELLIPTIC WHEELS FOR QUICK RETURN.

circular. The wheel for driving this has an elliptical part E C F, and a complete circle E H F G. Suppose B and F to be in contact. Then, as the lower wheel moves round, the two ellipses keep in contact until A and E come together. The two shafts thus make about half a revolution in the same time. The circular wheels now come into gear, and the lower wheel must make one and a half turns while the upper rotates through its second half. This will bring B and F again into contact and the whole process repeats itself. The quicker motion with the elliptical wheels is used for the upward stroke and the slower motion from the

circular wheels for the downward or cutting stroke. By looking at the side elevation, and at the view of the complete machine, it will be seen that the circular parts of the wheels D B and H G



VERTICAL SLOTTING MACHINE BY MESSRS. T. SHANKS & CO.,  
JOHNSTONE.

are placed to the left of the elliptical parts, B C and C F; and how, at each junction B and F of the two portions, one tooth stretches right across both of them so as to give a steady connected motion. Also, how the smooth part A K B of the upper elliptic wheel is cut away to clear the point of the other ellipse during

those revolutions in which the circles are in gear. From what has already been said about the other machines the student will easily understand the working of this one. He should, however, notice that the moving bar has a counterbalance to relieve the gearing from the weight of the sliding bar, also to reduce the driving force required for the upstroke and to prevent any sudden drop of the tool on the work which may be placed upon the table.

1. Illustrate and explain the form of reversing gear you would employ for a steam winch in which the engine shaft always runs one way.

2. Show a method of applying a self-acting motion for reversing the motion of the table of a planing machine when a screw is employed for driving it.

3. Describe, with a sketch and index of parts, some form of quick return gearing suitable for a planing machine, the movement being obtained by a combination of pulleys and spur wheels.

4. Sketch and describe an arrangement for driving the table of a planing machine by means of a screw, so that the table may travel 50 per cent. faster in the return than in the forward or cutting stroke. Why is a square threaded screw employed in such a machine?

5. Sketch and describe an arrangement of mechanism for reversing the table in a screw driven planing machine. In what way can a quick return of the table be obtained in such a machine?

6. Sketch and describe a good form of slow forward and quick return for a shaping machine.

7. Describe, with sketches, a planing, a slotting, or a shaping machine, showing clearly how the cutting and feeding motions are effected.

8. Sketch and describe the mechanism for feed motions:—(1) In a machine where there is a reciprocating movement, as in a planing machine; (2) where there is a continuous movement, as in a machine for boring cylinders.

9. Sketch and describe a vertical slotting machine with quick return elliptical gear. Give a separate diagram and explanation of the elliptical gear.

10. Sketch and describe the arrangement of mechanism by which the tool of a planing machine is traversed across the slide of the machine at each stroke of the table. (B. of E., 1900.)

11. Describe, with sketches, the mechanism for giving an automatic feed to the cutting tool of a lathe or shaping machine, and how it is put in or out of action, and the amount of feed varied. (B. of E., 1902.)

12. Describe, with sketches, two quick return motions for driving the table of a planing-machine—one in which the quick return is obtained by belting, and the other in which it is obtained by ordinary gearing.

(C. & G., 1903, O., Sec. A.)

13. The mechanism of the ordinary direct-acting engine is used as a quick return motion for a small shaping machine by simply placing the crank shaft centre below the line of stroke, instead of in the line of stroke produced. The crank radius is  $1\frac{3}{4}$  ins., the connecting rod is 7 ins. long, and the centre of the crank shaft is  $3\frac{1}{2}$  ins. below the line of stroke. Find, graphically, the stroke and the time ratio of the return and cutting strokes—the crank shaft being supposed to rotate uniformly.

Ans. Length of stroke =  $4\frac{1}{2}$  inches; Time ratio = 1 to 1.2.

(C. & G., 1903, O., Sec. A.)

14. Describe, with the aid of sketches, the mechanism known as the crank and slotted lever used to give a quick return motion to the tool of a shaping machine; and if the crank radius is half the distance between the fixed centres, find the ratio of the times occupied in performing the cutting and return strokes. The crank shaft may be supposed to rotate uniformly, and the obliquity of the connecting rod to the line of stroke of the tool-box neglected. Ans.

(C. & G., 1904, O., Sec. A.)

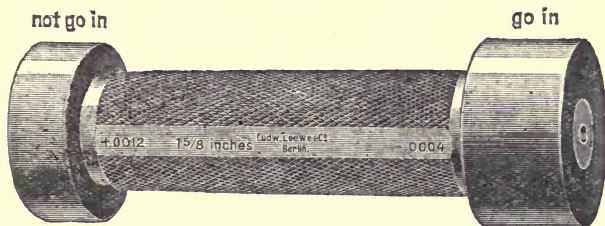
## LECTURE XXVI.

CONTENTS.—Measuring Tools and Gauges—Limit Gauges—Micrometer Screw Gauge—Sir Joseph Whitworth's Early Realisations of Mechanical Accuracy—Improved Equivalents Micrometer Gauge—A New Set of English Gauges—Whitworth Millionth Measuring Machine—Improved Standard Workshop Measuring Machine—Construction and Uses of the Tangentometer—Questions.

**Measuring Tools and Gauges.**—A few years ago, a pair of calipers and a foot-rule graduated to  $\frac{1}{32}$ -inch, formed an outfit for the British working mechanic. Anything more accurate than this was expressed by the somewhat elastic term *full* or *bare*! As far as each individual piece of fitting was concerned, the work turned out was probably of a good sound nature, and the same workman was able, through practice, to make a number of parts to within a fair degree of accuracy, when we consider the tools which were at his disposal. In this method of working, however, discrepancies due to individuality occur, and modern practice demands the suppression of these as far as possible. Again, good manual labour is not so cheap nowadays as it used to be. The result is, that machines, which lessen the cost of production and turn out work of a well-finished, accurate, and interchangeable nature, have greatly superseded the old rough-and-ready methods when dimensions were indicated by "sooking fits," "hair-breadths," and such like terms.

The introduction of the micrometer no doubt altered things to some extent; but this instrument, being of a delicate nature, requires careful handling, and that means lost time and consequently lost money. The micrometer is rather a tool-room instrument for the purpose of checking and comparing the various gauges and tools used in the shops.

**Limit Gauges.**—It is now found that the quickest and most accurate method of turning out interchangeable work is to make use of some system of limit gauges. In reality, a limit gauge consists of two gauges, one of which is larger and the other smaller by a minute fraction than the size intended. Thus, if we employ an internal limit gauge of the form

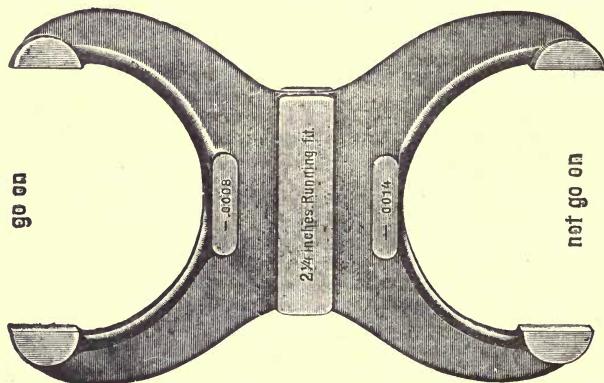


INTERNAL LIMIT GAUGE.

shown in the figure, for measuring a  $1\frac{5}{8}$ -inch hole, and, if we find that the right-hand end goes into the hole, but that the left-hand end does not, then we know for certain that the diameter of the hole cannot possibly differ from  $1\frac{5}{8}$ -inch by more than .0012 inch too large, or .0004 inch too small.

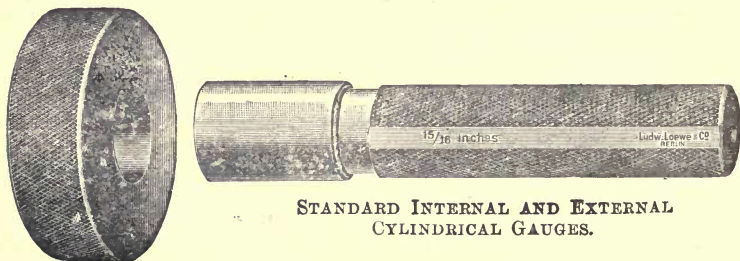
The difference limit varies, of course, with the class of work which it is intended to produce. It is quite evident, therefore, that for a machine which runs in a dust-laden atmosphere, a much greater difference limit will be required in the bearings, than for another similar machine which is designed to work in a comparatively clean atmosphere.

But, no matter how tight or how slack the working fits may be, it will be found, that work constructed on some system of limit gauges will have a much greater all-round efficiency than that made in the old haphazard trial and error fashion with an ordinary rule and calipers.



EXTERNAL LIMIT GAUGE.

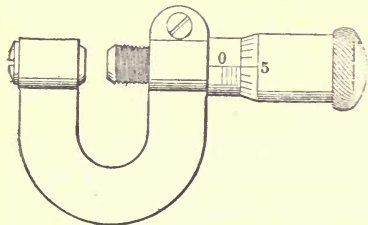
The above or second figure is an illustration of an External Limit Gauge for a  $2\frac{1}{4}$ " or 2.25 inch shaft. One end is marked to "go on" and is only 0.0008" or 8/10,000 of an inch less than the 2.25 in. shaft; whilst the other or smaller caliper end is marked to "not go on," for it is 0.0014 in. or 14/10,000 of an inch less than the 2.25 in. shaft. Consequently, the maximum difference between the "go on" fit and the "not go on" fit is  $(2.25 \text{ in.} - 0.0008 \text{ in.}) - (2.25 \text{ in.} - 0.0014 \text{ in.})$  or simply  $(0.0008 \text{ in.} - 0.0014 \text{ in.}) = 0.0006 \text{ in.}$ , that is 6/10,000 in., or say 1/1666 in. which is less than 1/1000th of an inch. If the wider end of the gauge just goes on, the shaft will be a "tight fit," but if the narrower end just goes on, the shaft will be a "running fit" in a bush bored exactly to 2.25 in. diameter.



STANDARD INTERNAL AND EXTERNAL  
CYLINDRICAL GAUGES.

The third figure shows a Standard Internal and External Cylindrical Gauge. This type of gauge is chiefly used as a standard of reference. It is hardened, ground, lapped by hand, and is accurate to within .0001 inch.

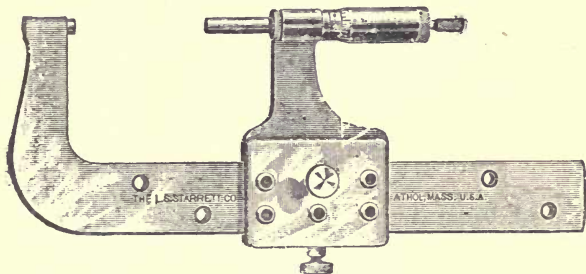
**The Micrometer Screw Gauge.**—This instrument is very useful for measuring diameters or thicknesses to within .001 inch. On turning the milled cap shown at the right-hand side of the illustration, the operator turns the screw of 40 threads to the inch, which also may be seen from the figure. It is clearly evident, that if one complete turn be given to the milled cap, the screw will advance or recede  $\frac{1}{40}$  inch. Therefore, if the edge of the sleeve (which forms part of the cap) be divided into 25 equal parts along its circumference, and the cap be rotated by a part of a revolution corresponding to one of these divisions, the screw will have advanced by  $\frac{1}{25}$  of  $\frac{1}{40}$  inch, i.e.,  $\frac{1}{1000}$  or .001 inch.



MICROMETER SCREW GAUGE.

FOR MEASURING ALL SIZES LESS THAN  
0.3 INCH BY THOUSANDTHS  
OF AN INCH.

It will be seen from the two figures that micrometer screw gauges



STARRETT MICROMETER GAUGE.

are made of various ranges and styles. If a mere portable gauge is required to suit different sizes, then one of the best forms is that made by the Starrett Co., U.S.A. In which the position of the movable end is determined by inserting a hardened steel tapered pin into hardened steel bushed holes. For fixed measuring machines of great accuracy the Whitworth Millionth Measuring Machine is still considered the standard in this country.

**Sir Joseph Whitworth's Early Realisations of Mechanical Accuracy.**—"Whitworth's Standard Measuring Machine" will be illustrated and described near the end of this Lecture. But, it may be mentioned here, that the whole subject of *accurate*, scientific, mechanical measurement and its standardisation had remained in great confusion and uncertainty until Whitworth *first* carefully considered and then made, about the year 1840, mechanically perfect flat-surface plates. *Second*, he made standard screw threads, screw taps and dies, as well as parent leading screws for lathes, &c. *Lastly*, a standard or parent Measuring Machine, which was to be an instrument of such extreme precision that it could detect the difference of one one-millionth of an inch in the end measurement of short standard bars.

These three early steps in his career were realised in succession. In fact, he could never have made the standard measuring machine if he had not previously made and drilled his men into producing the two former sets of tools.

All the present-day accurate machine-shop surface-plates, screw threads, standard bars, gauges and fixed measuring machines, may be said to be the outcome of Whitworth's skill, perseverance and forethought, in systematising the production of standard tools. He also advertised the results and sold accurate copies of his correct parent tools not only amongst British engineers but throughout the civilised world.

Sir Joseph Whitworth, before commencing the afore-mentioned difficult tasks, was satisfied that the most practical means of workshop measurement was to be founded on the truth of surface and the sense of touch. He maintained to his dying day, that the most delicate sense of the mechanic and mechanician was that of touch. In confirmation thereof, he showed that when a piece of metal had parallel end faces and was so held between the two fair-in line and parallel measuring planes of his machine, that the piece being tested just gravitated slowly downwards, due to its own weight between these two faces. Then, any good mechanic could at once detect the difference in bringing these two plane parts or distance faces nearer together or further apart by the minute difference of one one-millionth of an inch! Of course, such extreme accuracy is not required in ordinary tool-making and engine-building works, but it is required in some works.

**Improved Equivalents Micrometer Gauge.**—This new instrument has certain advantages over the ordinary screw Micrometers. By means of the screw on the hub and the two divided discs, readings can be taken up to  $\cdot300$  inch with the various equivalents. Or, measurements may be made in the same way as with the older style of gauge for all measurements less than one inch by  $\frac{1}{1000}$  of an inch between the jaws.

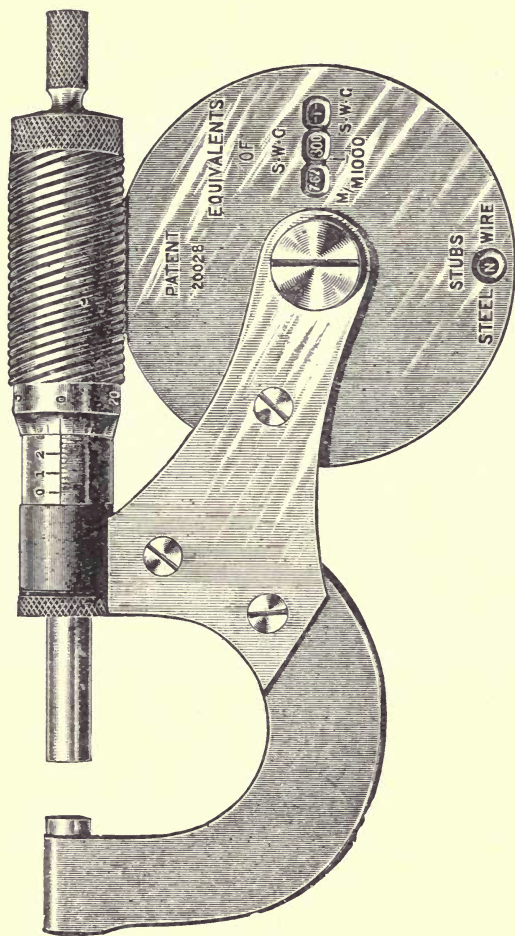
On one side of the disc appear decimals of an inch, decimals of millimetre standard wire gauge and Stubs' round hole wire gauge. On the other side, fractions of an inch by  $\frac{1}{16}$  inch, and screwing sizes for B.A. threads.

Equivalents to other gauges, such as the "British Standard Wire Gauge," "Stubs' round hole wire gauge," and fractions of an inch on the metric standard, can be read off without the necessity of reference tables; whilst at the same time the jaws are set to give an exact size in decimals of an inch. It will thus be seen, that by turning the disc until any decimal or other number appears, several equivalents can be read simultaneously, which will be especially convenient for telegraph and electrical engineers.

The hub is fitted with a small milled thumb piece, projecting beyond the ordinary hub, by means of which the speed of turning the screw

may be increased. The anvil end of the Micrometer is flush with the screw spindle, thus allowing of the close calipering of projections or ledges.

NOTE.—I am indebted for the figure and description of this improved gauge to the patentees and makers, Messrs. Grimshaw and Baxter, of 29 Goswell Road, London.



IMPROVED EQUIVALENTS MICROMETER GAUGE.

**A New Set of English Gauges** (*Windley's Patents*).—*Description of the Following Six Figures*.—Fig. I shows the lower half of a pocket-case ( $6" \times 5" \times 1\frac{1}{2}"$ ) containing a complete set of these Caliper Gauges. It will be observed that both the *external* and the *internal* measuring holders are divided in the centre, to allow of the insertion of one or more of the steel blocks (illustrated on the right hand of the case) between either of these two holders.

Fig. II indicates the arranging of the *External Gauge* to measure  $\frac{1}{8}$  inch. This can be effected in a very short time by means of the "quick-grip lock-nut device."

Fig. III gives a photographic proof of the perfectly flat surfaces and the very superior finish of the faces of the seven cast-steel blocks, by their clinging together after having been wrung together.

Professor Tyndall was the first scientist to prove that perfectly flat surfaces adhere together, due to the mere molecular attraction between the great number of bearing points when brought into close contact. He entirely disposed of the previously held theory that the adherence (of, say, two good surface plates) was due to the exclusion of the air between them, and, therefore, to atmospheric pressure.

Fig. IV shows the *internal* measuring holder arranged to measure  $3\frac{3}{8}$  inches. It is usually finished with flat ends, but it can be supplied with spherical ends having a radius slightly smaller than the smallest cylinder of  $2\frac{1}{2}$  inches which the gauge will enter.

Fig. V exemplifies how the  $\frac{3}{8}"$ ,  $\frac{1}{4}"$ ,  $\frac{3}{16}"$ ,  $\frac{1}{8}"$  and  $\frac{1}{16}"$  blocks, when wrung together, are used to check the  $1"$  block by the *external* gauge. This process may be reversed by placing the  $1"$  piece between the jaws and the five smaller pieces in the holder. This method of self-checking is one of the best points of these gauges.

Fig. VI is a view of the new 1910 pattern of a "Combined Limit and Double-ended Caliper Gauge." When the jaws marked (+, +) and (—, —) are opposite each other, as in the photo, the gauge is set for limit measurements, the (+, +) gap being slightly larger and the (—, —) gap slightly smaller than the exact measurement aimed at. But, when slackened, turned round and then readjusted (so that the (+ & —) markings come opposite each other), both ends become *exact* caliper or snap gauges. They now indicate the precise size due to the number and breadths of the round steel standard-sized blocks, which have been inserted in the centre, between the two lock-nuts.

It may be mentioned, that all the parts of these several gauges are made of the best British cast-steel, properly hardened and finished.

**Range of Measurements by the External and Internal Holders.**—In the up-to-date standard sets there are but seven steel blocks, viz.,  $1"$ ,  $\frac{1}{2}"$ ,  $\frac{3}{8}"$ ,  $\frac{1}{4}"$ ,  $\frac{3}{16}"$ ,  $\frac{1}{8}"$  and  $\frac{1}{16}"$ . This gives the *external* measuring holder a range from  $\frac{1}{16}"$  to  $2\frac{1}{2}"$ , by  $\frac{1}{16}"$  of an inch, or forty different sizes; and the *internal* measuring holder a range from  $2\frac{1}{2}"$  to  $4"$ , also by  $\frac{1}{16}"$ ths of an inch, or twenty-four different sizes. In addition to this, forty end sizes from  $\frac{1}{16}"$  to  $2\frac{1}{2}"$  are obtainable by wringing the blocks together in combinations. Thus, we get a range of over 100 separate standard gauge sizes with one small pocket instrument.

Mr. H. M. Budgett, of the Crown Works, Chelmsford, informs the author that he is now (1910) adding  $\frac{3}{32}"$  and  $\frac{5}{16}"$  measuring pieces to the above-mentioned seven blocks. This will greatly increase the range of the sizes obtainable with this instrument. In fact, the outside gauge will then give 160 sizes and the internal one 96 sizes, whilst the pieces

FOR DESCRIPTION AND DETAILS SEE PROFESSOR JAMIESON'S MANUAL  
OF APPLIED MECHANICS.

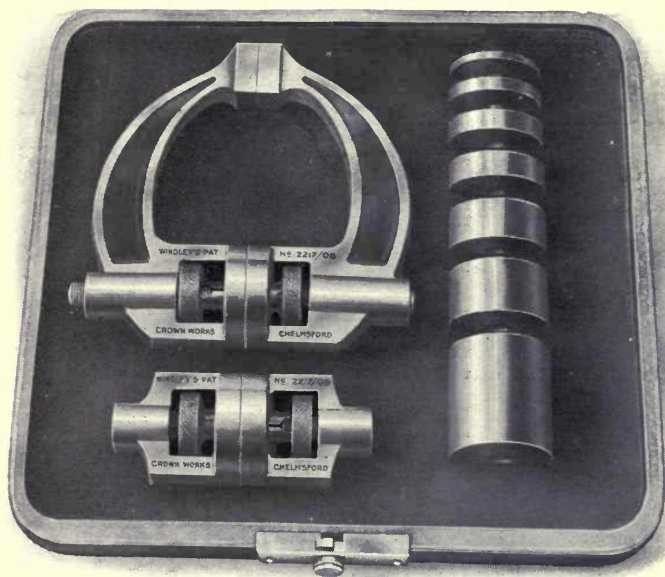


FIG. I.

SHOWING COMPLETE SET IN CASE, HALF ACTUAL SIZE.

*These Gauges are made by the Crown Works, Chelmsford, England.*

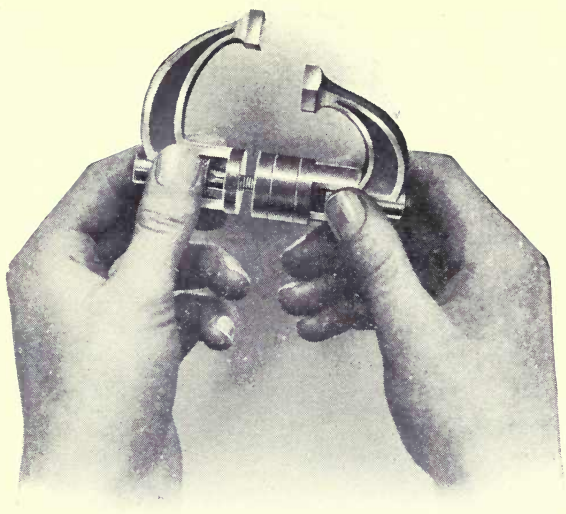


FIG. II.

SHOWING EXTERNAL GAUGE BEING ARRANGED TO MEASURE  $\frac{5}{8}$ -in.

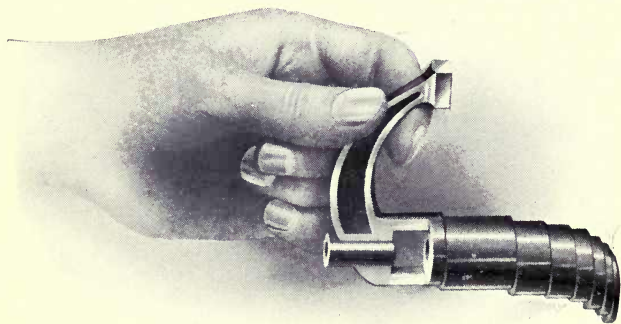


FIG. III.

SHOWING HOW THE VARIOUS PIECES WILL ADHERE WHEN WRUNG TOGETHER

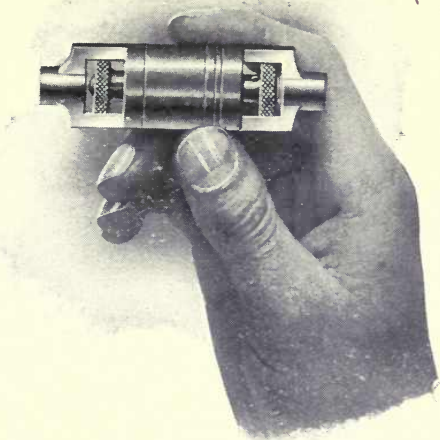


FIG. IV.  
SHOWING THE INTERNAL GAUGE ARRANGED TO MEASURE  $3\frac{3}{8}$  in.

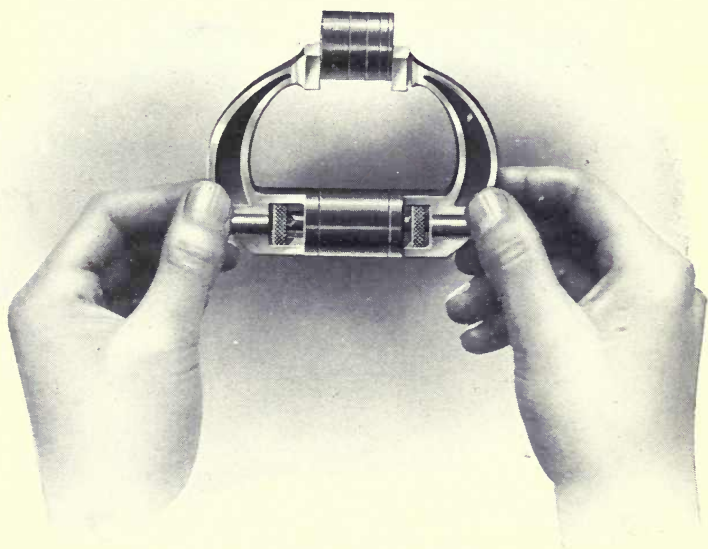


FIG. V.  
SHOWING HOW THE GAUGES ARE SELF-CHECKING,

SHOWING COMBINED LIMIT AND DOUBLE ENDED CALIPER GAUGE.

themselves will give 163 sizes; so that 419 different exact sizes could thereby be measured!

**Uses of the Gauges with a Surface-Plate.**—In every up-to-date tool-making and engineering workshop there should be a set of standard surface-plates. Then, the previously described gauges may be used in many ways with such a surface-plate. For example, an exact height gauge may be formed by wringing *one* arm of the *external* measuring holder with any desired number of blocks, and then using them on the surface-plate.

**The National Physical Laboratory Certificate.**—We finish the description of these British-made gauges in reproducing from a photograph the Certificate granted in August 1909 by The National Physical Laboratory, with two objects in view:

*First*, to show the extreme accuracy of the gauges, where none of the seven blocks had an error of one one hundred-thousandth of an inch!

*Second*, to bring to the notice of Engineering Students, that if ever they should be fortunate enough to devise anything new in regard to thermometers, pyrometers, barometers, micrometers, cyclometers, galvanometers, electrometers, or steam, vacuum and mechanical gauges, &c., they may have the accuracy of their invention or improvement tested by an absolutely impartial and reliable judge at a comparatively small cost.

Certificate of Examination of Seven End Gauges.

By the National Physical Laboratory, Teddington.

For: H. M. Budgett, Crown Works, Chelmsford.

Form: Cylindrical, 1" in diameter with a  $\frac{1}{4}$ " hole through the centre.

The end faces are perpendicular to the axis of the cylinder.

These gauges have been compared with the Laboratory Standards, and the mean lengths at 62° F. have been found to be:

Gauge.	Length at 62° F.	Gauge.	Length at 62° F.
$\frac{1}{16}$	0.06250 inch.	$\frac{1}{4}$	0.25000 inch.
$\frac{1}{8}$	0.12500 "	$\frac{3}{8}$	0.37500 "
$\frac{3}{16}$	0.18750 "	$\frac{1}{2}$	0.50000 "
Gauge 1 was 1.00000 inch at 62° F.			

Each gauge has been tested at several points, and the end faces in each case have been found to be parallel within 0".00001.

August 25, 1909.

J. A. HARKER,  
For the Director.

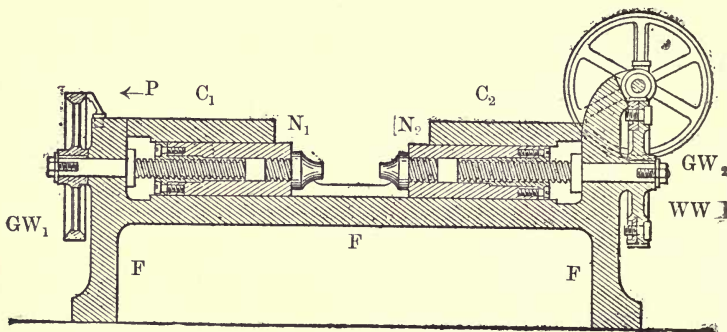
**The Whitworth Millionth Measuring Machine.**—From what has been said in this Lecture about Sir Joseph Whitworth's early realisations of mechanical accuracy, the student will be anxious to learn how he accomplished the wonderful task of constructing a machine capable of detecting a difference of only one-millionth of an inch in the length of two bars. Another object which he then had in view was to produce an exact fractional part of the "British Parliamentary Standard of Length," viz. the legal yard, or 36 inches. Consequently, he first of all made a machine to receive between its two measuring noses a bar of only 1 inch long, and thereafter he constructed the machine which we now illustrate and describe to measure bars of 3" and 4" in length, so that twelve of the

former or nine of the latter placed end to end would make up the British standard yard.

In 1851 he was awarded the Council Medal of the first Great Exhibition (which had been proposed and successfully carried through under the patronage of the late Queen Victoria and Prince Consort) for a precisely similar machine to take in a bar of 40" in length.

Referring to the figure, we see that it consists of a strong, rigid cast-iron framing F, which stands upon three legs, two at the right-hand end and one leg at the left-hand end. This frame F not only forms the bed of the machine, but it also includes the bases of the two fixed head-stocks, of which  $C_1$  and  $C_2$  are the upper or removable caps. Precisely in the centre-line of the two head-stocks the casting is moulded into two hollow right-angled or V grooves. These V grooves are then carefully planed and scraped perfectly fair and square and a rectangular steel bar  $\diamond$  is so very carefully fitted into each of them that they bear and can be moved evenly throughout the length of each end head-stock base, along the centre line. The ends of these two steel bars of  $\diamond$  or square section are then faced at right-angles to their length, and a truly central hole is bored and screwed, to receive a long steel screw at their outer ends and shorter screwed-in noses  $N_1$ ,  $N_2$  as shown in the figure. The inner faces of these two noses are so carefully turned and scraped that they are "dead parallel" to each other. It is between these noses  $N_1$  and  $N_2$  that the short standard bar or its copy is to be placed and its length measured in the following way.

It will be observed, that on the extreme left of the framing a graduated wheel  $GW_1$ , has been fitted to the outer turned end of the screw spindle



WHITWORTH MILLIONTH MEASURING MACHINE.

whose nose is  $N_1$ . Since the pitch of the screw has twenty threads to the inch, and the flat periphery surface of the graduated wheel  $GW_1$  is divided into 250 equal parts, with a fine pointer at P, it will be readily understood, that if this wheel  $GW_1$  is so turned that the pointer P shows a movement through one division, then the inner face of the nose  $N_1$  will *only* move one five-thousandth of an inch forward or backward, according to the direction in which the wheel is turned. This screw and wheel are therefore only suitable for comparatively rough to and fro adjustments of the nose  $N_1$ , since  $\frac{1}{20} \times \frac{1}{250} = \frac{1}{5000}$ th of an inch.

Now look at the other head-stock where the screw which actuates the nose  $N_2$  has also a pitch of twenty threads to the inch, and the wheel WW has 200 teeth. This wheel gears with the worm-screw WS, whose spindle is fixed to the centre of the large graduated wheel  $GW_2$ , which has 250 equal divisions marked upon its flat-faced periphery with a pointer not shown [in the figure. Consequently (since  $\frac{1}{210} \times \frac{1}{210} \times \frac{1}{250} = \frac{1}{1,002,000}$ ), if the wheel  $GW_2$  be so carefully turned that its pointer shows a movement of one division, the inner face of the nose  $N_2$  will *only* move to or fro by the *one-millionth of an inch*.

Such a small movement might bind or let the short bars 1", 2", 3" or 4" in length drop down as the case may be, and thus to a certain extent hamper the "finer feelings" of the operator. So Whitworth introduced what he called a "*feeler*" (which consists of a thin parallel slip of hardened polished steel) between one face of the standard bar (or the end of the bar to be measured and compared with the standard one) and the flat face of the nose  $N_1$  or  $N_2$ . When the adjustment of  $N_1$  and  $N_2$  is considered perfect, this "*feeler*" should just glide down gently by its own weight, between one of these nose faces and the nearest end of the bar under test.

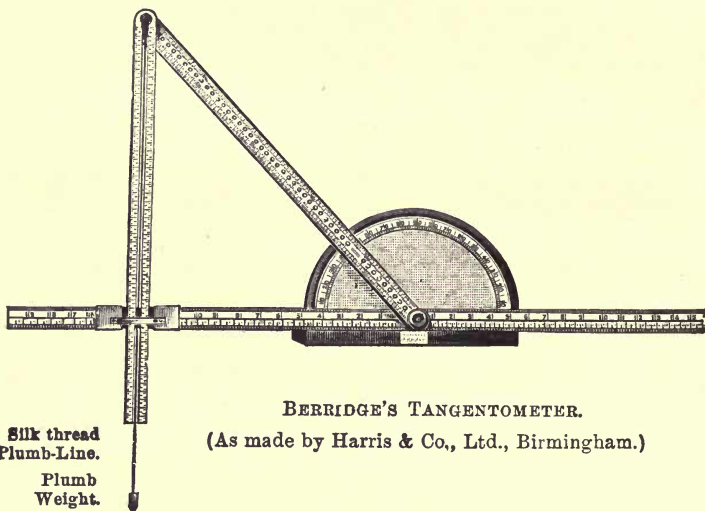
Such an accurate machine as we have just described must not only be brought to and kept at a fixed standard temperature, say 62° Fahr. for British measurements and 70° Fahr. for United States of America tests, but care must be taken to protect it from dust, moisture, and currents of air.

**Improved Standard Workshop Measuring Machine.**—For engineering workshops, a much less expensive and simpler fixed machine is now made. It is, however, sufficiently accurate to detect a difference of  $\frac{1}{10,000}$  of an inch in the length between one bar or gauge and another. It consists of a strong frame and bed- or lathe-shears, with a fixed head-stock at one end and another one which is movable. The latter is moved to or fro by means of a screw placed longitudinally between the whole length of the shears. This screw is rotated by an end wheel, and the screw gears with a nut fixed into the tongue of the movable head-stock. The spindle of this "poppit" or movable head is often round and parallel throughout. Consequently, all to-and-fro adjustments of the right-hand nose-face  $N_2$  are made in the same way as shown by the previous figure in connection with the fixed head-stock.

Now, supposing that you have to compare the length of a bar with that of a standard bar (whose length had been previously adjusted between the noses  $N_1$  and  $N_2$  and the pressure thereon noted), you place the new bar between  $N_1$  and  $N_2$ . If this bar is, say, longer than the standard bar, the free spindle of the movable head-stock will be pressed slightly outwards. Then, since its outer end now presses harder against the elastic centre of a circular metal case containing water (with a vertical graduated, clear glass tube extending therefrom), the water will rise higher in the tube than in the former case, that is when the standard bar was under measurement.

The graduated wheel of the fixed head-stock is now turned until the height or "head" of water in the glass tube is brought to the same position as in the first case. Then, the difference between the two readings upon the divided wheel at once indicates the difference in ten-thousandth of an inch between the two bars. We know, that the pressure upon the ends of each bar was the same because the hydrostatic "head" was the same. This replaces the personal error of the operator with his feeler.

**Construction of The Tangentometer.**—This instrument consists of a horizontal metre rule, divided into inches and parts of an inch along its upper scale, and into centimetres and millimetres along its lower scale. At the centre of this rule there is attached one end of a second rule, half a metre in length and perforated at various points. A small slot is cut in the second rule to enable the degrees marked on the semicircle to be read. Attached to this second rule by means of a pin, are (1) a third rule, (2) a plumb-line. The third rule is graduated into cm. and mm., and has a fine slot cut down its centre to enable the divisions on the first or horizontal rule to be read. In order to avoid parallax a cursor or fine dividing line connects the vertical with the horizontal rules. The whole instrument is made of boxwood and fixed on a firm central base.



**Uses of The Tangentometer.**—This instrument enables teachers to explain graphically the meaning of the several trigonometrical ratios to technical students. It also enables a student to measure for himself various angles and to prove his answers to questions by noting the ratio of the actual lengths of the sides of a right-angled triangle. He can then compare his results with the Table of Trigonometrical values for "Functions of Angles" at the end of this book.

FOR EXAMPLE, taking the above scale figure of the full-sized instrument, we see, that the centre line of the *second* rule (which is 50 cms. long) lies at an angle of  $48^\circ$  to the centre line of the *first* or horizontal rule. And, the *third* or vertical rule hangs at right angles to the first rule. The length of the plumb-line of the *third* rule is 37 cms. to where it cuts the centre line of the first rule at right angles at 33.5 cms. from its centre or zero mark.

Here, the *first* or horizontal rule is called the *base*; the *second* or 50 cm. rule is the *hypotenuse*; and the *third* or vertical rule is called the *perpendicular*. Then, we have the following ratios for the acute angle of  $48^\circ$  in the full-sized instrument.\*

$$\text{Sine of the Angle} = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{37 \text{ cms.}}{50 \text{ cms.}} = \cdot 74 \text{ see Table for } 48^\circ$$

$$\text{Cosine} \quad " = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{33\cdot 5 \text{ cms.}}{50 \text{ cms.}} = \cdot 67 \text{ see Table for } 48^\circ$$

$$\text{Tangent} \quad " = \frac{\text{Perpendicular}}{\text{Base}} = \frac{37 \text{ cms.}}{33\cdot 5 \text{ cms.}} = 1\cdot 11 \text{ see Table for } 48^\circ$$

$$\text{Cotangent} \quad " = \frac{\text{Base}}{\text{Perpendicular}} = \frac{33\cdot 5 \text{ cms.}}{37 \text{ cms.}} = 0\cdot 9 \text{ see Table for } 48^\circ$$

$$\text{Secant} \quad " = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{50 \text{ cms.}}{33\cdot 5 \text{ cms.}} = 1\cdot 5$$

$$\text{Cosecant} \quad " = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{50 \text{ cms.}}{37 \text{ cms.}} = 1\cdot 35$$

$$1 \text{ Radian} = \text{Unit angle in circular measure} = \frac{180^\circ}{\pi} = 57^\circ\cdot 29.$$

$\therefore 48^\circ \div 57^\circ\cdot 29 = 0\cdot 8378$  of a radian. See Table for the radian of  $48^\circ$ .

\* When resolving a force into its two component forces, the student must pay particular attention to the (+) or (−) sign of the value of the angle. See *Functions of Angles* in Castle's "Practical Mathematics for Beginners," chapter xvi., or other book on Trigonometry.

NOTE.—I am indebted to Ludw. Loewe and Co. Ltd., 30 Farringdon Road, London, for the use of their Electros of "Limit Gauges."

I have to thank Mr. H. M. Budgett, of the Crown Works, Chelmsford, for six excellent views of his company's new English Gauges, and the Louis Cassier Co., Limited, for the liberty to reproduce the Micrometer Gauges and the Whitworth measuring machine which appeared in *Cassier's Magazine*, September 1901. The attention of Students who are interested in gauges and accurate work is directed to "The Specification of the Engineering Committee on Standards"; and to comments upon the same in *The Electrician* for Aug. 1906.

## LECTURE XXVI.—QUESTIONS.

1. Describe any micrometer screw gauge with which you are acquainted suitable for measuring to the  $\frac{1}{1000}$  of an inch. Sketch and describe carefully the method of graduation and the position of the gauge when set to measure  $\cdot374$  inch.

2. Sketch and describe the construction and use of external and internal workshop gauges, by means of which the size of a spindle (say 2 inches diameter), and that of a hole into which it fits, may be ensured within specified limits of accuracy. State any advantages due to this system of working. (B. of E., 1902.)

3. Sketch a cylindrical *one inch* external gauge, and describe generally the measuring machine which you would require to employ, and the manner of using the same, in order to construct another gauge of the like kind, but measuring  $1\cdot005$  in diameter. How is the gauge worked down to the right size and finished?

4. Sketch and describe the construction and action of a "Whitworth Millionth Measuring Machine." For which purposes have you seen it used?

5. Sketch and describe an "Equivalents Micrometer Gauge" and state the advantages which it possesses over an ordinary gauge.

6. Sketch and describe fully an "External Limit Gauge" for a  $2\frac{1}{4}$  inch shaft, such as the one illustrated by the second figure in this Lecture. Explain concisely and clearly how you would use such a gauge. Show your calculations in full for the percentage error in  $2\frac{1}{4}$  inches diameter, if a turner *just* makes a shaft as a tight fit for the "*go on*" end of the gauge. Also for the percentage error if he should so reduce the diameter of a shaft, to let the "*not go on*" end of the gauge fit the shaft.

7. Describe by aid of sketches Sir Joseph Whitworth's early attempts towards realisations of mechanical accuracy with standard surface-plates, screws, gauges, and measuring machines.

8. Sketch and describe the new set of English Gauges as made by the Crown Works, Chelmsford, according to Windley's Patents.

9. Show how the gauges in the previous question are self-checking. Also, show by calculation, the exact range or number of different measurements which can be made with seven blocks and the external holders. If two extra blocks,  $\frac{1}{32}$ " and  $\frac{1}{16}$ ", were added, prove how many more measurements could be made.

10. Sketch neatly and describe concisely Sir Joseph Whitworth's Millionth Measuring Machine. State the pitch of screw, number of teeth on worm-wheel, and number of divisions on the graduated wheel to measure with such accuracy.

11. Sketch, index and describe, any fixed good workshop measuring machine and state how you would use it.

12. Find the sine, cosine, tangent, cotangent, secant and cosecant of an angle in a right-angled triangle whose base is 40 cms, and perpendicular 20 cms. Also express  $27^\circ$  in radian measure.

APPENDIX A (pages 368 to 370).

- (i) General Instructions by the Board of Education for their Examinations on Applied Mechanics, Stage I.
- (ii) General Instructions by the City and Guilds of London Institute for their Examination on Mechanical Engineering, Ordinary Grade.
- (iii) Rules and Syllabus of Examinations by the Institution of Civil Engineers for Admission of Students.

APPENDIX B (pages 371 to 401).

Board of Education's Exam. Papers in Applied Mechanics, Stage I. The City and Guilds of London Institute's Ordinary Exam. Papers in Mechanical Engineering. And the Institution of Civil Engineers' Exam. Papers: Elementary Mechanics, arranged in the order of the Lectures.

APPENDIX C (pages 403 to 410).

The latest Exam. Papers pertaining to Mechanics and set by the governing bodies enumerated under Appendix A.

APPENDIX D (pages 411 to 413).

- (i) Units of Measurement and their Definitions; Practical Electrical Units and their Symbol Letters.
- (ii) Examination Tables, Useful Constants, Logarithms, Antilogarithm and Functions of Angles.

**Appendix A.**  
**May Examination on Subject VII.**

**APPLIED MECHANICS.\***

BY THE BOARD OF EDUCATION, SECONDARY BRANCH,  
SOUTH KENSINGTON, LONDON

**Stage 1.**

**GENERAL INSTRUCTIONS.**

**If the regulations are not attended to, your paper  
will be cancelled.**

*Immediately before the Examination commences, the following  
REGULATIONS ARE TO BE READ TO THE CANDIDATES.*

Before commencing your work, you are required to fill up the numbered slip which is attached to the blank examination paper.

You may not have with you any books, notes, or paper other than that supplied to you for use at this examination.

You are not allowed to write, draw, or calculate on your paper of questions.

You must not, under any circumstances whatever, speak to or communicate with another candidate. Those superintending the examination are not at liberty to give any explanation bearing upon the paper.

You must remain seated until your papers have been collected, and then quietly leave the examination room. No candidate will be allowed to leave before the expiration of one hour from the commencement of the examination, and none can be re-admitted after having once left the room.

All papers, not previously given up, will be collected at 10 o'clock.

If any of you break any of these regulations, or use any unfair means, you will be expelled, and your paper cancelled.

**Before commencing your work, you must carefully  
read the following instructions.**

Put the number of the question before your answer.

You are to confine your answers *strictly* to the questions proposed.

Such details of your calculations should be given as will show the methods employed in obtaining arithmetical results.

The value attached to each question is shown in brackets after the question.

A table of logarithms and functions of angles and useful constants and formulæ is supplied to each candidate.

*The examination in this subject lasts for three hours.*

\* See Appendix C for the latest Exam. Papers.

## CITY AND GUILDS OF LONDON INSTITUTE.

## DEPARTMENT OF TECHNOLOGY.

## Technological Examinations.

## MECHANICAL ENGINEERING.\*

## ORDINARY GRADE—PART I.

## (FIRST YEAR'S COURSE.)

## INSTRUCTIONS.

No Certificates will be given on the results of this Examination (First Year's Course), but the Candidates' successes will be notified to the Centre where they were examined.

To obtain a Certificate, it is essential that Candidates should pass both in Part I. and Part II. ; *the Examination in Part II. will be held on Thursday, May 3,† at 7 p.m.* Candidates may take both Parts I. and II. in the same year.

The class of Certificate and the order of Prize will be determined by the results of the Examination in Part II. only.

The maximum number of marks obtainable is affixed to each question.

The number of the question must be placed before the answer in the worked paper.

*Three hours allowed for this paper.*

The Candidate is at liberty to use divided scales, compasses, set squares, calculators, slide rules, and tables of logarithms.

A piece of squared paper to be given to each Candidate, if required.

The Candidate is not expected to answer more than *nine* questions, which must be selected from *two* Sections only.

\* See Appendix C for the latest Exam. Papers.

† This date is only approximate, and subject to a slight alteration each year.

## The Institution of Civil Engineers' Rules for Admission of Students.

### SYLLABUS OF THE EXAMINATIONS.\*

1. **ENGLISH** (one Paper, *time allowed, 3 hours*). A general Paper comprising questions in Geography, History and Literature.

2. **MATHEMATICS** (two Papers, *time allowed, 3 hours for each*). Papers comprise questions in Arithmetic; Algebra; Geometry (Euclid I.-IV.); and Trigonometry.

3. Two subjects, to be selected by the Candidate from the following ten: a language is not compulsory, but in any case not more than one language may be taken (*time allowed, 3 hours for each Paper*):

LATIN, GREEK, FRENCH, GERMAN, ITALIAN, SPANISH.

**Elementary Mechanics of solids and fluids.**† **ELEMENTARY PHYSICS**, including heat, light, **Electricity and Magnetism**. **ELEMENTARY CHEMISTRY**. **GEOMETRICAL AND FREEHAND DRAWING**.

\* See Appendix C for the latest Exam. Papers.

† My Manuals on Applied Mechanics, Magnetism and Electricity, are suitable for Young Engineers desiring to prepare by correspondence or otherwise for these subjects, of which the most recent examples in Mechanics are given in Appendix C, whilst those on Electricity and Magnetism are printed in the Appendix of the Eighth Edition of the latter work. Candidates should write direct at once to the Author of this book for his C.E. Prospectus, which gives full details of Tuition for these Examinations.

### RULES OF THE EXAMINATIONS.

The Examinations are held in London in February and October annually, on four days beginning on the second Tuesday in each of those months. The February Studentship Examination may, in the discretion of the Council, be held also in Manchester, Glasgow and Newcastle-on-Tyne.

The Council will consider an Application from a person who is duly recommended for Admission as a Student of the Institution, to present himself for the Studentship Examination.

Applications to attend the Associate Membership Examination will be received from Students of the Institution who are not less than 21 years nor more than 26 years of age on the last day for entry.

Arrangements may be made for the examination of Candidates in India or in the Colonies, after submitting duly completed proposals for Election. All applications for Rules, Forms and Admission, &c., must be made through the Secretary, the Institution of Civil Engineers, Great George Street, Westminster, London, S.W.

## Appendix B.

*See Appendix D for Practical Electrical Units*

## LECTURE II.—ORDINARY QUESTIONS.

1. In a shale mine in order to drain one of the pits a treble ram pump, driven by an electric motor, is employed. The rams are  $9\frac{1}{2}$  inches in diameter by 12-inch stroke, they each make  $34\cdot75$  strokes per minute, and the height to which the water is lifted is 393 feet.

Find :

- How many gallons of water this pump can lift per minute.
- How many foot-pounds of useful work are done per minute.
- The useful horse-power when the pumps are running steadily.

(B. of E., 1906.)

2. A windmill is employed to drive a pump which has to lift water from a well and deliver it into an overhead tank. It was found that when the windmill works steadily under the action of a uniform wind for a period of 1 hour, 5000 gallons of water are raised from the well and delivered into the tank—the average height of lift is 60 feet. What under these conditions is the useful horse-power of the windmill? (B. of E., 1907.)

3. An electrical hoist is employed in raising coal from the hold of a ship and delivering it into railway cars, the amount of lift being 125 feet. If the coal is raised at the rate of 2400 lbs. per minute, what is the useful horse-power?

Convert this into watts.

If the current is supplied at a voltage of 250, and if the efficiency of the whole arrangement is 50 per cent., how many amperes of current must be supplied to the motor working the hoist? (B. of E., 1907.)

4. Two closely coiled spiral springs were made out of round steel wire,  $\frac{1}{4}$ -inch diameter. The one spring, A, had a mean diameter of coil of 4 inches and the other, B, had a mean diameter of coil of 5 inches; both springs had 12 complete coils. These two springs were tested by loads extending them axially, and the results of the tests are shown in the table below:—

Axial load in pounds } 2 4 6 8 10 12 14 16 18 20										
Extension of the spring A. Inches. } 0·26 0·52 0·79 1·06 1·32 1·59 1·86 2·12 2·39 2·66										
Extension of the spring B. Inches. } 0·51 1·02 1·53 2·04 2·55 3·06 3·57 4·09 4·60 5·12										

Plot the results on squared paper.

Given that the law connecting the extension of these springs with their mean diameter of coil is of the form—

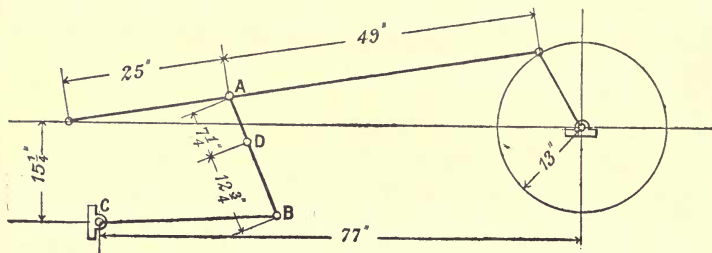
$$\frac{\text{Extension of } B}{\text{Extension of } A} = \left( \frac{\text{Mean diameter of coil of } B}{\text{Mean diameter of coil of } A} \right)^n$$

what is the probable value of  $n$ ?

(B. of E., 1908.)

5. Show how to determine the work done by a variable force moving in its own direction. A cage weighing 1200 lbs, is raised 300 ft, by a windlass having a wire rope weighing  $1\frac{1}{2}$  lbs. per foot run. Show, by a diagram to scale, the work done at any stage, and mark on it the numerical values for a lift of 100, 200, and 300 feet respectively. (C. & G., 1909, O., Sec. A.)

6. An engine cylinder, fitted with a Joy valve gear, has a stroke of 26 inches, and a connecting-rod 74 inches long. In the accompanying



LINE DIAGRAMS OF PROBLEM ON JOY'S VALVE GEAR.

sketch the link AB is pivoted to the connecting-rod at A, and to the free end B of a swinging link BC centred at C. The motion for operating the valve is taken from a point D on the link AB. Draw the path of the point D for a complete revolution of the crank. Take a scale of 1 in. = 1 ft.

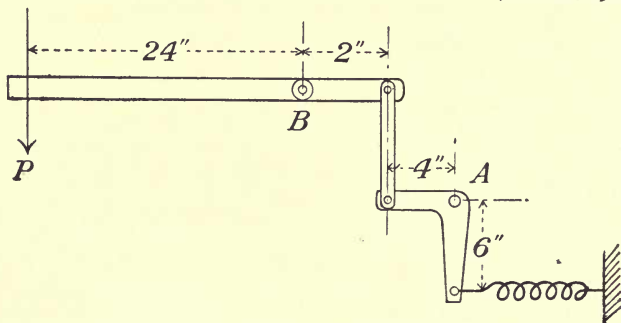
(C. & G., 1909, O., Sec. A.)

#### LECTURE IV.—ORDINARY QUESTIONS.

1. The right-angled bell crank lever, centred at A, shown in the sketch is attached to a spring by one of its arms, and to another lever, centred at B, by the other arm.

If the spring requires a direct pull of 20 lbs. in order to stretch it 2 inches, find what force P, applied as shown, will stretch the spring this amount.

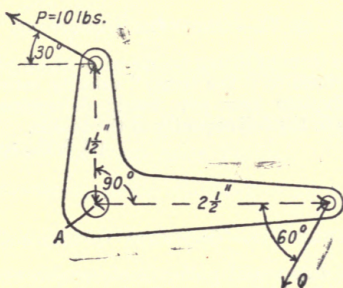
(B. of E., 1906.)



BELL CRANK LEVER AND SPIRAL SPRING.

2. An ordinary bell-pull, shown in the sketch, is in equilibrium. Determine in any way you please the magnitude of the force  $Q$  and the magnitude and direction of the resultant thrust upon the supporting pin  $A$ .

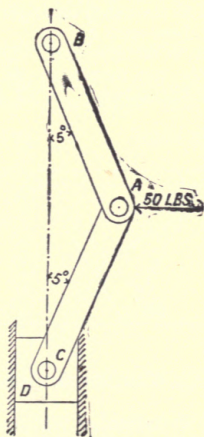
(B. of E., 1907.)



ORDINARY BELL-PULL LEVER.

3. The figure shows the mechanism known as a knuckle joint. A force of 50 lbs. is applied at the point  $A$ , its line of action being perpendicular to the line  $BC$ . Determine graphically, or in any other way, the vertical thrust delivered by the block  $D$ . Neglect friction.

How will this thrust vary as the block  $D$  descends? (B. of E., 1908.)



TOGGLE OR KNUCKLE JOINT.

## LECTURE V.—ORDINARY QUESTIONS.

1. Describe how you would determine experimentally the coefficient of sliding friction between two pieces of metal of any convenient size when the speed of rubbing is low. (B. of E., 1906.)

## LECTURE VI.—ORDINARY QUESTIONS.

1. Describe any one form of lifting tackle with which you are acquainted and explain with reference to it the terms "velocity ratio," "force ratio," and "efficiency." Explain how you would determine their numerical values for all loads up to the full capacity of the tackle. (C. & G., 1908, O., Sec. A.)

## LECTURE VII.—ORDINARY QUESTIONS.

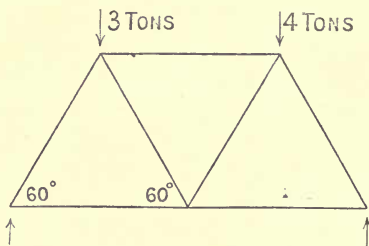
1. The sheave of a differential pulley block consists of two parts which have diameters of 8 and 9 inches respectively. What is the velocity ratio of the mechanism when a load is being raised?

If the mechanical efficiency of the pulley is 32 per cent., what pull must be exerted in order to raise a load of 2 tons? (B. of E., S. 1, 1909.)

## LECTURE VIII.—ORDINARY QUESTIONS.

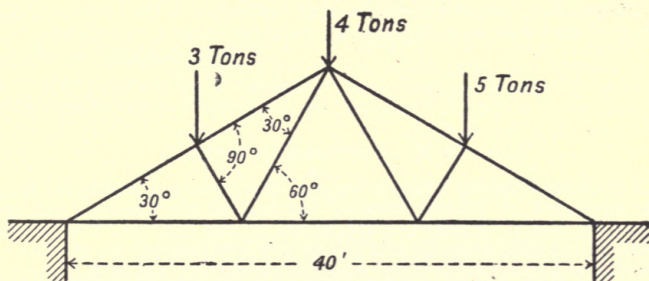
1. The weight of a span of telegraph wire is 12.7 lbs. At one end the wire makes an angle of  $5^\circ$  and at the other an angle of  $7^\circ$  with the horizontal, what are the pulling forces at these ends? (B. of E., S. 1, 1909.)

2. A simple Warren girder is as sketched. Loads of 3 and 4 tons are carried at the two joints of the top member. Find, analytically or otherwise, the forces in the different members. (C. & G., 1906, O., Sec. B.)



SIMPLE WARREN GIRDER.

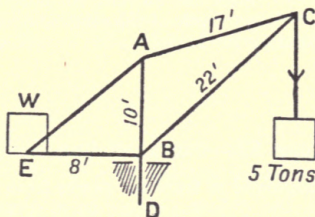
3. Explain one method of determining the stresses in the members of a pin-jointed frame.



LOADED ROOF TRUSS.

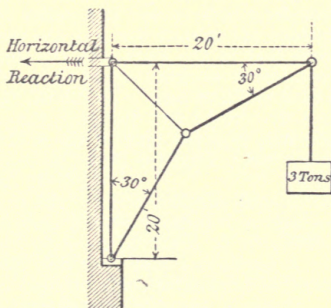
Determine the stresses in all the members of the roof truss loaded, as shown in the figure. (C. & G., 1908, O., Sec. B.)

4. A simple crane is of the form shown in the diagram, and carries 5 tons at C. Determine the stress in AB, AC, and BC, and find the magnitude of the balance weight W so that there shall be no bending moment on the post BD. (C. & G., 1908, O., Sec. D.)



LINE DIAGRAM OF A SIMPLE CRANE.

5. A crane of the form shown by the accompanying sketch carries a load of 3 tons. The reaction at the upper bearing is horizontal. Determine the stresses in the members.



LINE DIAGRAM OF A WALL CRANE.

mine the resultant pressure on the footstep bearing, and the stresses in the members of the crane, assuming that all the connections are pin-joints.

(C. & G., 1909, O., Sec. D.)

#### LECTURE X.—ORDINARY QUESTIONS.

1. A machine weighing 8 tons is dragged slowly along a horizontal floor. If the coefficient of friction between the base of the machine and the floor is 0.35, find in pounds the magnitude of the pull, and the normal pressure on the floor when (a) the line of pull is horizontal, (b) the line of pull makes an upward angle of  $30^\circ$  with the horizontal. (B. of E., 1907.)

2. The length of a journal is 9 inches, and diameter 6 inches, and it carries a load of 3 tons. What horse-power is absorbed when making 100 revolutions per minute, taking the coefficient of friction as 0.15, and how many thermal units are radiated away per minute when the temperature of the bearing remains constant? (C. & G., 1907, O., Sec. A.)

3. An electric locomotive draws a train of 700 tons up an incline of 1 in 100 at a steady speed of 10 miles per hour. If the frictional resistances are equal to 15 lbs. per ton, what is the total pull exerted on the train and what is the horse-power? Find the current consumption in amperes, if the voltage is 625, and if 60 per cent. of the electric energy supplied to the locomotive is spent in hauling the load. (B. of E., S. I., 1909.)

#### LECTURE XI.—ORDINARY QUESTIONS.

1. Hemp ropes are employed to transmit power from the engine shaft to the driving pulleys on the different floors in a spinning factory. The maximum tension in a rope is twice the minimum tension, the breaking strength of one rope is 5700 lbs., and it is desired to have a factor of safety of 30.

Find the maximum horse-power which can be safely transmitted by one of these hemp ropes at a speed of 70 feet per second. (B. of E., 1906.)

2. It is required to transfer 3 horse-power to a pulley 16 inches diameter by belting. The revolutions per minute are 100, the tension in the tight side of the belt is  $1\frac{1}{2}$  times that in the slack side, the thickness of the belt is  $\frac{7}{16}$  inches, and the maximum working stress allowable is 320 lbs. per square inch. Find the least width of belt. (C. & G., 1906, O., Sec. A.)

3. In a rope brake on a fly-wheel 8 feet diameter, the ropes being 1 inch diameter the load is 500 lbs., and the pull on the spring balance varies from 10 to 20 lbs. during a test. Find the brake horse-power, the revolutions being 105 per minute. (C. & G., 1907, O., Sec. A.)

4. Two shafts, which are not parallel, and do not intersect, are to be connected by a belt passing over suitably placed pulleys. Explain what are the necessary conditions to be observed in order that the belt shall remain on the pulleys. A horizontal shaft, running along one side of a machine shop, drives another horizontal shaft at right angles to the first shaft and 20 ft. below. Sketch a suitable arrangement for the belt-drive if both shafts are to revolve at the same speed.

(C. & G., 1908, O., Sec. A.)

## LECTURE XII.—ORDINARY QUESTIONS.

1. An engine having a stroke of 12 inches, and a connecting-rod 24 inches long, centre to centre, makes 300 revolutions per minute. Find graphically the velocity of the piston at six intermediate positions of the stroke, and draw a curve showing the velocity of the piston at any instant.  
(C. & G., 1908, O., Sec. A.)

## LECTURE XIII.—ORDINARY QUESTIONS.

1. A crane, tested in the usual way, and in which the velocity ratio is 40, gave the following results :

Weight lifted (W)	100	300	500	700
Force applied (P)	8.5	17.0	25.6	34.2

Plot a curve showing the relation between P and W on a W base, and, on the same base, plot a curve of efficiency.  
(C. & G., 1906, O., Sec. A.)

## LECTURE XV.—ORDINARY QUESTIONS.

1. Taking the mean diameter of the thread of a 1-inch bolt to be 0.92 inches, the number of the threads to the inch being 8, and the coefficient of friction 0.17; find the turning couple required to overcome an axial force of  $2\frac{1}{2}$  tons, and the efficiency under this load.  
(C. & G., 1906, O., Sec. A.)

## LECTURE XVI.—ORDINARY QUESTIONS.

1. To do the cutting work in a small screw cutting lathe it is found that 0.47 H.-P. is required, and that the frictional losses in the gearing, bearings, &c., absorb another 0.21 H.-P. How many foot-pounds of work per minute is the driving-belt giving to the lathe ?

The countershaft is driven by an electric motor, and the countershaft and belts absorb 0.17 H.-P. How many watts must the motor give off in order to keep the lathe running ?

If the voltage is 220, how many amperes will the motor require, assuming that its own efficiency is 89 per cent. ?

1 H.-P. = 746 watts, and amperes multiplied by volts = watts.

(B. of E., 1906.)

2. The table of a drilling machine is raised by a hand-wheel, to the spindle of which is attached a single-threaded worm which meshes with a worm-wheel having 40 teeth. Compound with the worm-wheel is a spur-pinion, having 19 teeth of 1-inch pitch, which meshes with a rack on the frame of the machine. Sketch the arrangement and find how many turns of the handle are required to raise the table through 2 feet.

(C. & G., 1906, O., Sec. A.)

3. In the feed gear of a drilling machine, in which a rack is used to give the traverse of the spindle, the spindle is rotated by a bevel-wheel of 18 teeth keyed on the driving shaft, gearing with one of 32 teeth on the sleeve surrounding the spindle. The greatest and least diameters of the pulleys on the speed cone for the driving shaft are 7 inches and 4 inches, and this cone drives a similar speed cone on the horizontal speed shaft. On this shaft is a single-threaded worm which gears with a worm-wheel of 45 teeth on the vertical feed shaft. A single-threaded worm on this shaft gears with a wheel of 30 teeth, turning on a horizontal stud, and to which is attached a pinion of 15 teeth gearing with a rack of  $\frac{1}{2}$ -inch pitch, which gives the required feed. Find the least and greatest number of revolutions of the drill spindle per inch of feed.

(C. & G., 1906, O., Sec. A.)

4. The traverse shaft of a lathe is driven from the headstock mandrel by belting, the greatest diameter of the speed cone at the extremity of the mandrel being 5 inches. This drives a similar cone on the transverse shaft, and the smallest diameter is 2 inches. A worm on the traverse shaft meshes with a single-threaded worm-wheel, having 40 teeth, turning on a stud carried by the saddle. At the front end of this spindle is a spur-wheel of 15 teeth, meshing with a wheel of 45 teeth, which turns on a stud carried by the apron; and compound with this last wheel is a pinion of 12 teeth, which meshes with the rack of  $\frac{1}{2}$ -inch pitch, attached to the lathe bed. Sketch the mechanism and find the traverse of the saddle per revolution of the headstock mandrel.

(C. & G., 1907, O., Sec. A.)

5. A lathe is driven by a belt running on the 12-inch diameter pulley of its speed cone, which then revolves at 200 revolutions per minute, and the back gear of the lathe reduces this speed in the ratio of 9 to 1. Under these working conditions it is found that when a certain cut is being taken off a bar, 6 inches diameter, the horse-power transmitted by the belt is 0.60. What is the pressure on the cutting tool in a direction tangential to the turned surface, if we assume that 75 per cent. of the power transmitted through the belt is lost in frictional and other wasteful resistances?

(B. of E., 1908.)

6. Show how screws, differing in pitch from the leading screw, can be cut in a lathe. If the leading screw of a lathe has three threads per inch, and is right-handed, what arrangements of change would you use to cut (i) a right-handed screw of four threads to the inch, (ii) a left-handed screw of eleven threads to the inch? You may assume that you have a set of change wheels with teeth varying from 20 to 100 by differences of five teeth.

(C. & G., 1908, O., Sec. A.)

7. Describe, with the help of neatly-drawn sketches which should be roughly to scale, any form of loose head-stock or poppet-head for a small lathe, with which you have had practical experience. Show how the spindle or poppet is advanced or withdrawn, and how it is clamped.

(B. of E., S. 1., 1909.)

8. A small machine tool is driven direct by an electric motor. How would you determine the horse-power absorbed in the process of cutting the material?

(B. of E., S. 1., 1909.)

9. The countershaft of a drilling machine makes 240 revolutions per minute, and it carries a stepped pulley, the diameters of which are 12, 9 and 6 inches respectively. This drives an intermediate shaft by a belt and pulley with similar steps. The intermediate shaft drives the drill spindle by a bevel wheel of 30 teeth gearing with one of 40 teeth on the drill spindle. Calculate the possible speeds of the drill spindle, and also determine the diameter of the largest drill you can use if the circumferential cutting speed is limited to 240 inches per minute.

(C. & G., 1909, O., Sec. A.)

### LECTURE XVIII.—ORDINARY QUESTIONS.

1. The following results were obtained during an experiment to determine the quantity of water which would be discharged through a small circular orifice in the side of a tank. The diameter of the orifice, which had sharp edges, was 1 inch.

Number of experiment	Duration of experiment	Actual discharge	Head of water above centre of orifice
	Minutes.	Lbs.	Inches.
1	15	576	1'5
2	15	660	2'0
3	15	733	2'5
4	15	827	3'27
5	15	915	4'01
6	15	1,011	5'0
7	10	737	6'0
8	10	788	7'0

Plot on squared paper a curve to show the relation between the discharge in lbs. per minute, and the head of water above the centre of the orifice.

From your curve determine the discharge in gallons per hour when the head of water was  $5\frac{1}{2}$  inches.

(B. of E., 1907.)

2. A straight balk of timber is 20 feet long and 12 inches square in cross-section: its weight per cubic foot is 43'5 lbs. If a weight of 112 lbs. is placed on the centre of the balk when it is floating in water, find the depth to which the balk will be immersed.

(B. of E., S. 1, 1909.)

### LECTURE XIX.—ORDINARY QUESTIONS.

1. The rim of a turbine is going at 50 feet per second; 100 lbs. of fluid enter the rim each second, with a velocity in the direction of the rim's motion of 60 feet per second, leaving it with no velocity in the direction of the wheel's motion. What is the momentum lost per second by the fluid? This is force. What work is done per second upon the wheel?

(B. of E., 1907.)

2. A hydraulic press has a ram 6 inches in diameter: water is supplied to the press from a single-acting pump, which has a plunger 1 inch in diameter with a stroke of  $1\frac{1}{2}$  inches. Neglecting frictional and other losses in the pump and press, find the average rate (in foot-pounds per minute) at which the pump works, if it makes 100 working strokes per minute, while the press is exerting a force of 70 tons. (B. of E., 1907.)

3. A centrifugal pump, driven by an electric motor directly coupled to it, is found during a test to deliver 320 gallons of water per minute into an overhead tank, the mean height of lift being 65 feet. What useful horse-power is the motor doing?

If during the test the electric motor takes 35 amperes of current at 440 volts, what is the combined efficiency of the whole plant?

(B. of E., 1908.)

4. A turbine, which gives off 50 horse-power to a belt running on a pulley on its shaft, is supplied with water which, as it enters the turbine, is under a head of 125 feet. If 75 per cent. of the total energy of the entering water is thus utilised, what work is done per pound of water, and how many gallons of water pass through the turbine per working day of 10 hours?

(B. of E., S. I., 1909.)

#### LECTURE XX.—ORDINARY QUESTIONS.

1. Describe, with the help of neatly drawn sketches, which should be roughly to scale, a hydraulic jack, showing clearly all valves.

(B. of E., 1908.)

2. In a hydraulic crane, with a ram 8 inches diameter, the water pressure is 800 lbs. per square inch, and the velocity of lift is increased eight-fold by the use of a four-sheaved pulley block. What load can this crane lift if its mechanical efficiency is 40 per cent.?

How many gallons of power water will be used in lifting the load 50 feet?

(B. of E., 1908.)

#### LECTURE XXI.—ORDINARY QUESTIONS.

1. A cycle track is approximately elliptical in shape, the maximum radius of curvature being 150 yards and the minimum 50 yards.

Find at each of these two places, the ratio which the centrifugal force bears to the weight, if the speed of the racing cyclist is 25 miles per hour. What would be the two inclinations of the track to the horizontal if the track is laid so as to be perpendicular to the resultant force in each case?

(B. of E., 1906.)

2. A railway truck weighing 10 tons starts from rest down an incline  $\frac{1}{2}$  mile long of 1 in 250. If the frictional and other resistances are equivalent to 8 lbs. per ton weight of the truck, with what velocity will the truck be moving when it gets to the end of the incline?

How far would it then run along a level stretch of the line before coming to rest?

(B. of E., 1906.)

3. Two adjacent positions,  $G_1$ ,  $G_2$  of the centre of mass  $G$  of a balance weight were obtained by geometrical construction from a skeleton diagram of the mechanism. These positions, measured in feet from two perpendicular axes, were found to be as follows :

	$x$	$y$
$G_1$	0.167	0.078
$G_2$	0.352	0.146

The displacement  $G_1$   $G_2$  took place in  $1/50$  second. Find the  $x$  and  $y$  components of the mean velocity of  $G$  for this interval.

Plot the points  $G_1$ ,  $G_2$ , on squared paper. (B. of E., 1906.)

4. A traction engine travels at 6 miles per hour; the road wheels are 6 feet in diameter and are driven through 5 to 1 gearing. Find the angular velocity in radians per second of the fly-wheel on the engine shaft.

(B. of E., 1907.)

5. The rim of a cast-iron pulley has a mean radius of 12 inches; the rim is 6 inches broad, and  $\frac{1}{2}$  inch thick, and the pulley revolves at the rate of 150 revolutions per minute; what is the centrifugal force on the pulley rim per inch length of rim?

One cubic inch of cast-iron weighs 0.26 lb. (B. of E., 1907.)

7. With an automatic vacuum brake a train, weighing 170 tons and going at 60 miles an hour on a down gradient of 1 in 100, was pulled up in a distance of 596 yards. Find the total resistance per ton in pounds, and the time taken to stop the train. (C. & G., 1907, O., Sec. A.)

8. A horizontal jet of water issues at a velocity of 20 feet per second from the 2-inch diameter nozzle of a hose pipe, and strikes a vertical wall. What is the mass of water, in engineers' units, which strikes the wall per second? What is the momentum of this quantity of water? What is the force on the wall? It is assumed that no water splashes back.

(B. of E., 1908.)

9. A man, whose weight is 14 stone, stands on the floor of a lift. What force does he exert on it (i) when the lift is stationary, (ii) when it is descending with an acceleration of 10 feet per second per second, and (iii) when it is ascending with the same acceleration?

(B. of E. 1908.)

10. A train is running round a circular curve of 2000 feet radius at a speed of 50 miles per hour. A weight is suspended by a thin cord from the roof of one of the carriages; at what inclination to the vertical will the cord hang?

(B. of E., 1908.)

11. The fly-wheel of a punching machine weighs  $1\frac{1}{2}$  tons, and has a radius of gyration of 3 feet. It is turning at the rate of 130 revolutions per minute when the punching of a hole is started, but at the completion of the operation of punching it is found to be turning at the rate of only 125 revolutions per minute. How many foot-pounds of work have been expended in punching the hole and overcoming the frictional resistances of the machine?

*Note.*—If  $k$  is radius of gyration, this means that we may imagine the mass of the whole wheel to be at the distance  $k$  from the axis, and this enables us to calculate its kinetic energy.

(B. of E., 1908.)

12. Explain how to determine the velocity of a moving body by considering the space described in a given time. A motor car, starting from rest, travels a distance of  $s$  feet in  $t$  seconds, in accordance with the following table :

$t$	0	1	2	3	4	5	6	7	8
$s$	0	5	16	33	56	85	120	161	208

Draw a curve showing the distance travelled at any time within this period, and from this curve determine the velocity of the car at the ends of the third, fourth, and fifth seconds respectively. (C. & G., 1908, O., Sec. A.)

13. A pulley, 3 feet in diameter, has a peripheral speed of 2000 feet per minute. It is unbalanced to an amount which may be represented by a mass of 0.5 lbs. at a radius of 1 foot. Calculate the unbalanced force on the pulley-shaft, and determine the positions at the pulley-rim of two masses of 0.4 lbs. to give a perfect balance. (C. & G., 1908, O., Sec. A.)

14. An experiment with a small Pelton water-wheel gave results shown in the annexed table :

Mean Revs. per Min.	Cubic Feet of Water passing through Wheel per sec.	Speed of Jet. Feet per sec. $v$ .	Peripheral Speed of Vane. Feet per Sec. $V$ .	Ratio. $\frac{v}{V}$	Efficiency of Wheel. Per Cent.
1090	0.870				44.0
975	0.824				63.8
885	0.834				72.4
645	0.840				72.8
540	0.840				66.8
460	0.847				61.5
385	0.834				56.1
265	0.860				40.1

The cross-sectional area of the nozzle in square feet was 0.001043. The mean diameter of the bucket was 10.7 inches.

Fill in the third, fourth, and fifth columns of the table.

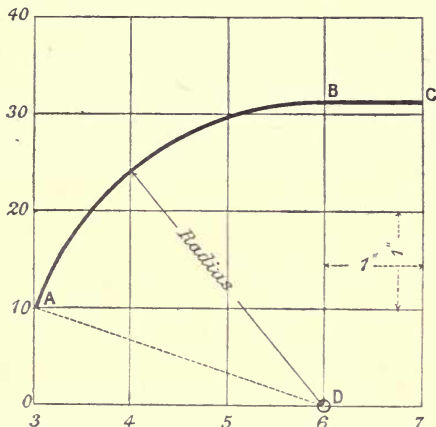
Plot a curve to show the variation of efficiency with variation of ratio

$\frac{v}{V}$ , taking efficiency in vertical ordinates and  $\frac{v}{V}$  in horizontal abscissæ.

(B. of E., S. 1, 1909.)

15. Define the term "horse-power." The tension on the draw-bar of a locomotive is 800 lbs. when the speed is 45 miles per hour. The weight of the train, excluding the locomotive, is 200 tons. If the efficiency of the locomotive is 65%, find the horse-power of the engine. Also find the accelerating force exerted at the draw-bar to change the speed from 45 to 50 miles per hour in one minute. (C. & G., 1909, O., Sec. A.)

16. Define the terms "velocity" and "acceleration," and show how to determine the acceleration of a body moving in a straight line when its velocity is known at each instant. The accompanying diagram is to be



CURVE REPRESENTING THE VELOCITY OF BODY.

drawn so that each unit square is of 1 inch side. The curve ABC represents the velocity of a body during a given interval of time, it is circular from A to B, with centre at D, and from B to C it is a straight line parallel to the horizontal axis. Draw to scale a curve showing the acceleration of the body at any instant, and mark on it the numerical values of the acceleration at the end of each second. (C. & G., 1909, O., Sec. A.)

17. Show how the resultant velocity of a body may be obtained when the component velocities impressed upon it are known. A goods engine moving at the rate of 20 miles per hour, has equal coupled wheels 54 inches in diameter, and the crank-pins move in circles 24 inches in diameter. Determine the velocities of the crank-pins relative to the rails for three equally spaced positions reckoned from the lowest positions of the crank-pins. (C. & G., 1909, O., Sec. A.)

18. Assuming the expression for the law of centrifugal force, show how to obtain a formula for the stress  $f$  in the rim of a fly-wheel in the form

$$f = \frac{wv^2}{g}$$

where  $w$  is the weight of a unit volume of the rim,  $v$  is the velocity, and  $g$  is the gravitation constant. Determine the limiting speed in revolutions per minute of a cast-iron fly-wheel rim having a mean diameter of 10 feet, when the allowable stress is 2400 lbs. per square inch. The weight of one cubic inch of cast-iron may be taken as 0.28 lbs.

(C. & G., 1909, O., Sec. A.)

19. A hammer head weighing 3·22 lbs. moving at 30 feet per second is stopped in 0·001 second : what is the average force of this blow in pounds ?

(B. of E., S. 1, 1909.)

20. A locomotive is travelling at 60 miles per hour. The driving-wheels are 6 feet 6 inches in diameter. What is the angular velocity of the driving-wheels in radians per second ?

If the stroke of the piston is 26 inches, what is the mean speed of the piston relatively to the cylinder in feet per minute, assuming there is no slip of the wheels ?

(B. of E., S. 1, 1909.)

21. A fly-wheel, which weighs 18 tons, when mounted on its axis and rotated, is found to be out of balance, and, in order to bring it into balance it is found necessary to fix a counter-weight of 420 lbs. to the wheel, at a distance of 90 inches from the axis of the shaft. What was the distance of the centre of gravity of the unbalanced wheel from the axis of the shaft ?

Show by a sketch where you would fix the counter-weight.

(B. of E., S. 1, 1909.)

22. A fly-wheel weighs 8 tons, its radius of gyration is 5 feet 5 inches, and it is rotating at a speed of 90 revolutions per minute. How many foot-pounds of energy are stored up in it ?

If this wheel were supported in two bearings, each 12 inches in diameter, and if the coefficient of friction were 0·01, how much energy is wasted in overcoming friction in one revolution, and how many revolutions would this fly-wheel make before coming to rest after the turning force was cut off ?

(B. of E., S. 1, 1909.)

## LECTURE XXII.—ORDINARY QUESTIONS.

1. A tie-bar in a roof is made of steel angle bar ; the section of the steel angle bar is 4 inches by 4 inches by  $\frac{5}{8}$  inch, and the tie-bar when finished in the workshop is 20 feet in length. When in position in the roof the tie-bar may during a gale have to resist a total pull of 22½ tons ; what is the tensile stress per square inch in the metal of the tie-bar under these conditions, and how much would the tie-bar lengthen under this load ?

Young's modulus of elasticity is 12,500 tons per square inch.

(B. of E., 1906.)

2. A knuckle joint is required to withstand a tensile force of 10 tons. The safe working stress, both in tension and shear, may be taken as 9000 lbs. per square inch. Find the diameters of the rod and pin, and sketch the joint, roughly, to scale.

(C. & G., 1906, O., Sec. B.)

3. A rectangular test-bar, in tension, gave the following results :

Total load in lbs. }	8,000	16,000	24,000	32,000	34,000	40,000	48,000	56,000	60,000	55,000
Extension in inches. }	·002	·0044	·0070	·0103	·016	·190	·470	1·36	2·5	2·9

Sketch a curve showing the relation between force and extension on any suitable scale—squared paper may be used—and infer the stress at the

elastic limit and the maximum stress, the original dimensions of the bar being 1.763 inches by .611 inches. If the distance between the gauge points is 10 inches, find the coefficient of elasticity ( $E$ ) of the bar.

(C. & G., 1906, O., Sec. B.)

4. Explain what you mean by the efficiency of a riveted joint, and point out on what it depends. In a marine boiler the diameter is 12 feet, the working pressure is 200 lbs. per square inch, and the longitudinal joints are butt joints with double straps treble riveted. If the ultimate stress is 62,000 lbs. per sq. inch, the factor of safety 5, and the efficiency of the joint .8, find the thickness of the plate required, and make a rough sketch of the joint.

(C. & G., 1906, O., Sec. B.)

5. In order to connect together the two halves of a long tie-rod, an eye is forged at the end of one half, and a fork (into which the eye enters at the end of the other half, and a pin is passed through the two sides of the fork and through the eye. If the total pull in the tie-rod is 16 tons, and if the shearing stress in the metal of the bolt is not to exceed 8000 lbs. per square inch, what diameter would you make the pin?

(B. of E., 1907.)

6. A strut is built up out of two pieces of T-steel, each 6 inches by 3 inches by  $\frac{3}{8}$  inch, riveted back to back. If this strut supports a load of 22.3 tons, what is the compressive stress per square inch?

If a total load of 105 tons would destroy this strut, what is the factor of safety?

(B. of E., 1907.)

7. A bar, of rectangular section, 1.75 inches wide and 0.61 inches thick, is found under a load of 20,000 lbs. to have stretched 0.0056 inch. Find the stress induced, and, if the length be 10 inches, find Young's modulus.

(C. & G., 1907, O., Sec. B.)

8. Find the thickness of the plates of a cylindrical boiler 50 inches in diameter to sustain a pressure of 50 lbs. per square inch, the working stress being 4000 lbs. per sq. inch and the efficiency of the joint being 0.60.

(C. & G., 1907, O., Sec. B.)

9. If in the last question the joint is a lap joint double riveted, and the diameter of the rivets is  $\frac{3}{4}$  inch, find the pitch, the shear stress of rivets, being 4000 lbs. per square inch.

(C. & G., 1907, O., Sec. B.)

10. A piece of steel is to be tested in tension; show how you would proceed to make a test, and indicate, by means of a diagram, how the force and extension vary with each other.

(C. & G., 1907, O., Sec. B.)

11. A copper trolley wire, which is 0.45 inch in diameter and 60 feet in length, is found to elongate 0.075 inch under a certain pull. If the modulus of elasticity (Young's modulus) of this quality of copper is known to be 15,000,000 lbs. per square inch, what is the total pull in the trolley wire?

(B. of E., 1908.)

12. Explain the meanings of the terms "stress," "strain," and "modulus of elasticity," by reference to the case of a rod under tensional stress. A piece of boiler plate, 2 inches by  $\frac{3}{8}$  inch in cross-section, has a load of 12,000 lbs. applied to it in a testing machine. The modulus of elasticity of the material expressed in inches and pounds is 31,000,000. Calculate the values of the stress and strain and determine the increase of length, in a length of 12 inches, due to the applied load.

(C. & G., 1908, O., Sec. B.)

13. Make a sketch of a knuckle joint connecting an eccentric rod to a valve spindle, and assuming that the total load on the latter is 4000 lbs., determine the dimensions of the various parts, and design the joint. Show your calculations clearly, and state what working stresses you have assumed.

(C. & G., 1908, O., Sec. B.)

14. A tension member, 8 inches by  $\frac{3}{4}$  inch in cross section, has a riveted butt-joint with cover plates on each side. The total load on the member is 72,000 lbs. Design and draw a joint for this member, and show all your calculations. (C. & G., 1908, O., Sec. B.)

15. A hollow cylinder, 10 inches mean diameter, 10 feet long, and  $1\frac{1}{2}$  inches thick, is to be cast with its axis vertical. Taking the specific gravity of cast iron as 7.5, find the pressure on the bottom of the mould when it is full of metal. One side of a mould for a cast-iron casting is a rectangle, 3 feet deep by 2 feet wide. Find the whole pressure on the side of the mould. (C. & G., 1908, O., Sec. D.)

16. In an experiment with a hollow cast-iron column, 6 feet long, 5 inches in external diameter, and 4 inches in internal diameter, it was found that under a compressive load of 30 tons the column shortened by 0.063 inch; what is the value of Young's Modulus (E) in pounds per square inch for this cast-iron?

When the load was increased to 192 tons, the column broke; what was the compressive stress in tons per square inch at the instant of fracture.

(B. of E., S. 1, 1909.)

### LECTURE XXIII. ORDINARY QUESTIONS.

1. Describe how you would determine experimentally the modulus of rigidity of *either* a block of india-rubber *or* a steel rod. (B. of E., 1906.)

2. In a direct-acting steam-engine mechanism the stroke of the piston is 2 feet and the crank shaft makes 150 revolutions per minute.

What is the speed of the crank shaft in radians per second? What is the speed of the crank pin in feet per second? What is the mean speed of the piston in feet per minute? (B. of E., 1906.)

3. A solid cylindrical shaft is 5 inches in diameter. Find the external diameter of a hollow shaft of same material, the internal diameter of which is two-thirds the external and which shall have the same strength. Compare the weights in the two cases. If the safe working stress be 4 tons per square inch, and the revolutions per minute 100, find the greatest horse-power which can be safely transmitted. (C. & G., 1906, O., Sec. B.)

4. A motor having a turning moment T is coupled directly to a shaft making N revolutions per minute. Show how to calculate the work transmitted by the shaft, and obtain a formula for the horse-power transmitted in terms of N and T and a constant. Determine the horse-power transmitted by a shaft making 800 revolutions per minute if the turning moment is 16,000, measured in pounds and inches. (C. & G., 1908, O., Sec. B.)

5. A chain is to be used for lifting a load of 5 tons. Assuming a safe working stress on the chain of 4 tons per square inch, find the diameter of the iron of the chain. (C. & G., 1908, O., Sec. D.)

6. A shaft 3 inches in diameter transmits a twisting moment of 66,000-lb. inches and the flange couplings are bolted together by four bolts spaced on a circle of 5 inches diameter. Determine the nature and amount of stress on each bolt and determine its diameter if the allowable stress is 12,000 lbs. per square inch. (C. & G., 1909, O., Sec. B.)

7. Explain what kind of stress is produced in a shaft by a twisting

moment, and make a diagram showing how the stress varies across the section of a shaft. A piece of tubing, 2 inches in external diameter, and  $\frac{1}{8}$  inch thick, is used as a shaft. Assuming that the stress upon it is uniformly distributed, determine the twisting moment it will transmit if the allowable stress is 12,000 lbs. per square inch. (C. & G., 1909, O., Sec. B.)

## LECTURE XXIV.—ORDINARY QUESTIONS.

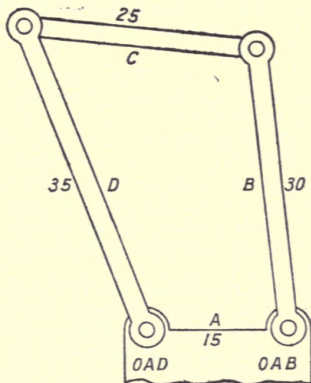
1. Sketch a single Hooke's joint, and explain in general terms, how the angular velocity ratio varies during a revolution. What is the object of a double Hooke's joint? (C. & G., 1906, O., Sec. A.)

## LECTURE XXV.—ORDINARY QUESTIONS.

1. Sketch, and describe the action of, the pin and slot mechanism as applied to shaping machines.

In such a mechanism, the distance between the two centres of rotation is 3 inches, and the time ratio has to be *two*. If the line of stroke produced pass through the centre of the variably rotating crank, and is perpendicular to the line of centres, find the length of the crank radius, and also of the slotted link, for a stroke of 10 inches. (C. & G., 1906, O., Sec. A.)

2. In the four-bar mechanism shown in the sketch, the bar *A* is a fixed bar; the bars *B* and *D* rotate about the fixed centres *OAB* and *OAD*, and they are coupled together at their outer ends by the bar *C*; the bar *B*



FOUR BAR MECHANISM.

revolves with uniform velocity round its fixed axis *OAB* at 50 revolutions per minute. Find in any way you please the position of the bar *D* when

the bar  $B$  is turned in a clockwise direction through angles of  $30^\circ$ ,  $60^\circ$ , and  $90^\circ$  from the position shown in the sketch.

Prepare a table similar to the one shown, and obtain and enter up the results required to complete the table :

Angle turned through by the bar $B$ .	Angle turned through by the bar $D$ .	Mean angular velocity of the bar $D$ in radians per second during each interval.
$30^\circ$ . . .		
$60^\circ$ . . .		
$90^\circ$ . .		

(B. of E., S. 1, 1907.)

3. Sketch the arrangement in a planing machine in which bevel gears are used, explaining how the motion is reversed, and how a quick-return motion is obtained. Sketch also a "shipper" mechanism in which cams or lugs are used.

(C. & G., 1907, O., Sec. A.)

4. The piston of a vertical engine has a travel of 18 inches, and the connecting-rod is 36 inches in length between centres of bearings. The line of action of the piston cuts the horizontal position of the crank 4 inches from the centre of the crank-shaft. Draw the curve of position of the mid-point of the connecting rod for a complete revolution of the crank. Use a scale of  $\frac{1}{8}$ th.

(C. & G., 1908, O., Sec. A.)

5. Describe, with the help of neatly-drawn sketches which should be roughly to scale, a belt gear for giving a slow cutting speed and a quick return motion suitable for use in a planing machine, the table of which is traversed to and fro by a screw or rack.

(B. of E., S. 1, 1909.)

## LECTURE II.—STUD. I.C.E. EXAM. QUESTIONS.

1. Explain the use of diagrams for determining the results of experiment. Plot the following experimental values of P and W and obtain the relation between them :

P pounds	.	.	.	8	16	24	40
W „	.	.	.	0	200	400	800

(Stud. I. C. E., Feb. 1906.)

2. Show how the work done by a variable force can be represented graphically. Assuming that the resistance of a spiral spring is proportional to its extension, and that a load of 24 lbs. extends the spring 0.25 inch, determine the work done in extending the spring 1 inch.

(Stud. I. C. E., Oct. 1906.)

3. Define the terms “force” and “work.”

A spring is compressed and the relation between the compressive force and the compression is as given below. Find graphically the work done in the process.

Force in lbs.	.	.	.	0	15	35	65	110	170	300
Compression in inches	.	.	.	0	1	2	3	4	5	6

(Stud. I. C. E., Oct. 1907.)

## LECTURE III.—STUD. I.C.E. EXAM. QUESTIONS.

1. A metre rule (weight 50 grams) rests on the edge of a table with 20 centimetres projecting over the edge. On the other end rests a 20-gram weight. How far from the edge of the table may a 500-gram weight be hung before the rod tilts ?

(Stud. I. C. E., Feb. 1905.)

2. Prove that the moment about any axis of three forces in equilibrium is zero, and extend the theorem to any number of coplanar forces in equilibrium.

Determine the tension of the rope wound on a capstan 2 feet in diameter when 10 men, each weighing 12 stone, are pushing horizontally on the capstan-bars 4 feet from the deck at a radial distance of 8 feet, the vertical through a man's centre of gravity overhanging his toes a distance of 1-foot.

(Stud. I. C. E., Oct. 1905.)

3. Prove that the centre of gravity of a triangular plate of uniform thickness is on the line joining an apex to the centre of the opposite side, and at a distance from this apex of two-thirds of the length of this median line. Also show how to determine by experiment the centre of gravity of an irregular plate of uniform thickness.

(Stud. I. C. E., Feb. 1906.)

4. Show how to determine the resultant of two parallel forces. A horizontal bar, 6 feet long, is supported at each end by rings depending from spring balances. Determine the position of the centre of gravity of the bar if the spring balances indicate loads of 40 lbs. and 50 lbs. respectively.

(Stud. I. C. E., Oct. 1906.)

5. Explain what is meant by the moment of a force. Show that the sum of the moments of two forces in a plane with respect to a point in that plane is equal to the moment of their resultant. Also show that the moment of a couple, with respect to any axis at right angles to the plane of the couple is invariable.

(Stud. I. C. E., Oct. 1906.)

6. Show how to determine experimentally the centre of gravity of an irregular body, and in particular explain how you would proceed to determine the centre of gravity of a metal plate shaped to the section of a tram-way rail.

(Stud. I. C. E., Feb. 1907.)

7. What do you understand by the "centre of gravity" of a body?

A balk of timber weighs 800 lbs. One end rests on the ground, and the other on a "V" support placed on a weigh-bridge. The weight recorded is 320 lbs. The weigh-bridge is then moved so that the "V" is 1 foot nearer the end that is resting on the ground and the weigh-bridge registers 360 lbs. Find how far the centre of gravity of the balk is from that end.

(Stud. I. C. E., Oct. 1907.)

8. What is a "couple"? How is a couple specified, and how can it be represented?

A pair of compasses is opened so that the legs are at  $90^\circ$ . Couples are applied to the legs whose moments are respectively 3 lb.-foot units and 4 lb.-foot units, and they twist in opposite ways. Find what couple must be applied at the hinge to equilibrate the two, and the axis of that couple.

(Stud. I. C. E., Oct. 1907.)

9. Define the term "centre of gravity." A cylindrical vessel is 5 feet deep and weighs 100 pounds; when it is empty its centre of gravity is 2 feet above its base. It is gradually filled with water. Plot to scale a curve showing the relation between the depth of water in the vessel and the height of the new centre of gravity, if the vessel when just full can contain 500 pounds of water.

(Stud. I. C. E., Feb. 1908.)

#### LECTURE IV.—STUD. I.C.E. EXAM. QUESTIONS.

1. Answer, giving reasons, the following questions on the balance:

(i) What conditions must be satisfied in order that a balance may be true?

(ii) If the scale-pan knife-edges are above the middle knife-edge show that the sensitiveness of the balance increases with the load.

(iii) Why must the scale-pans be suspended freely from the beam?

(Stud. I. C. E., Feb. 1905.)

2. Explain the action of one form of lever weighing-machine. A 100-ton testing machine, using a single lever for weighing the pull on a test-piece, is arranged so that the line of action of the pull is 4 inches distant from the fulcrum, and this pull is balanced by a weight of 5000 lbs. on the long arm of the lever. Calculate the distance the weight moves from its zero position to balance the full load of 100 tons.

(Stud. I. C. E., Feb. 1907.)

#### LECTURE V.—STUD. I.C.E. EXAM. QUESTIONS.

1. Explain how work is computed when a force moves its point of application in any direction.

Find the work done per minute by a force pulling a body weighing 400 lbs. over a rough plane at the rate of 5 miles an hour, if the coefficient of friction is 0.25.

(Stud. I. C. E., Feb. 1906.)

#### LECTURE VI.—STUD. I.C.E. EXAM. QUESTIONS.

1. Write a short essay on the use and principle of a machine, bringing in the meaning of the terms: velocity or displacement ratio, effort, load, advantage, efficiency.

Describe some experiment you have made with a machine, and illustrate the meaning of the above terms by numerical examples.

(Stud. I. C. E., Feb. 1905.)

2. Give sketches of three systems of pulleys, and state their mechanical advantage.

Explain which system is preferable for a long pull, as in hoisting a weight; and which is to be preferred for a strong pull, as in setting up a backstay.

(Stud. I. C. E., Oct. 1905.)

#### LECTURE VII.—STUD. I.C.E. EXAM. QUESTIONS.

1. Explain the principle of action of the "Weston" or differential pulley, and show how to determine the displacement or velocity ratio. In such a machine the velocity ratio was found to be 20, and in order to lift a weight of 420 lbs. a pull of 40 lbs. was exerted. Determine the efficiency of the machine for this load.

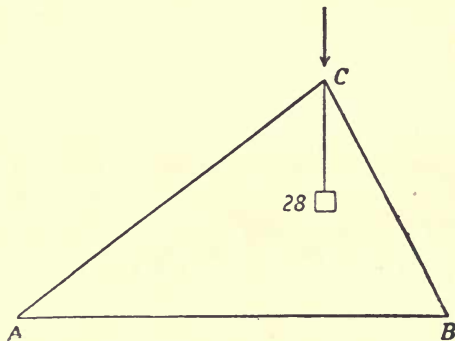
(Stud. I. C. E., Feb. 1906.)

#### LECTURE VIII.—STUD. I.C.E. EXAM. QUESTIONS.

1. A rod AB, the weight of which may be neglected, is hinged at A, and a weight of 14 lbs. is suspended from its middle point. A string is fastened to the end B, and when the system is at rest the rod is inclined at  $30^\circ$  to the horizontal, and the string makes an angle of  $90^\circ$  with the rod. Find the pull along the string.

(Stud. I. C. E., Oct. 1904.)

2. Two rods AC, BC are freely jointed together at C, and a load of 28 lbs. is suspended from C. The two ends A and B are connected by a horizontal string. If the system be placed vertically with A and B on a smooth



TWO RODS AC AND BC WITH STRING AB.

floor, find by a graphic method the thrusts along AC, BC, and the pull of the string, when  $AB = 5$  feet,  $AC = 4$  feet,  $BC = 3$  feet.

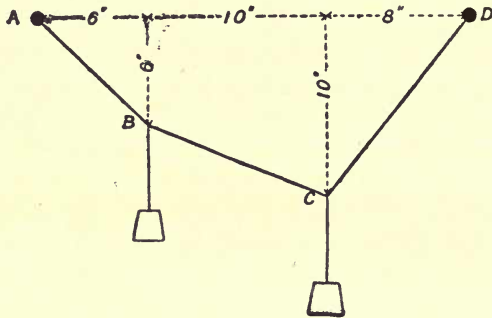
(Stud. I. C. E., Feb. 1905.)

3. Show in a diagram the forces which maintain equilibrium in a book held horizontally between a finger and thumb, and draw a graphical determination of their magnitude.

A boat is propelled by two sculls, each 9 feet long and 6 feet from the rowlock to the blade, and the sculler pulls each hand with a force of 20 lbs. Prove that the thrust on each rowlock is 30 lbs., but the propulsive force on the boat is 20 lbs., and that the boat moves about double as fast as the hands pull.

(Stud. I. C. E., Oct. 1905.)

4. A flexible cord is carried by two pegs A and D in the same horizontal line and 24 inches apart. A weight of 8 lbs. hangs at B and an unknown weight at C, thereby causing the cord to assume the form shown in the figure.



FLEXIBLE CORD WITH ATTACHED WEIGHTS.

Find by a graphical construction the magnitude of the unknown weight and the tensions in the parts AB, BC and CD of the cord.

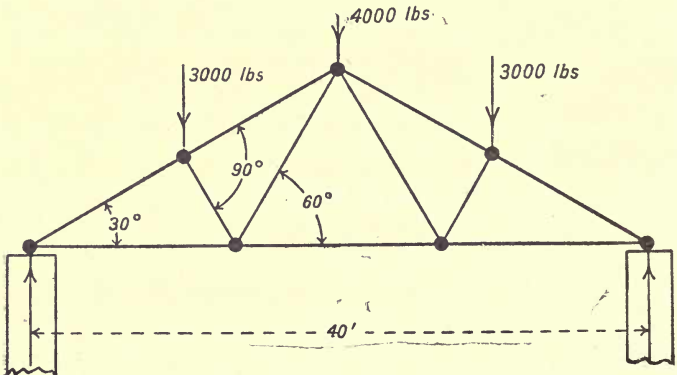
(Stud. I. C. E., Feb. 1906.)

5. Show how to determine the resultant of a number of forces meeting at a point. Six forces acting at a point are parallel to the sides of a regular hexagon taken in order, and their magnitudes are 4, 6, 7, 9, 8 and 3 pounds respectively. Find the resultant, assuming that all the forces are directed towards the point.

(Stud. I. C. E., Oct. 1906.)

6. Obtain the graphical condition for the equilibrium of a number of forces acting at a point. A jointed frame is loaded as shown in the figure. Determine the stresses in the members of the frame.

(Stud. I. C. E., Feb. 1907.)



A JOINTED LOADED FRAME.

7. Give the conditions that forces acting on a rigid body may be in equilibrium.

A uniform plate, weighing 5 lbs., is made in the form of a right-angled area whose sides are 3, 4, and 5 feet. It is hung up by means of a string and a peg so that the 5-foot side is horizontal. The peg is at the angle joining the 4- and 5-foot sides; the string is at the other acute angle and makes an angle of  $60^\circ$  with the 5-foot side. Find graphically the tension in the string. (Stud. I. C. E., Oct. 1907.)

8. What condition must be fulfilled in order that a system of forces acting on a body, which are all in one plane, but not acting at one point, may be in equilibrium. Show how the construction can be used to find the supporting forces required for a structure when loaded.

(Stud. I. C. E., Oct. 1907.)

9. If three forces act on a rigid body, what conditions must be fulfilled in order that equilibrium may be maintained?

A uniform beam of timber weighs 200 pounds. One end rests on the ground, the other has a cord attached to it. This cord is pulled till the beam makes  $30^\circ$  with the horizontal, and then the cord makes  $60^\circ$  with the horizontal. Find (graphically or otherwise) the tension in the cord and the force on the ground.—*Ans.*: Tension in cord = 100 lbs; reaction = 173.2 lbs. (Stud. I. C. E., Feb. 1908.)

#### LECTURE IX.—STUD. I.C.E. EXAM. QUESTIONS.

1. Define horse-power, and prove that a locomotive of  $H$  horse-power can draw a train of  $W$  tons against a resistance of  $r$  lbs. per ton at a speed of  $\frac{375 H}{r W}$  miles an hour.

Calculate the horse-power of a locomotive drawing a train of 200 tons up an incline of 1 in 200 at 50 miles per hour, taking the road and air resistance at this speed as 28 lbs. per ton. (Stud. I. C. E., Oct. 1905.)

2. Define the terms "work" and "power." A car weighing 3 tons is running at 20 miles per hour up a uniform slope of 1 in 50. The frictional resistances are 50 pounds per ton. Find the H.-P., and the work done in 20 minutes.—*Ans.*, H.-P. = 13.8; work done in 20 mins. = 9,100,800 ft. lbs. (Stud. I. C. E., Feb. 1908.)

#### LECTURE X.—STUD. I.C.E. EXAM. QUESTIONS.

1. What is meant by coefficient of friction? Show how the coefficient of friction between wood and wood can be determined.

(Stud. I. C. E., Oct. 1904.)

If a machine, such as a screw-jack, does not overhaul, show that the work done against the friction must be more than 50 per cent. of the total work the man does.

(Stud. I. C. E., Oct. 1904.)

2. Define the coefficient of limiting friction, and prove that it is the tangent of the slope of the incline on which the body is on the point of sliding.

Determine geometrically the greatest slope on which a four-wheeled carriage can be held by the brakes, applied to the hind pair of wheels.

(Stud. I. C. E., Oct. 1905.)

#### LECTURE XI.—STUD. I.C.E. EXAM. QUESTIONS.

1. A belt-pulley has a diameter of 5 feet, and it is delivering 12 H.-P. to a line of shafting. If the pulley make 120 revolutions per minute, find the force along the tight part of the belt. (Stud. I. C. E., Oct. 1904.)

2. How can the brake horse-power of an engine be determined. If the diameter of the brake-pulley is 3 feet, speed 250 revolutions per minute, load 24 lbs., and the spring-balance which takes up the rope at the slack end registers 4 lbs., find the horse-power. (Stud. I. C. E., Feb. 1905.)

3. Explain the meanings of the terms "work" and "energy," and define the practical unit of work used by British engineers.

Find the brake horse-power of an engine making 300 revolutions per minute if the tensions on the tight and slack sides of the brake strap are 72 lbs. and 6 lbs. respectively, and the brake wheel is 5 feet in diameter.

(Stud. I. C. E., Feb. 1907.)

4. Define the term "horse-power."

A pulley whose diameter is 4 feet is making 250 revolutions per minute. A belt is put on the pulley and a weight of 50 lbs. is hung from one end. A spring balance attached to the other end reads 12 lbs. Find the horse-power delivered to the pulley. If the angle of embrace of the belt is  $120^\circ$  find the extra force on the bearing due to the brake.

(Stud. I. C. E., Oct. 1907.)

#### LECTURE XIII.—STUD. I.C.E. EXAM. QUESTIONS.

1. Describe any experiment made by you to determine the efficiency of a machine.

In a test of a hand-crane, with gear having a velocity ratio of 150 : 1, it was found that an effort of 25 lb. at the handles raised a load of 1 ton. Determine the efficiency of the machine for this load.

(Stud. I. C. E., Feb. 1907.)

2. Explain the terms "velocity ratio," "force ratio," and "efficiency," as applied to a machine. A lifting crab has the following relation between the force on the handle in pounds and the weight lifted in tons. Draw curves connecting the force ratio and the efficiency with the load, and find the efficiency of the crab at loads of 5 tons and 15 tons. The velocity ratio is 500.

Force in pounds	.	.	.	30	60	90	120
Load in tons	.	.	.	3'3	7'4	12'5	17'5

Ans. : Efficiencies are 0'558 and 0'6987 respectively.

(Stud. I. C. E., Feb. 1908.)

## LECTURE XVII.—STUD. I.C.E. EXAM. QUESTIONS.

1. Show that the total pressure upon a flat plate immersed in water is proportional to the depth of its centre of figure below the surface. Calculate the pressure per square inch on a horizontal plate at a depth of 10 feet, assuming that a cubic foot of water weighs 62·4 lbs.

(Stud. I. C. E., Oct. 1906.)

2. Show how to find the total pressure on any submerged plane surface.

A sluice gate is 4 feet wide and 6 feet high; the bottom is 20 feet below the surface of the water; find the total pressure on the gate.

[One cubic foot of water weighs 62½ lbs.] (Stud. I. C. E., Oct. 1907.)

## LECTURE XVIII.—STUD. I.C.E. EXAM. QUESTIONS.

1. At the bottom of a barometer there is a bubble of air, of which the density is 0·0013 grams per centimetre, and diameter 0·3 millimetre. If this rises up the tube its volume increases. Explain why. At what height will its volume be doubled?

(Stud. I. C. E., Feb. 1905.)

2. Show how to determine the total pressure and the centre of pressure of a vertical rectangular plate immersed in water with one edge at the surface. Find the total pressure and the depth of the centre of pressure for a plate 12 feet square forming one side of a tank full of water.

(Stud. I. C. E., Feb. 1906.)

3. Define the term "specific gravity," and show how to determine the specific gravity of (i) a solid, (ii) a liquid. (Stud. I. C. E., Oct. 1906.)

4. Explain how you would proceed to demonstrate experimentally that a body floating in water displaces a quantity of liquid equal to itself in weight.

The displacement of a tug-boat in sea-water was found to be 1560 cubic feet. Calculate the weight of the boat, assuming that a cubic foot of sea-water weighs 64 lbs.

(Stud. I. C. E., Feb. 1907.)

5. Define the term "centre of pressure" for a surface immersed in a fluid.

A rectangular sectioned water-channel has a board put vertically across it, which is held up against the water-pressure by two horizontal bars, one at the bottom of the channel, the other 18 inches up the sides. Find the depth of water in the channel which will just upset the board, and the pressure then existing on the board if the breadth of the channel is 5 feet.

(Stud. I. C. E., Feb. 1908.)

## LECTURE XIX.—STUD. I.C.E. EXAM. QUESTIONS.

1. Describe by aid of a sketch the action of an ordinary suction pump for raising water from a well. Calculate the work required to raise 80 gallons of water through a height of 20 feet, if the pump has an efficiency of 64 per cent.

(Stud. I. C. E., Feb. 1907.)

2. Sketch and describe a force pump suitable for raising water from a well.

It is required to raise water through a total height of 80 feet by a pump of 6 inches stroke with a barrel 3 inches in diameter. If 20 strokes per minute are made, find the gallons pumped per hour, and the horse-power required.

(Stud. I. C. E., Oct. 1907.)

3. Describe, with sketches, some kind of double-acting force-pump.

A pump is delivering water into a boiler in which the pressure is 120 lbs. per square inch above atmospheric pressure. Find the work done in foot-pounds per pound of water delivered to the boiler. Find also the horse-power of the pump if it delivers 2,000 gallons per hour and its efficiency is 60 per cent. *Ans.*: Work done per lb. of water delivered = 276 ft.lbs.; and H.-P. of pump = 4.7.

(Stud. I. C. E., Feb. 1908.)

#### LECTURE XX.—STUD. I.C.E. EXAM. QUESTIONS.

1. The pressure of water in a high-pressure main is 700 lbs. weight per square inch. A load of 2 tons is to be lifted by means of a ram driven from the main. Find the sectional area of the ram, and the cubic feet of water used per H.-P.-hour.

(Stud. I. C. E., Oct. 1904.)

#### LECTURE XXI.—STUD. I.C.E. EXAM. QUESTIONS.

1. What is the difference between acceleration and velocity?

Plot to scale the following velocities in terms of the time, and write down the times from the start, when the acceleration is zero; also state the period during which the acceleration is negative. Find by the method of equidistant co-ordinates the average velocity.

Time in Seconds.	Velocity in Feet per Second.	Time in Seconds.	Velocity in Feet per Second.
0	5.0	6	15.8
1	10.0	7	15.5
2	13.0	8	16.7
3	14.8	9	18.3
4	15.5	10	20.3
5	15.8	11	22.5

(Stud. I. C. E., Oct. 1904.)

2. A train of 120 tons is found to increase its speed from 20 miles per hour to 40 miles per hour in ten minutes. If the frictional resistance is 2000 lbs. weight, find the force which must be pulling the train—assuming it to be a constant force. Find average and final horse-power.

(Stud. I. C. E., Oct. 1904.)

3. Water is projected horizontally from a nozzle. If the point at which it strikes the floor is 6 feet below the nozzle and 5 feet from the vertical line drawn through the nozzle, find the velocity with which the water is projected. ( $g = 32.2$  feet per sec. per sec.)

(Stud. I. C. E., Oct. 1904.)

4. A 2-ton fly-wheel drops in speed from 100 revolutions per minute to 90 revolutions per minute. If the mean radius is 5 feet, find the work given up by the fly-wheel.

(Stud. I. C. E., Oct. 1904.)

5. A 10-ton truck, moving at the rate of 4 feet per second strikes an 8-ton truck which is standing at rest. If the two move off together after the impact, find the velocity they start with. If the resistance due to friction, &c., is 200 lbs. weight, find how far they will run before coming to rest.  
(Stud. I. C. E., Oct. 1904.)

6. A ball weighing 10 lbs. is making 50 revolutions per minute in a horizontal circle of 5 feet radius. Find the force in lbs. weight acting upon it towards the centre of the circle.  
(Stud. I. C. E., Oct. 1904.)

7. The distances passed over from rest in 1, 2, 3, 4, 5 . . . etc., units of time are respectively 0.05, 0.3, 0.58, 0.95, 1.4, 2.0, 2.7, 3.5, 4.4, 5.35, 6.5, 7.7, 9.0, 10.4, 11.6, 13.2, 14.9, 16.6, 18.4, 20.3, 22.3, 24.3, 26.7, 28.8, 31.2, 33.7 units of length.

Plot the space-time curve. The unit of time is  $\frac{1}{100}$  second, and the unit of length 1 centimetre. Find the acceleration and show how far it is constant.  
(Stud. I. C. E., Feb. 1905.)

8. Give examples of how a diagram can be used—

- (i) to correct the observation or measurement of an experiment  
(See, for example, Question 1.)
- (ii) to find the relation between two quantities, *e.g.*, the effort and load in a machine.
- (iii) to find the average value of a quantity, *e.g.*, to find the average velocity of a body.  
(Stud. I. C. E., Feb. 1905.)

9. A shot is projected from a gun. Explain why—

- (i) The momentum of the shot is equal (under certain conditions) to the momentum of the gun.
- (ii) The energy of the shot when it leaves the gun is greater by far than the energy communicated to the gun.

Example: shot 100 lbs.; gun 1 ton; velocity of shot 1,200 feet per second. Find velocity of recoil of the gun, and the energy of the gun.  
(Stud. I. C. E., Feb. 1905.)

10. In the car of a balloon a piece of iron is hung from a spring balance. The balance registers 5 lbs. when the car is at rest. What will it register when the car is rising with an acceleration of 2 feet per second per second?  
(Stud. I. C. E., Feb. 1905.)

11. Give some account of the advance made in dynamics by either Galileo or Newton.

Show how to find the acceleration towards the centre of a circle of a body moving with uniform velocity in the circle.

(Stud. I. C. E., Feb. 1905.)

12. Continuous brakes are now capable of reducing the speed of a train of  $3\frac{3}{4}$  miles an hour every second, and take 2 seconds to be applied; show in a tabular form the length of an emergency stop at a speed of  $3\frac{3}{4}$ ,  $7\frac{1}{2}$ , 15, 30, 45, 60 miles an hour.

Compare the resistance with gravity; express the resisting force in lbs. per ton; calculate the coefficient of adhesion of the brake-shoe and rail with the wheel, and sketch the mechanical arrangement.

(Stud. I. C. E., Oct. 1905.)

13. If  $W$  tons is transported from rest to rest a distance  $s$  feet in  $t$  seconds, being accelerated for a distance  $s_1$  and time  $t_1$  by a force of  $P_1$  tons up to

velocity  $v$  feet per second, and then brought to rest by  $P_2$  tons acting for  $t_2$  seconds through  $s_2$  feet, prove the formulas—

$$(i) \frac{Wv}{g} = P_1 t_1 = P_2 t_2 = \frac{P_1 P_2}{P_1 + P_2} t;$$

$$(ii) \frac{Wv^2}{2g} = P_1 s_1 = P_2 s_2 = \frac{P_1 P_2}{P_1 + P_2} s;$$

$$(iii) v = 2 \frac{s_1}{t_1} = 2 \frac{s_2}{t_2} = 2 \frac{s}{t}.$$

Supposing  $r$  in  $m$  is the steepest incline a train can crawl up, and  $r$  in  $n$  is the steepest incline on which the brakes can hold the train, prove that the quickest run up an incline of  $r$  in  $p$  from one station to stop at the next, a distance of  $a$  feet, can be made in

$$\sqrt{\left\{ \frac{\left( \frac{1}{m} + \frac{1}{n} \right)}{\left( \frac{1}{m} - \frac{1}{p} \right) \left( \frac{1}{n} + \frac{1}{p} \right)} \frac{2a}{g} \right\}} \text{ seconds.}$$

Calculate for  $m = 50$ ,  $n = 5$ ,  $p = 100$ ,  $a = 5280$ .

(Stud. I. C. E., Oct. 1905.)

14. Determine the motion of a circular hoop of radius  $a$  feet, whirling in a vertical plane on a round stick held horizontally, if released when the centre is moving with velocity  $V$  feet per second at an angle  $a$  with the horizon, and prove that it will make  $\frac{V}{2\pi a}$  revolutions per second in the air.

Prove that the tension in the hoop will be the weight of a length  $\frac{V}{g}$  feet of the rim.

(Stud. I. C. E., Oct. 1905.)

15. Explain how velocities may be compounded. Determine the apparent velocity and direction of rain-drops falling vertically with a velocity of 20 feet per second with reference to a bicyclist moving at the rate of 12 miles an hour.

(Stud. I. C. E., Feb. 1906.)

16. Show that if a body starting from rest and moving in a straight line is accelerated  $f$  feet per second per second it will describe a distance  $s$  in  $t$  seconds expressed by the formula

$$s = \frac{1}{2} ft^2$$

A train starting from rest receives a uniform acceleration of 0.25 foot per second per second for one minute. Calculate the distance travelled.

(Stud. I. C. E., Feb. 1906.)

17. Define potential and kinetic energy.

Find the gain of potential energy of a train weighing 320 tons after mounting an incline 4 miles long of  $r$  in 200, and find its kinetic energy when moving at 30 miles an hour.

(Stud. I. C. E., Feb. 1906.)

18. Explain how forces are measured, and distinguish between the mass of 1 lb. and the weight of 1 lb.

Determine what force will be necessary to change the velocity of a mass of 400 lbs. from 15 to 25 feet per second in 8 seconds.

(Stud. I. C. E., Feb. 1906.)

19. Prove that the acceleration  $a$  of a body moving with velocity  $v$  in a circular path of radius  $r$  is expressed by the formula

$$a = \frac{v^2}{r}$$

Calculate the force required to constrain a locomotive weighing 50 tons to move in a circle of 400 feet radius when its velocity is 30 miles an hour.

(Stud. I. C. E., Feb. 1906.)

20. Explain the terms "moment of inertia," and "radius of gyration," and determine their values for the case of a circular disk of mass  $m$  and radius  $r$ , when rotating about an axis passing through the centre and perpendicular to the plane of the disk. Give numerical values when the mass is 20 lbs. and the radius is 2 feet.

(Stud. I. C. E., Feb. 1906.)

21. Define the terms "velocity" and "acceleration" in the case of a body moving in a straight line. A motor-car starting from rest and uniformly accelerated acquires in 2 minutes a velocity of 30 miles an hour. Find the acceleration.

(Stud. I. C. E., Oct. 1906.)

22. A heavy ball, attached to a string 30 inches long, is whirled round in a horizontal circle with constant velocity. Make a diagram showing the forces acting on the ball, and calculate the velocity when the string is so inclined that the ball moves in a circle of 24 inches radius.

(Stud. I. C. E., Oct. 1906.)

23. Explain what is meant by kinetic energy and deduce an expression for the kinetic energy of a circular disk, rotating about an axis passing through its centre of figure and perpendicular to the plane of the disk. Calculate the kinetic energy of a disk having a radius of 2 feet and weighing 400 lbs. when revolving at the rate of 240 revolutions per minute.

(Stud. I. C. E., Oct. 1906.)

24. Show how velocities may be compounded. A stone is dropped from a balloon 80 feet above the ground and moving horizontally at the rate of 12 miles an hour. Determine the velocity and direction of the stone when it strikes the ground.

(Stud. I. C. E., Feb. 1907.)

25. Define the terms "velocity" and "acceleration." A tram-car starting from rest covers  $s$  feet in  $t$  seconds in accordance with the following Table:—

$t$	1	2	3	4	5	6	7	8	9	10
$s$	4	11	21	34	50	69	91	116	144	175

Plot the space-time curve and from it determine the velocity of the body at the end of each second, and show your results to scale upon a time base. Explain how to determine the acceleration from this latter curve, and determine its value at the end of the fifth second.

(Stud. I. C. E., Feb. 1907.)

26. Define "angular velocity" and "angular acceleration" for a body revolving about a fixed axis, and deduce a formula for the angle turned through by a shaft starting from rest and accelerated uniformly.

The spindle of a dynamo is uniformly accelerated, and in 10 seconds from starting it is found to be revolving at the rate of 600 revolutions per minute. Find the acceleration and the number of revolutions it has made. (Stud. I. C. E., Feb. 1907.)

27. Prove that the acceleration ( $a$ ) of a body moving in a circle of radius  $r$  with velocity  $v$  is expressed by the formula  $a = \frac{v^2}{r}$ . A pulley is found to be out of balance to an amount which may be represented by a mass of 4 oz. at a radius of 1 foot. Determine the unbalanced force when the shaft is making 1200 revolutions per minute.

(Stud. I. C. E., Feb. 1907.)

28. Explain what you understand by "acceleration."

A railway carriage is accelerating at 3 feet per sec. per sec. Find the acceleration possessed by a stone dropping from its roof. If the carriage is 8 feet high, find the time taken to fall and the distance the stone travels.

(Stud. I. C. E., Oct. 1907.)

29. Define "angular acceleration," and show how it is related to linear acceleration.

A hoop whose diameter is 3 feet, is rolling along the ground and comes to rest in 10 seconds, after rolling 240 feet. If it is retarded uniformly, find the value of the angular retardation.

(Stud. I. C. E. Oct. 1907.)

30. A weight is suspended by a string and rotates in a horizontal circle. Find the forces acting on the weight.

Such a weight rotates at 20 revolutions per minute when the radius of its circular path is 3 feet. Find the length of the suspending string.

(Stud. I. C. E. Oct. 1907.)

31. Define "kinetic energy" and "potential energy."

A steamer weighing 2000 tons is proceeding at 20.1 miles per hour. When steam is cut off its speed drops to 19.9 miles per hour after it has moved through 200 feet. Find the mean force retarding it. If it speeds up again to its first speed in half a minute, find approximately the work done in foot-tons, and the horse-power required.

(Stud. I. C. E., Oct. 1907.)

32. Explain how velocities can be represented and combined.

Two men, A and B, are 5 miles apart, A being due west of B. They start walking at the same moment: A walks to the south-east at 4 miles per hour, and B walks at 3 miles per hour in such a direction as to meet A on his road. Find graphically the two possible times taken for the two men to meet.—Ans.: 53 minutes and 1 hour 46 minutes.

(Stud. I. C. E., Feb. 1908.)

33. Define the term acceleration, and show that if a curve be plotted connecting the velocity of a body and the time, the area under the curve is the distance traversed, and the slope of the curve measures the acceleration.

A body starts from rest with a uniform acceleration. After the lapse of a certain time it is found that in successive intervals of 5 seconds and 7 seconds it traverses  $62\frac{1}{2}$  feet and  $129\frac{1}{2}$  feet respectively: find the acceleration at the above time—Ans.: 3 feet per sec. per sec.

(Stud. I. C. E., Feb. 1908.)

34. Explain what you understand by kinetic and potential energy.

A weight of 420 lbs. is lifted by a force which varies as follows:

Height above ground	}	0	1	2	3	4	5	6	7
in feet									
Force in pounds		700	610	490	390	380	450	650	800

Plot a curve connecting the force and height, and hence find the potential and kinetic energy of the body, and the work done by the force when the body is  $6\frac{1}{2}$  feet from the ground.—*Ans.*:  $E_P = 2730$  ft.-lb.:  $E_K = 2359$  ft.-lb., and work done by the force = 5089 ft.-lb.

(Stud. I. C. E., Feb. 1908.)

35. Explain how energy is stored in a fly-wheel, and obtain an expression for this energy. If such a wheel stores 1,000 foot-lb. when rotating at 1 revolution per second, find the work that must be done to change its speed from 10 revolutions per second to 20 revolutions per second. *Ans.*:  $E_K = \frac{1}{2}I\omega^2$ . Work done in changing the speed = 300,000 foot-lb.

(Stud. I. C. E., Feb. 1908.)

36. Explain what you understand by the term "centripetal force."

A weight of 20 pounds is hung by a string 10 feet long. It is pulled to one side so as to be 6 feet horizontally away from the vertical. If the weight is then let go, find from the energy equation the velocity of the weight at the moment it passes through the lowest point, and deduce the total tension in the string at that moment. *Ans.*:  $V = 11.32$  feet per sec.;  $F = 8$  lb.

(Stud. I. C. E., Feb. 1908.)

## LECTURE XXII.—STUD. I.C.E. EXAM. QUESTIONS.

1. Prove that the increase of pressure per foot vertically downwards in a liquid of specific gravity  $s$  is  $0.433 \times s$  lbs. per square inch.

Mining in ground of uniform density at a depth of  $h$  feet, determine the percentage of coal that can be won, leaving sufficient as pillars for the support of the roof, supposing the coal to crush under its own weight in a column  $h$  feet high.

(Stud. I. C. E., Oct. 1905.)

2. Investigate the mechanical advantage of the smooth screw, and explain generally how the wind drives a windmill, and a screw propeller propels a steamer.

Prove that a platelayer who can apply a force of 28 lbs. will be apt to break the screw-bolts if provided with a lever more than 3 feet long; the screw having 8 threads to the inch, and the breaking tension of the bolt being 30 tons per square inch.

(Stud. I. C. E., Oct. 1905.)

3. Explain the meanings of the terms "stress," "strain" and "modulus of elasticity" with reference to a bar in tension. A tie-bar, 3.5 square inches in section and 16 feet long, stretches 0.05 inch under a load of 28,000 lbs. Find the values of the stress, strain, and modulus of elasticity.

(Stud. I. C. E., Oct. 1906.)

## LECTURE XXIII.—STUD. I.C.E. EXAM. QUESTIONS.

1. A machine is operated by a shaft making  $N$  revolutions per minute and transmitting a twisting moment  $T$ . Deduce an expression for the horse-power delivered to the machine and calculate its numerical value if the shaft makes 110 revolutions per minute, and the twisting moment is 2000, the units being pounds and feet.

(Stud. I. C. E., Feb. 1907.)



## May, 1910, Examination on Subject VII.

## APPLIED MECHANICS.

## STAGE I.

GENERAL INSTRUCTIONS.—See APPENDIX A.

*You must not attempt more than EIGHT questions; EITHER No. 1 OR No. 22 must be one of these eight, but not both. The remaining seven questions may be selected from Nos. 2 to 21. The questions in Series A are framed to be more particularly suitable for the Building Trades, and those in Series B for Mechanical Engineers.*

## SERIES A.

1. Describe, with the help of good sketches, only *one* of the following, (a), (b), (c), or (d):

- (a) A mortar-mixing machine.
- (b) The method of securing the cutting chisels into the cutter blocks of a wood-planing machine.
- (c) Any form of friction clutch suitable for use with a speed cone or reversing pulleys.
- (d) Any form of vernier calliper suitable for measuring the dimensions of a test-bar to the nearest thousandth of an inch.

(B. of E., 1910.)

2. Describe, with sketches, an apparatus to verify the rule for finding the compressive and tensile forces in the jib and tie of a crane. Do the experimental results exactly agree with the rule, and if not, what is the probable reason?

(B. of E., 1910.)

3. Answer only *one* of the following, (a), (b), or (c):

- (a) Two of the tests specified in order to determine the quality of Portland cement are the determination of the tensile strength of (i) a mortar of neat cement, (ii) a mortar with sand. Describe carefully how the specimens would be made and tested.
- (b) You are supplied with a length of steel-wire one-eighth of an inch in diameter. You are asked to find (i) Young's Modulus for the material, (ii) the limit of elasticity, (iii) the breaking stress. Explain how you would carry out the test. (See my *Adv.* Vol. II.)
- (c) You wish to know the strength and stiffness of a large timber beam. It is to be built in at the ends; to be about 20 feet long between supports; to be, say, 10 inches broad and 12 inches deep, and it is to be loaded uniformly all over. You therefor test a small beam of the same kind of timber; describe exactly how you would make the test, and how would you use your results? (See my *Adv.* Vol. II.)

(B. of E., 1910.)

4. A wooden beam is built into a wall at one end. Eight feet from the wall there is a hook in the beam, and from this hook is suspended a weight of 1 ton. What is the bending moment in the beam (i) at the wall, (ii) at 3 feet from the wall? Describe the nature of the compressive and tensile stresses throughout any section. (B. of E., 1910.)

5. Let the length of a strut divided by the diameter of its section be called  $x$ .  $W$  is the maximum load carried. Tests were made on a set of cast-iron struts all of the same section but of different lengths, with the following results:

$x$	10	15	20	25	30
$W$	64,000	53,500	44,800	33,700	24,100

Plot a curve showing how the strength depends upon  $x$ . What is the maximum load when the length is 18 times the diameter? (See my *Adv. Vol. II.*) (B. of E., 1910.)

6. There is a triangular roof-truss  $ABC$ ;  $AC$  is horizontal and 10 feet long. The angle  $BCA$  is  $25^\circ$  and  $BAC$  is  $55^\circ$ ; there is a vertical load of 5 tons at  $B$ . What are the compressive forces in  $BA$  and  $BC$ . What are the vertical supporting forces at  $A$  and  $C$ . Find these answers any way you please. (B. of E., 1910.)

7. A man's hand on the handle of a crane moves 120 feet when the weight is lifted 1 foot; 35 per cent. of the total energy given by the man is wasted in friction. A load of 1.5 tons is being lifted. What force is being exerted by the hand? (B. of E., 1910.)

8. Roughly, what is the weight of a cubic foot of brickwork? There is a brick building 80 feet long and 50 feet wide. The foundations carry the following weight:—First, the volume of brickwork is 24,000 cubic feet, the roof and floors weigh altogether 200 lb. per horizontal square foot of the area; the machinery weighs altogether 150 tons. What is the total weight to be carried? What is the breadth of the foundation wall at the footings, if the load there is not to exceed  $1\frac{1}{2}$  tons per square foot? (B. of E., 1910.)

9. It is necessary to keep the "surface level" of water in a shaft at a depth of 30 feet. When left to itself the level rises 4 feet in 1 minute. The shaft is circular, 6 feet in diameter. What is the weight of water entering per minute? This water is lifted by a pump whose efficiency is 0.45. What horse-power must be supplied to the pump? (B. of E., 1910.)

10. Answer only *one* of the following questions, (a) or (b):

(a) Describe briefly, sketches are hardly needed, how Portland cement is manufactured. Give a reason for each part of the process. What is your notion of what occurs (i) when cement sets, (ii) when it slowly hardens as it gets older?

(b) Describe briefly, sketches are hardly needed, how any kind of steel used for girders is manufactured. Give a reason for each part of the process. (B. of E., 1910.)

11. You are given a 4-ton screw-jack. How would you experimentally determine its efficiency under various loads? What sort of results would you expect to obtain? (B. of E., 1910.)

### SERIES B.

12. The depth of water outside the gate of a dry dock is 25 feet. What is the total water pressure on the gate if the width of the gate is 40 feet? The weight of 1 cubic foot of salt water is 64 lb. (B. of E., 1910.)

13. In a hydraulic cylinder, 1 square foot in cross-section, the piston moves through a distance of 1 foot. The pressure of the water is 1400 lb. per square inch. What work is done on the piston? What is the work done per gallon of water used? (B. of E., 1910.)

14. The pull in the draw-bar between locomotive and train is 13 lb. per ton when on the level; the train weighs 200 tons, what is the total force? If the train is being pulled up an incline of 1 in 80 (a vertical rise of 1 foot in a rail distance of 80 feet), what is the additional pull required? What is the total pull? The speed is 2,500 feet per minute. What is the horsepower exercised in drawing the train? (B. of E., 1910.)

15. As a particle of water flows without friction, its height  $h$  feet above datum level, its pressure  $p$  (in lb. per sq. ft.), and its speed  $v$  (in feet per second), may all alter, but the sum

$$\frac{v^2}{2g} + \frac{p}{w} + h$$

remains constant. Here  $g = 32.2$  and  $w = 62.3$ . The particle flows from a place  $A$  where  $v = 0$ ,  $p = 0$ , and  $h = 80$ , to a place  $B$  where  $p = 0$ , and  $h = 40$ ; what is the value of  $v$  at the place  $B$ . (B. of E., 1910.)

16. The motion of a body of 3,220 lb. is opposed by a constant frictional resistance of 2,000 lb. It starts from rest under the action of a varying force  $F$  lb. whose value is here given at the instants at which the body has passed  $x$  feet from rest:

$F$	3,140	2,870	2,630	2,700
$x$	0	5	10	15

As more work is being done upon the body than what is being wasted in friction, what is the speed of the body when it has moved 15 feet from rest? (B. of E., 1910.)

17. An electric motor is employed for lifting purposes. In lifting 80 tons of grain 100 feet high it is found that 20 Board of Trade units of Energy have to be paid for. A Board of Trade unit is 1 kilowatt for 1 hour, and a horse-power is 0.746 kilowatt. The cost is twopence per unit. What is the cost of 1 horse-power hour usefully done? What is the ratio of useful work to the electric energy supplied? (See Appendix D of this book.) (B. of E., 1901.)

18. The radial speed of the water in the wheel of a centrifugal pump is 6 feet per second. The vanes are directed backwards at an angle of 35 degrees to the rim. What is the real velocity of the water relatively to the vanes? What is the component of this which is tangential to the rim?  
(B. of E., 1910.)

19. A projectile leaves the muzzle of a gun at 2,000 feet per second, its path being inclined at  $20^\circ$  upwards. What are the horizontal and vertical components of its velocity? In 3 seconds how far has it travelled horizontally? What is its vertical height above the gun? Neglect resistance of the atmosphere.  
(B. of E., 1910.)

20. A block of cast-iron, 3 inches by 4 inches by 3 inches, is fastened to the arm of a wheel at the distance of 3 feet from the axis. The wheel makes 2,000 revolutions per minute. What is the force tending to fracture the fastening? One cubic inch of cast-iron weighs 0.26 lb.  
(B. of E., 1910.)

21. A fan drives air vertically downward through an opening, 8 feet in diameter, with a velocity of 30 feet per second? The air weighs 0.08 lb. per cubic foot.

What weight of air is driven downward per second? What is its momentum?  
(B. of E., 1910.)

22. Describe, with sketches, only *one* of the following, (a), (b), (c), or (d):

- (a) The shaft bearing of any modern fast running machine such as a water turbine or dynamo machine.
- (b) An hydraulic appliance in use by hydraulic companies or their customers, such as an accumulator, or a motor, or a force pump.
- (c) Any form of sensitive drilling machine of the "pillar" type. What are the advantages of this type of machine for small accurate work?
- (d) Any form of quick return motion suitable for use on a shaping machine. Show how the stroke of the tool may be varied.

(B. of E., 1910.)

## Appendix C.

### The Institution of Civil Engineers' Examinations for Admission of Students, October 1909.

#### (vii.) ELEMENTARY MECHANICS.

*Not more than EIGHT questions to be attempted by any Candidate.*

1. Define the terms *relative velocity* and *absolute velocity*.

A bicycle has 28-inch wheels, and is being ridden at 20 miles an hour. Find the velocity of a point on the rim 14 inches from the ground, (1) relative to the rider, (2) relative to the ground. (Stud. I. C. E., Oct. 1909.)

2. Explain the term *acceleration*, and show how it is to be measured when non-uniform. The velocity of a body at given times is as given in the schedule below. Draw the curve connecting the two quantities, and find the space the body has moved.

(Stud. I. C. E., Oct. 1909.)

Time in seconds. .	0	1	2	3	4	5	6	7	8
Velocity in feet per second . . . . }	0	0.95	3.80	5.00	4.60	3.15	1.65	0.75	0

3. Give the equations for the motion of a body which has a uniform acceleration.

An airship is travelling in a horizontal line at 30 miles per hour towards an object on the ground on which it is desired to drop a shell. The ship is 1600 feet above the ground. Find where it must let the shell go in order to hit the object.  
(Stud. I. C. E., Oct. 1909.)

4. Show how to find the acceleration produced by a force acting on a body.

A tramcar whose weight is 14 tons is being pulled horizontally along a track by a force of 2000 lbs.; the track friction is 20 lbs. per ton. Find the acceleration produced in miles per hour per second.

(Stud. I. C. E., Oct. 1909.)

5. Enunciate the conditions that must be fulfilled in order that three non-parallel forces may be in equilibrium.

A uniform plank AB is pivoted smoothly at the end A, and has a rope attached to the end B. The plank is 10 feet from A to B, and weight 100 lbs. The rope is 8 feet long and is fixed to a point C, such that C is

at the same height as A and the angle ABC is a right angle. Determine graphically the tension in the rope and the pressure on the pivot.

(Stud. I. C. E., Oct. 1909.)

6. Show that a body which is moving with uniform angular velocity  $\omega$  in a circle of radius  $r$  has an acceleration  $\omega^2 r$  towards the centre of the circle.

A flywheel has an internal trough turned on it to contain cooling water. Find the least possible number of revolutions per minute that will permit the retention of the water if the diameter of the trough is 8 feet.

(Stud. I. C. E., Oct. 1909.)

7. What is the *moment* of a force? A timber balk is 42 feet long: a small cross-bar is placed underneath, and 2 feet from one end; the force required to lift the other from the ground is 600 lbs.: the cross-bar is moved to 6 feet from the end, and the force is then 500 lbs.; find the weight of the balk, and the distance of its centre of gravity from the end.

(Stud. I. C. E., Oct. 1909.)

8. Give the condition that a body may stand in equilibrium on a plane. A triangle ABC is cut out of a thick board: the side AB is 10 inches, and the angle ABC is  $30^\circ$ . Find the other sides if the triangle is just stable when standing on the side AB on a horizontal plane.

(Stud. I. C. E., Oct. 1909.)

9. Define the terms *work*, *power*, *horse-power*.

A car weighs  $1\frac{1}{2}$  ton and its engine is working at a constant horse-power. It is running on a road for which the total frictional resistance is 50 lbs per ton. A five-mile run takes 20 minutes, and in the 5 miles the total rise is 400 feet. The car's speed is the same at each end of the run. Find its HP.

(Stud. I. C. E., Oct. 1909.)

10. What is the meaning of the *efficiency* of a machine?

A screwjack has a screw whose pitch is  $\frac{1}{4}$  inch, and the force is applied by a lever 15 inches long. It is found that 5 tons is lifted by a force of 56 lbs. applied at right angles to the lever. Find the efficiency of the jack.

(Stud. I. C. E., Oct. 1909.)

11. What is the *specific gravity* of a body?

A thin metal tube is 32 inches long, and one square inch in sectional area. The bottom inch is filled with a metal. When the tube is put in a vessel of water, it floats with 2 inches out of the water. Find the specific gravity of the metal if the tube weighs 0.6 lb.

(Stud. I. C. E., Oct. 1909.)

12. A plane surface is immersed in a fluid: find an expression for the pressure exerted on the same.

A cube of 3 feet side is placed in water 4 feet deep, with one face of the cube horizontal. Find the pressure on each side.

(Stud. I. C. E., Oct. 1909.)

## Appendix C.

### The Institution of Civil Engineers' Examinations for Admission of Students, February, 1910.

#### ELEMENTARY MECHANICS.

*Not more than EIGHT questions to be attempted by any Candidate.*

*(The weight of a cubic foot of water may be taken as  $62\frac{1}{2}$  lbs. and  $g$  as 32.)*

1. Explain how two velocities can be combined.

A cyclist is riding along a straight road which runs at  $30^\circ$  to a straight piece of railway line. He sees an engine on that line when he is looking in a direction making a constant angle of  $45^\circ$  with his direction of motion. He is travelling at 12 miles an hour: find the velocity of the engine.

2. Establish the equation for the space traversed by a body moving with constant acceleration.

A balloon has a total weight of 1 ton and is at rest 900 feet above the ground. It suddenly lets fall 1 cwt. of ballast: neglecting friction, find how high it will have risen when the ballast reaches the ground.

3. A curve is drawn connecting time and velocity of a moving body: show how the space traversed in a given interval can be found from it.

Such a curve is drawn with a horizontal scale of 1 inch = 1 second and a vertical scale of 1 inch = 1 foot per second. It is a semi-circle of 3 inches radius with its centre at the 3 seconds point on the horizontal line. Find the total space traversed, and the acceleration at the end of the first second.

4. Give the theorem known as the triangle of forces.

A uniform heavy bar is 5 feet long and weighs 20 lbs. Two strings 3 and 4 feet long are attached to the ends, the other extremities of the strings being fixed to a peg. Draw the position when the bar hangs freely; determine the triangle of forces and the tensions in the strings.

5. If a body be moving in a circular path of radius  $r$  with a velocity  $v$ , show that there must be a constraining force of the amount  $\frac{v^2}{r}$  acting along the radius.

A cyclist and his machine together weigh 12 stone; he is turning a corner with a velocity of 15 miles per hour and the radius is 80 feet. Find the force acting.

6. What is the condition that must be fulfilled in order that a body acted on by parallel forces shall be at rest?

Two heavy uniform rods are fixed at an angle and pivoted at the junction point. The shorter rod is 6 feet long and weighs 10 lbs. : the longer rod is 8 feet long and weighs 15 lbs. When suspended by the pivot the shorter rod is horizontal. Find the angle at which the rods are fixed together.

7. Explain the meaning of the terms *work* and *power*, and give the usual units in which they are measured.

A locomotive is steadily pulling a train weighing 500 tons (total) at 25 miles an hour up a slope of 1 in 200. The frictional resistance is 10 lbs. a ton. Find the HP. exerted in traction.

8. Explain the meaning of the term *momentum*.

A vessel of 2000 tons is starting to tow one of 1000 tons. The instant before the rope becomes taut, the vessels are moving on the same line with respective velocities of 7 and 3 knots. Find the common velocity at the moment the rope becomes taut.

9. What do you understand by the *velocity ratio* and the *efficiency* of a machine?

A hand crane has a handle radius of  $1\frac{1}{2}$  foot. It is found that forty-two turns are required to raise the load one foot, and that a tangential force of 37 pounds will just raise 5 tons. Find the efficiency of the crane.

10. Define the *centre of gravity* of a body, and show how you would experimentally find the centre of gravity of a lamina.

A triangle has sides which are respectively 5, 4 and 3 inches. To each a square is attached externally, and in the plane of the triangle, the squares being all cut from the same sheet. Draw the figure to scale and mark on it the centre of gravity of the area formed by the three squares.

11. Define the term *specific gravity* and show how you would measure it for a body.

A yard of wire weighs 70 grams in air and 61 grams in water. Find the specific gravity and the section of the wire [yard =  $91\frac{1}{2}$  centimetres. 1 cubic centimetre of water weighs 1 gram.]

12. How would you find the pressure exerted by a head of water on a dam?

A dam is 50 feet high and has a vertical internal face. Find the total pressure per foot run and the position at which it may be considered to act.

## APPENDIX D.

THE CENTIMETRE, GRAMME, SECOND, OR C.G.S. SYSTEM OF  
UNITS OF MEASUREMENT AND THEIR DEFINITIONS.\*

**I. Fundamental Units.**—The C.G.S. and the practical electrical units are derived from the following mechanical units.

The *Centimetre* as a unit of *length*; the *Gramme* as a unit of *mass*; and the *second* as a unit of *time*.

The *Centimetre* (cm) is equal to 0.3937 inch in length, and nominally represents one thousand-millionth part, or  $\frac{1}{1,000,000,000}$  of a quadrant of the earth.

The *Gramme* (gm) is equal to 15.432 grains, and represents the mass of a cubic centimetre of water at 4° C. Also, 1 lb. of 16 oz. is equal to 453.6 grammes. *Mass* (*M*) is the quantity of matter in a body.

The *second* (s) is the time of one swing of a pendulum making 86,164.09 swings in a sidereal day, or the  $\frac{1}{86,400}$  part of a mean solar day.

**II. Derived Mechanical Units.**—

*Area* (*A* or cm<sup>2</sup>).—The unit of area is the *square centimetre*.

*Volume* (*V* or cm<sup>3</sup>).—The unit of volume is the *cubic centimetre*.

*Velocity* (*v* or cm/s) is rate of change of position. It involves the idea of direction as well as that of magnitude. *Velocity* is *uniform* when equal distances are traversed in equal intervals of time. The unit of velocity is the velocity of a body which moves through unit distance in unit time, or the *velocity of one centimetre per second*.

*Momentum* (*Mv*, or gm × cm/s) is the quantity of motion in a body, and is measured by mass × velocity.

*Acceleration* (*a* or cm/s<sup>2</sup>) is the rate of change of velocity, whether that change takes place in the direction of motion or not. The unit of acceleration is the acceleration of a body which undergoes unit change of velocity in unit time, or an acceleration of one centimetre-per-second per second. The acceleration due to gravity is considerably greater than this, for the change of velocity imparted by gravity to falling bodies in one second is about 981 centimetres per second (or about 32.2 feet per second). The value differs slightly in different latitudes. At Greenwich the value of the acceleration due to gravity is  $g = 981.17$ ; at the Equator  $g = 978.1$ , and at the North Pole  $g = 983.1$ .

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\* The Author is indebted to his Publishers, Charles Griffin and Co., for liberty to abstract the following pages on this subject from the latest edition of Munro and Jamieson's "Pocket-book of Electrical Rules and Tables for Electricians and Engineers," to which the student is referred for further values and definitions, and values of Practical Electrical Units of Measurement and Testing Rules, &c.—A.J.

*Force* ( $F$  or  $f$ ) is that which tends to alter a body's natural state of rest or of uniform motion in a straight line.

*Force* is measured by the rate of change of momentum which it produces, or mass  $\times$  acceleration.

*The Unit of Force*, or *Dyne*, is that force which, acting for one second on a mass of one gramme, gives to it a velocity of one centimetre per second. The force with which the earth attracts any mass is usually called the "weight" of that mass, and its value obviously differs at different points of the earth's surface. The force with which a body gravitates—i.e. its weight (in dynes), is found by multiplying its mass (in grammes) by the value of  $g$  at the particular place where the force is exerted.

*Work* is the product of a force and the distance through which it acts. The unit of work is the work done in overcoming unit force through unit distance, i.e. in pushing a body through a distance of one centimetre against a force of one dyne. It is called the *Erg*. Since the "weight" of 1 gramme is  $1 \times 981$ , or 981 dynes, the work of raising one gramme through the height of one centimetre against the force of gravity is 981 ergs or  $g$  ergs. One kilogramme-metre = 100,000 ( $g$ ) ergs. One foot-pound = 13,825 ( $g$ ) ergs =  $1.356 \times 10^7$  ergs.

*Energy* is that property which, possessed by a body, gives it the capability of doing work. *Kinetic energy* is the work a body can do in virtue of its motion. *Potential energy* is the work a body can do in virtue of its position. The unit of energy is the *Erg*.

*Power or Activity* is the rate of working. The unit is called the *Watt* ( $W_P$ ) =  $10^7$  ergs per second, or the work done at the rate of one *Joule* ( $J$ ) per second.

One *Horse-power* (H.P.) = 33,000 ft.-lbs. per minute = 550 ft.-lbs. per second. But as seen above under *Work*, 1 ft. lb. =  $1.356 \times 10^7$  ergs, and under *Power*, 1 *Watt* =  $10^7$  ergs per second.

Hence, a *Horse-power* =  $550 \times 1.356 \times 10^7$  ergs per sec. = 746 Watts.

If  $E$  = volts,  $C$  = amperes, and  $R$  = ohms. ; then, by Ohm's Law  $C = E/R$ , Also,  $EC = C^2R = E^2 R = \text{Watts}$ .

$$\text{Therefore,} \quad \text{H.P.} = \frac{EC}{746} = \frac{C^2R}{746} = \frac{E^2}{746R}.$$

# PRACTICAL ELECTRICAL UNITS.

1. As a **Unit of Resistance (R)**, the **International Ohm** ( $\omega$  or  $\Omega$ ), based upon the ohm which is  $10^9$  units of resistance in the C.G.S. system of electro-magnetic units, is represented by the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grammes in mass, of a constant cross-sectional area and of the length of 106.3 centimetres.

2. As a **Unit of Current (C)**, the **International Ampere (A)**, which is one-tenth of the unit of current of the C.G.S. system of electro-magnetic units, and which is represented sufficiently well for practical use by the unvarying current which, when passed through a solution of nitrate of silver in water, and in accordance with the International specifications, deposits silver at the rate of 0.001118 grammes per second.

3. As a **Unit of Electro-motive Force (E)** the **International Volt (V)**, which is the E.M.F. that, steadily applied to a conductor whose resistance is one International Ohm, will produce a current of one International Ampere, and which is represented sufficiently well for practical use by  $\frac{1.1000}{1.1334}$  of the E.M.F. between the poles or electrodes of the voltaic cell known as Clark's cell, at a temperature of  $15^\circ$  Centigrade, and prepared in the manner described in the International specification.

4. As the **Unit of Quantity (Q)** the **International Coulomb (A  $\times$  s)**, which is the quantity of electricity transferred by a current of one International Ampere in one second.

5. As the **Unit of Capacity (K)** the **International Farad (Fd)**, which is the capacity of a conductor charged to a potential of one International Volt by one International Coulomb of electricity.

6. As a **Unit of Work** the **Joule (J)** (or Watt-second ( $W \times s$ )), which is  $10^7$  units of work in the C.G.S. system, and which is represented sufficiently well for practical use by the energy expended in 1 second in heating an International Ohm.

7. As the **Unit of Power ( $P_w$ )** the **International Watt ( $W_P$ )**, which is equal to  $10^7$  units of power in the C.G.S. system, and which is represented sufficiently well for practical use by the work done at the rate of one Joule per second. The Kilowatt (K.W.) = 1000 Watts =  $1\frac{1}{2}$  horse-power.

8. As the **Unit of Induction (L)** the **Henry ( $H_1$ )**, which is the induction in the circuit when the E.M.F. induced in this circuit is one International Volt, while the inducing current varies at the rate of one ampere per second.

9. The **Board of Trade Commercial Unit of Work or (B.T.U.)** is the Kilowatt-hour (K.W.-hr.) = 1000 Watt-hours =  $1\frac{1}{2}$  H.P. working for one hour. Or say 10 amperes flowing in a circuit for 1 hour at a pressure of 100 volts.

*Note.*—For further simple explanations with Examples, see 7th Edition of Prof. Jamieson's "Manual of Magnetism and Electricity," pp. 87 to 94, and 222 to 224. Also latest Edition of Munro and Jamieson's Electrical Pocket Book, both published by Charles Griffin & Co. Ltd., London.

## EXAMINATION TABLES.

## USEFUL CONSTANTS.

1 Inch = 25·4 millimetres.

1 Gallon = ·1605 cubic foot = 10 lbs. of water at 62° F.

1 Naut = 6080 feet.

1 Knot = 6080 feet per hour.

Weight of 1 lb. in London = 445,000 dynes.

One pound avoirdupois = 7000 grains = 453·6 grammes.

1 Cubic foot of water weighs 62·3 lbs. at 65° F.

1 Cubic foot of air at 0° C. and 1 atmosphere, weighs ·0807 lb.

1 Cubic foot of Hydrogen at 0° C. and 1 atmosphere, weighs ·00557 lb.

1 Foot-pound =  $1·3562 \times 10^7$  ergs.

1 Horse-power-hour = 33000 × 60 foot-pounds.

1 Electrical unit = 1000 watt-hours.

Joule's Equivalent to suit Regnault's H, is  $\left\{ \begin{array}{l} 774 \text{ ft.-lbs.} = 1 \text{ Fah. unit.} \\ 1393 \text{ ft.-lbs.} = 1 \text{ Cent. } \end{array} \right.$

1 Horse-power = 33000 foot-pounds per minute = 746 watts.

Volts × amperes = watts.

1 Atmosphere = 14·7 lb. per square inch = 2116 lbs. per square foot = 760 m.m. of mercury =  $10^6$  dynes per sq. cm. nearly.

A Column of water 2·3 feet high corresponds to a pressure of 1 lb. per square inch.

Absolute temp.,  $t = \theta^\circ \text{ C.} + 273^\circ\cdot7$ .

Regnault's H =  $606\cdot5 + \cdot305 \theta^\circ \text{ C.} = 1082 + \cdot305 \theta^\circ \text{ F.}$

$p u^{1\cdot0646} = 479$

$\log_{10} p = 6\cdot1007 - \frac{B}{t} - \frac{C}{t^2}$

where  $\log_{10} B = 3\cdot1812$ ,  $\log_{10} C = 5\cdot0871$ ,

$p$  is in pounds per square inch,  $t$  is absolute temperature Centigrade,  
 $u$  is the volume in cubic feet per pound of steam.

$\pi = 3\cdot1416$ .

One radian = 57·3 degrees.

To convert common into Napierian logarithms, multiply by 2·3026.

The base of the Napierian logarithms is  $e = 2\cdot7183$ .

The value of  $g$  at London, = 32·182 feet per sec. per sec.

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4 8 12	17 21 25	29 33 37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4 8 11	15 19 23	26 30 34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3 7 10	14 17 21	24 28 31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3 6 10	13 16 19	23 26 29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3 6 9	12 15 18	21 24 27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3 6 8	11 14 17	20 22 25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3 5 8	11 13 16	18 21 24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2 5 7	10 12 15	17 20 22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2 5 7	9 12 14	16 19 21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2 4 7	9 11 13	16 18 20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2 4 6	8 11 13	15 17 19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2 4 6	8 10 12	14 16 18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2 4 6	8 10 12	14 15 17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2 4 6	7 9 11	13 15 17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2 4 5	7 9 11	12 14 16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2 3 5	7 9 10	12 14 15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2 3 5	7 8 10	11 13 15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2 3 5	6 8 9	11 13 14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2 3 5	6 8 9	11 12 14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1 3 4	6 7 9	10 12 13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1 3 4	6 7 9	10 11 13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1 3 4	6 7 8	10 11 12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1 3 4	5 7 8	9 11 12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1 3 4	5 6 8	9 10 12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1 3 4	5 6 8	9 10 11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1 2 4	5 6 7	9 10 11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1 2 4	5 6 7	8 10 11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1 2 3	5 6 7	8 9 10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1 2 3	5 6 7	8 9 10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1 2 3	4 6 7	8 9 10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1 2 3	4 5 6	8 9 10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1 2 3	4 5 6	7 8 9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1 2 3	4 5 6	7 8 9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1 2 3	4 5 6	7 8 9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1 2 3	4 5 6	7 8 9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1 2 3	4 5 6	7 8 9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1 2 3	4 5 6	7 7 8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1 2 3	4 5 5	6 7 8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1 2 3	4 4 5	6 7 8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1 2 3	4 4 5	6 7 8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1 2 3	3 4 5	6 7 8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1 2 3	3 4 5	6 7 8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1 2 2	3 4 5	6 7 7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1 2 2	3 4 5	6 6 7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1 2 2	3 4 5	6 6 7

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1 2 2	3 4 5	5 6 7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1 2 2	3 4 5	5 6 7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1 2 2	3 4 5	5 6 7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1 1 2	3 4 4	5 6 7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1 1 2	3 4 4	5 6 7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1 1 2	3 4 4	5 6 6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1 1 2	3 4 4	5 6 6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1 1 2	3 3 4	5 6 6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1 1 2	3 3 4	5 5 6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1 1 2	3 3 4	5 5 6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	81 9	1 1 2	3 3 4	5 5 6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1 1 2	3 3 4	5 5 6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1 1 2	3 3 4	5 5 6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1 1 2	3 3 4	4 5 6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1 1 2	2 3 4	4 5 6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1 1 2	2 3 4	4 5 6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1 1 2	2 3 4	4 5 5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1 1 2	2 3 4	4 5 5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1 1 2	2 3 4	4 5 5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1 1 2	2 3 4	4 5 5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1 1 2	2 3 3	4 5 5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1 1 2	2 3 3	4 5 5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1 1 2	2 3 3	4 4 5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1 1 2	2 3 3	4 4 5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1 1 2	2 3 3	4 4 5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1 1 2	2 3 3	4 4 5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1 1 2	2 3 3	4 4 5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1 1 2	2 3 3	4 4 5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1 1 2	2 3 3	4 4 5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1 1 2	2 3 3	4 4 5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1 1 2	2 3 3	4 4 5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1 1 2	2 3 3	4 4 5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0 1 1	2 2 3	3 4 4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0 1 1	2 2 3	3 4 4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0 1 1	2 2 3	3 4 4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0 1 1	2 2 3	3 4 4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0 1 1	2 2 3	3 4 4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0 1 1	2 2 3	3 4 4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0 1 1	2 2 3	3 4 4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0 1 1	2 2 3	3 4 4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0 1 1	2 2 3	3 4 4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0 1 1	2 2 3	3 4 4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0 1 1	2 2 3	3 4 4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0 1 1	2 2 3	3 4 4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0 1 1	2 2 3	3 3 4

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0 0 1	1 1 1	2 2 2
01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0 0 1	1 1 1	2 2 2
02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0 0 1	1 1 1	2 2 2
03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0 0 1	1 1 1	2 2 2
04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0 1 1	1 1 2	2 2 2
05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0 1 1	1 1 2	2 2 2
06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0 1 1	1 1 2	2 2 2
07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0 1 1	1 1 2	2 2 2
08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0 1 1	1 1 2	2 2 3
09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0 1 1	1 1 2	2 2 3
10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0 1 1	1 1 2	2 2 3
11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0 1 1	1 2 2	2 2 3
12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0 1 1	1 2 2	2 2 3
13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0 1 1	1 2 2	2 3 3
14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0 1 1	1 2 2	2 3 3
15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0 1 1	1 2 2	2 3 3
16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0 1 1	1 2 2	2 3 3
17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0 1 1	1 2 2	2 3 3
18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0 1 1	1 2 2	2 3 3
19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0 1 1	1 2 2	3 3 3
20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0 1 1	1 2 2	3 3 3
21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0 1 1	2 2 2	3 3 3
22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0 1 1	2 2 2	3 3 3
23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0 1 1	2 2 2	3 3 4
24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0 1 1	2 2 2	3 3 4
25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0 1 1	2 2 2	3 3 4
26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0 1 1	2 2 3	3 3 4
27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0 1 1	2 2 3	3 3 4
28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0 1 1	2 2 3	3 4 4
29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0 1 1	2 2 3	3 4 4
30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0 1 1	2 2 3	3 4 4
31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0 1 1	2 2 3	3 4 4
32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0 1 1	2 2 3	3 4 4
33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0 1 1	2 2 3	3 4 4
34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1 1 2	2 3 3	4 4 5
35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1 1 2	2 3 3	4 4 5
36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1 1 2	2 3 3	4 4 5
37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1 1 2	2 3 3	4 4 5
38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1 1 2	2 3 3	4 4 5
39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1 1 2	2 3 3	4 5 5
40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1 1 2	2 3 4	4 5 5
41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1 1 2	2 3 4	4 5 5
42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1 1 2	2 3 4	4 5 6
43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1 1 2	3 3 4	4 5 6
44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1 1 2	3 3 4	4 5 6
45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1 1 2	3 3 4	5 5 6
46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1 1 2	3 3 4	5 5 6
47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1 1 2	3 3 4	5 5 6
48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1 1 2	3 4 4	5 6 6
49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1 1 2	3 4 4	5 6 6

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1 1 2	3 4 4	5 6 7
51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1 2 2	3 4 5	5 6 7
52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1 2 2	3 4 5	5 6 7
53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1 2 2	3 4 5	6 6 7
54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1 2 2	3 4 5	6 6 7
55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1 2 2	3 4 5	6 7 7
56	3631	3639	3648	3656	3664	3673	3.81	3690	3698	3707	1 2 3	3 4 5	6 7 8
57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1 2 3	3 4 5	6 7 8
58	3802	3811	3819	3.28	3837	3846	3855	3864	3873	3882	1 2 3	4 4 5	6 7 8
59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1 2 3	4 5 5	6 7 8
60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1 2 3	4 5 6	6 7 8
61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1 2 3	4 5 6	7 8 9
62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1 2 3	4 5 6	7 8 9
63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1 2 3	4 5 6	7 8 9
64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1 2 3	4 5 6	7 8 9
65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1 2 3	4 5 6	7 8 9
66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4567	1 2 3	4 5 6	7 9 10
67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1 2 3	4 5 7	8 9 10
68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1 2 3	4 6 7	8 9 10
69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1 2 3	5 6 7	8 9 10
70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1 2 4	5 6 7	8 9 11
71	5129	5140	5152	5164	5175	5188	5200	5212	5224	5236	1 2 4	5 6 7	8 10 11
72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1 2 4	5 6 7	9 10 11
73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1 3 4	5 6 8	9 10 11
74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1 3 4	5 6 8	9 10 12
75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1 3 4	5 7 8	9 10 12
76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1 3 4	5 7 8	9 11 12
77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1 3 4	5 7 8	10 11 12
78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1 3 4	6 7 8	10 11 13
79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1 3 4	6 7 9	10 11 13
80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1 3 4	6 7 9	10 12 13
81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2 3 5	6 8 9	11 12 14
82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2 3 5	6 8 9	11 12 14
83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2 3 5	6 8 9	11 13 14
84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2 3 5	6 8 10	11 13 15
85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2 3 5	7 8 10	12 13 15
86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2 3 5	7 8 10	12 13 15
87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2 3 5	7 9 10	12 14 16
88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2 4 5	7 9 11	12 14 16
89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2 4 5	7 9 11	13 14 16
90	7943	7962	7980	7998	8017	8035	8054	8072	8891	8110	2 4 6	7 9 11	13 15 17
91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2 4 6	8 9 11	13 15 17
92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2 4 6	8 10 12	14 15 17
93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2 4 6	8 10 12	14 16 18
94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2 4 6	8 10 12	14 16 18
95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2 4 6	8 10 12	15 17 19
96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2 4 6	8 11 13	15 17 19
97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2 4 7	9 11 13	15 17 20
98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2 4 7	9 11 13	16 18 20
99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2 5 7	9 11 14	16 18 20

Angle.		Chord.	Sine.	Tangent.	Co-tangent.	Cosine.			
Degrees.	Radians.								
0°	0	000	0	0	∞	1	1.414	1.5708	90°
1	.0175	.017	.0175	.0175	57.2900	.9998	1.402	1.5533	89
2	.0349	.035	.0349	.0349	28.6363	.9994	1.389	1.5359	88
3	.0524	.052	.0523	.0524	19.0811	.9986	1.377	1.5184	87
4	.0698	.070	.0698	.0699	14.3007	.9976	1.364	1.5010	86
5	.0873	.087	.0872	.0875	11.4301	.9962	1.351	1.4835	85
6	.1047	.105	.1045	.1051	9.5144	.9945	1.338	1.4661	84
7	.1222	.122	.1219	.1228	8.1443	.9925	1.325	1.4486	83
8	.1396	.140	.1392	.1405	7.1154	.9903	1.312	1.4312	82
9	.1571	.157	.1564	.1584	6.3138	.9877	1.299	1.4137	81
10	.1745	.174	.1736	.1768	5.6713	.9848	1.286	1.3963	80
11	.1920	.192	.1908	.1944	5.1446	.9816	1.272	1.3788	79
12	.2094	.209	.2079	.2126	4.7046	.9781	1.259	1.3614	78
13	.2269	.226	.2250	.2309	4.3315	.9744	1.245	1.3439	77
14	.2443	.244	.2419	.2493	4.0108	.9703	1.231	1.3265	76
15	.2618	.261	.2588	.2679	3.7321	.9659	1.218	1.3090	75
16	.2793	.278	.2756	.2867	3.4874	.9613	1.204	1.2915	74
17	.2967	.296	.2924	.3057	3.2709	.9563	1.190	1.2741	73
18	.3142	.313	.3090	.3249	3.0777	.9511	1.176	1.2566	72
19	.3316	.330	.3256	.3443	2.9042	.9455	1.161	1.2392	71
20	.3491	.347	.3420	.3640	2.7475	.9397	1.147	1.2217	70
21	.3665	.364	.3584	.3839	2.6051	.9336	1.133	1.2043	69
22	.3840	.382	.3746	.4040	2.4751	.9272	1.118	1.1868	68
23	.4014	.399	.3907	.4245	2.3559	.9205	1.104	1.1694	67
24	.4189	.416	.4067	.4452	2.2460	.9135	1.089	1.1519	66
25	.4363	.433	.4226	.4663	2.1445	.9063	1.075	1.1345	65
26	.4538	.450	.4384	.4877	2.0503	.8988	1.060	1.1170	64
27	.4712	.467	.4540	.5095	1.9626	.8910	1.045	1.0996	63
28	.4887	.484	.4695	.5317	1.8807	.8829	1.030	1.0821	62
29	.5061	.501	.4848	.5543	1.8049	.8746	1.015	1.0647	61
30	.5236	.518	.5000	.5774	1.7321	.8660	1.000	1.0472	60
31	.5411	.534	.5150	.6009	1.6643	.8572	.985	1.0297	59
32	.5585	.551	.5299	.6249	1.6003	.8480	.970	1.0123	58
33	.5760	.568	.5446	.6494	1.5399	.8387	.954	.9948	57
34	.5934	.585	.5592	.6745	1.4826	.8290	.939	.9774	56
35	.6109	.601	.5736	.7002	1.4281	.8192	.923	.9599	55
36	.6283	.618	.5878	.7265	1.3764	.8090	.908	.9425	54
37	.6458	.635	.6018	.7536	1.3270	.7986	.892	.9250	53
38	.6632	.651	.6157	.7813	1.2799	.7880	.877	.9076	52
39	.6807	.668	.6293	.8098	1.2349	.7771	.861	.8901	51
40	.6981	.684	.6428	.8391	1.1918	.7660	.845	.8727	50
41	.7156	.700	.6561	.8693	1.1504	.7547	.829	.8552	49
42	.7330	.717	.6691	.9004	1.1106	.7431	.813	.8378	48
43	.7505	.733	.6820	.9325	1.0724	.7314	.797	.8203	47
44	.7679	.749	.6947	.9657	1.0355	.7193	.781	.8029	46
45°	.7854	.765	.7071	1.0000	1.0000	.7071	.765	.7854	45°
			Cosine.	Co-tangent.	Tangent.	Sine.	Chord.	Radians.	Degrees.
								Angle.	



# INDEX

## A

- ACCELERATION, definition, 260, 421  
 „ due to gravity, 260  
 „ unit of, 260  
 Accumulated work, 280  
 „ work in a fly-wheel, 281  
 „ work in a rotating body, 281  
 Accumulator, hydraulic, 253-255  
 Action and reaction, 3  
 „ of the common suction-pump, 227-229  
 Activity, definition of, 12, 412  
 Actual or working advantage, 67  
 Admission of Students to the Institution of Civil Engineers, rules and syllabus of Exams. for, 370  
 Air vessel, action of an, 232  
 „ „ force pump with, 231  
 Ampere, 413  
 Angle of repose, 106; resistance, 107  
 Angular and linear motion, 272  
 „ velocity, 260  
 Anti-friction wheels, 109  
 Anti-logarithms, Appendix D, 417  
 Applied Mechanics, definition of, 1  
 Archimedes, law of, 217-219  
 Atmospheric pressure, 220  
 Atwood's machine, 262-269  
 Axle, wheel and, 55-57  
 „ „ „ compound, 72-74
- BACKLASH, in wheel and screw-gearings, 158  
 Back-motion gear in a lathe, 174-185  
 Balance, Roman, 35  
 „ bent lever, 43  
 Balancing fast speed machinery, 279  
 Bales, screw for passing, 165  
 Ball bearings, 110  
 Barometer, the mercurial, 221
- Bear, hydraulic, or portable punching machine, 251  
 Bearings, ball, 110  
 „ roller, 183  
 Bed or shears of a lathe, 183, 187  
 Bell crank lever, 43  
 Belt-gearing reversing motions, 122-124  
 „ „ shape of pulley faces for, 127  
 „ „ velocity ratio of pulleys in, 119-121  
 Belts, brake horse-power transmitted by, 118  
 „ difference of tension in, 116  
 „ open and crossed, 121  
 „ tendency of, to run on highest parts of pulleys, 127  
 Bench vice, screw, 165-167  
 Bent lever balance, 43  
 „ levers, 42  
 „ „ duplex, 43  
 Bevel wheel and clutch reversing gear, 345  
 Bicycle and railway curves, motion on, 287  
 Block and tackle, 65-68  
 „ snatch, 65  
 Block, Weston's differential pulley, 74-77  
 Board of Education Exam., instructions for, 368  
 Board of Trade unit, 413  
 Bodies, formulæ for falling, 261  
 „ path of projected bodies which fall under the action of gravity, 273  
 „ proof by Atwood's machine of formulæ for falling, 262-269  
 Boiler plates, large hydraulic press for flanging, 245-247  
 „ „ single riveted lap joints, 310  
 Bolt, holding down, 156  
 Bon-accord centrifugal pump, 236

Bottle screw-jack, 162-164  
 Brake, horse-power transmitted by belts, 118  
 Bramah's hydraulic press, 241-245  
 „ leather collar packing, 243  
 Bucket pump, combined plunger and, 234  
 Buttress screw thread, 155

## C

CALCULATIONS, note regarding engineering, 5  
 Cams, 330-334  
 Capstan, ships, 57-59  
 Carpenter's turkus, 44  
 Centimetre, 411  
 Centre of gravity, 28  
 „ of parallel forces, 20  
 „ of pressure, 217  
 Centrifugal force, 275  
 „ force machine, 277  
 „ pumps, 236-238  
 „ stress in fly-wheels, 279  
 Centripetal force, 276  
 Chains, stresses in, 316  
 Change wheels in a lathe, 176-179, 183, 184, 187  
 „ „ reversing plate or quadrant, 183, 187  
 Chinese windlass, 72  
 Circle, definition of pitch, 131  
 Circular discs, velocity ratio of, 130  
 City and Guilds exam., instruction for, 369  
 Civil Engineers, Students' Exam., rules and syllabus of, 370  
 Clarke's patent adjustable curve, 16, 17  
 Click, reversible, 335  
 Co-efficient of friction, 105, 111  
 Cohesion of matter, 224, 298  
 Combined lever, screw, and pulley gear, 160  
 „ plunger and bucket pump, 234  
 Comparison of dynamical formulæ for linear and angular motion, 272  
 Components and resultant of forces, 4  
 „ of a force at right angles to each other, 84

Composition and resolution of forces, 82, 84; velocities, 260  
 Compound, Weems', screw and hydraulic jack, 249-251  
 Compound wheel and axle, 72-74  
 Compressibility of matter, 298  
 Compressing a bar within the elastic limit, work done in, 307  
 Compressive stress and strain, 301  
 Cones, stepped speed, 124  
 Constants—appendix D, 414  
 Coulomb, 413  
 Couple, definition of, 25  
 Coupling joint, Hooke's, 329  
 „ screw, for carriages, 156  
 Crab, double purchase, 143, 144  
 „ single purchase, 140-142  
 Cranes, stresses in various members and jib arrester, 85, 92  
 Crank, bell, lever, 43  
 Curve, focus and directrix of a, 272  
 Curves, motion on bicycle and railway, 287  
 Cylinder, forming a screw thread on a, 149

## D

DEAD load, definition of, 300  
 Density of matter, 298  
 Differential pulley blocks, Weston's, 74-77  
 Dilatibility of matter, 298  
 Directrix of a curve, 272  
 Discs, velocity ratio of, 130  
 Distinction, solids, liquids, gases, fluids, &c., 224  
 Double acting force pump, 234  
 „ Hooke's joint, 329  
 „ purchase winch or crab, 143, 144  
 „ threaded screws, 157  
 Driving belt, difference of tension in a, 116  
 Ductility, definition of, 299  
 Duplex bent levers, 43  
 Dynamical formulæ for linear and angular motion, comparison of, 272

## E

EFFICIENCY, apparatus for determining, of screw gear, 160

- Efficiency of combined lever,  
     screw, and pulley  
     gear, 160  
     ,, of a machine, defini-  
         tion of, 53  
     ,, of screws, 151
- Elasticity, definition of, 300  
     ,, limits of, 303  
     ,, modulus of, 304-306  
     ,, safe loads and, 302  
     ,, table of moduli of, 305
- Ellipse, 273
- Elliptic wheels for quick return,  
     351-353
- Endless screw and worm-wheel, 168
- Energy, kinetic, potential, 280, 412
- English gauges, 360
- Equilibrant of parallel forces, 25
- Equilibrium, conditions of, in case  
     of floating bodies,  
     218
- Equilibrium, forces in, 3  
     ,, graphic demonstration  
         of three forces in, 80
- Erg, 6, 412
- Exams. B. of E. C and G ; Inst. C.E.,  
     Appendix C, 403
- Experimentally determining the  
     energy stored up in a rotating  
     fly-wheel, 285
- Extending a bar within the elastic  
     limit, work done in, 307
- Extension of matter, 297
- F**
- FACTORS of safety, 302
- Falling bodies, formulæ for, 261  
     ,, proof of formulæ for, by  
         Atwood's machine, 262-269
- Farad, 413
- Feed, silent, 340-342
- Floating bodies, conditions of equi-  
     librium, 218
- Fluids, solids, gases, 224
- Fly-press, the, 283
- Fly-wheels, centrifugal stress in, 279  
     ,, energy stored up in, 281
- Fly-wheel, to find experimentally  
     the energy stored up in a  
     rotating, 285
- Focus of a curve, 272
- Force, definition of, 1, 412  
     .. elements of a, 2
- Force, moment of a, 21  
     ,, pump, single-acting, 229-231  
     ,, pump, double-acting, 234  
     ,, pump with air vessel, 231  
     ,, resolution of a, into two com-  
         ponents at right angles, 84  
     ,, unit of, 2, 412
- Forces, centrifugal, centripetal, 275  
     ,, graphic representation of, 3  
     ,, in equilibrium, 3  
     ,, parallel, 25-28  
     ,, parallelogram of, 82  
     ,, resultant of, 4  
     ,, straight lever acted on by  
         inclined, 42  
     ,, three equal, in equilibrium,  
         83  
     ,, triangle of, 82  
     ,, two, at right angles, 83
- Forging and hardening turning  
     tools, 202
- Friction, angle of, 106  
     ,, anti-, wheels, 109  
     ,, circular discs, velocity  
         ratio of, 130  
     ,, co-efficient of, 105, 111  
     ,, definition of, 101  
     ,, heat developed by, 102  
     ,, inclined plane with, 110-  
         112  
     ,, inclined plane without, 93  
     ,, laws of, 103-109  
     ,, cone reversing gear, 344
- Fulcrum, position of, in a lever, 30  
     ,, pressure on, and reaction  
         from, 25
- Fusce, the, 59-61
- Fusibility, definition of, 300
- G**
- GALILEO's and Kater's pendulum  
     experiments, 269
- Gas gauge, 222 ; gases, 224
- Gauge limits, micrometer, &c., 357  
     ,, Stubs' wire, 358
- Gauges, low pressure, &c., 221
- Gear, back motion in lathe, 174-  
     192  
     ,, change wheels in lathe, 176-  
         195  
     ,, efficiency of combined lever,  
         screw and pulley, 160  
     ,, screw and pulley, 160

- Gear, starting and stopping, 124  
 „, worm-wheel lifting, 170  
 Gearing, backlash in wheel and screw, 158  
 „, belt, reversing motions, 122-124  
 „, belt, shape of pulley faces for, 127  
 „, pitch of teeth in wheel, 132  
 „, principle of work applied to wheel, 135  
 „, velocity ratio of pulleys in belt, 119-121  
 „, velocity ratio in wheel, 123  
 „, wheel, in jib-cranes, 144-146  
 Grain, screw for moving, 148  
 Gramme, 411  
 Graphic representation of forces, 3  
 „, „, „, velocities, 260  
 Gravity, acceleration due to, 260  
 „, centre of, 28  
 „, specific, definition of, 214  
 Gyration, radius of, 282

## IH

- HARDENING the tools for a lathe, 202  
 Head or pressure of a liquid, 209  
 Headstock, fast or fixed, of a lathe, 184, 185  
 „, movable, for a common lathe, 179, 180  
 „, movable, for a screw-cutting lathe, 181  
 Heart-shaped cam, 331  
 Heat developed by friction, 102  
 „, relation between, and work, 104  
 Helix of a screw thread, 148  
 Henry, 413  
 Herbert's hexagon turret lathe, 188-195  
 Hollow round shafts, strength of, 324  
 Homogeneous materials, 299  
 Hooke's coupling joint, 329  
 „, double „, 329  
 „, low, 303  
 Horse-power brake, transmitted by belts, 118  
 „, „, definition of, 12, 412  
 „, „, of working agent, 13

- Huyghen's pendulum experiments, 269  
 Hydraulic accumulator, 253-255  
 „, bear, 251  
 „, jacks, 247-251  
 „, machines, 227-255  
 „, press, Bramah's, 241-245  
 „, press, large, 245-247  
 Hydraulics, 207-258  
 Hyperbola, 273

## I

- IDLE wheel (note on), 138  
 Immersion of solids in fluids, 217  
 Impenetrability of matter, 297  
 Improved Standard Measuring Machine, 363  
 Inclined forces, straight lever acted on by, 42  
 „, plane, the screw as an, 149  
 „, planes, principle of work, 93-99, 270-272  
 „, applied to, 97, 110, 112  
 Indicator, motion for Richard's, 344  
 Inertia definition, 290  
 Intermittent motion for cam, 332  
 Internal and external limit gauges, 355-357  
 Isotropic material, 299  
 Instructions for Board of Education Exams., 368  
 „, „, City and Guilds Exam., 369  
 „, „, Institution of Civil Engineers Students' Exam., 370

## J

- JACK, bottle screw, 162-164  
 „, traversing screw, 164  
 Jacks, hydraulic, 247-251  
 Jib cranes, stresses in, 85-92  
 „, „, wheel gearing in, 144-146  
 Joint, Hooke's coupling, 329  
 „, „, double, 329  
 Joints, single-riveted lap, 310  
 Joule's relation between heat and work, 102, 413

## K

- KATER's and Galileo's pendulum experiments, 269

Kelvin's, Lord, wire-testing machine, 210  
 Kilowatt, 413  
 Kinetic and potential energy, definitions of, 280  
 Kinetic energy imparted to a falling body, 271  
 Knuckle-joint, 46-49

L

LAP-JOINTS, single riveted, 310  
 Lathe, back motion gear of, 174-185  
 „ bed of a, 183, 187  
 „ change wheels in a, 176-179, 183, 187  
 „ cutting forces and H.P. for, 199-202  
 „ forging and hardening the tools, 202  
 „ hexagon turret, 188-195  
 „ „ „ feed motion, 192  
 „ fixed headstock of a, 184, 185, 187  
 „ leading screw of a, 175, 186  
 „ mechanism in a screw-cutting, 174-188  
 „ motions of saddle and slide-rest of a, 176  
 „ movable headstock for a, 179, 180, 181  
 „ reversing plate for change of wheels of a, 184, 187  
 „ saddle of a, 176  
 Law, Archimedes', 217-219  
 „ Hooke's, 303  
 „ Pascal's, 208  
 Laws of friction, 103-109  
 „ „ motion, Newton's, 260  
 Leading screw of a lathe, 175, 186  
 Leather collar packing, Bramah's, 243  
 Lever, bell crank, 43  
 „ bent, 42  
 „ combined with screw and pulley, 160  
 „ definition of a, 22  
 „ duplex bent, 43  
 „ machine for testing tensile strength of materials, 41  
 „ position of fulcrum of a, 30

Lever, practical applications of the, 35-46  
 „ pressure on, and reaction from, the fulcrum of a, 25  
 „ principle of moments applied to the, 22  
 „ principle of work applied to the, 53  
 „ safety valve, 38-40  
 „ straight, acted on by inclined forces, 42  
 „ when its weight is taken into account, 29

Lifting gear, worm-wheel, 170

Limiting angle of resistance, 107

„ stress, 301

Limit and micrometer gauges, 355

Limits in calculations, 5

„ of elasticity, 303

Linear motion, comparison of dynamical formulæ for angular and, 272

„ velocity, 259

„ „ formulæ for, with uniform acceleration, 261

Liquid, definition of a, 207, 224

„ immersion of solids in a, 217

„ pressure due to head of a, 209

„ „ on any immersed surface, 209, 211

„ transmission of pressure by a, 208

Loads, definition of live and dead, 300

„ safe, and elasticity, 302

Lockfast lever and safety valve, 39

Logarithms—Appendix D, 415

Low pressure gauges, 221

Lubrication, 109

Lumberer's tongs, 43

M

MACHINE, efficiency of a, 53

„ measuring, 363

„ modulus of a, 53, 136

„ testing, 40

Machinery, importance of balancing high speed, 279

Malleability, definition of, 298

Mass, definition of, 289, 290

Materials, machine for testing tensile strength of, 47 42

Materials, properties of, 297-311  
 Matter, definition of, 1  
 Measuring tools and gauges, 355-366  
 Mechanical advantage, 66, 68  
 Mechanics, definition of applied, 1  
 Mercurial barometer, the, 221  
 Metals, melting points of, 300  
 Micrometer screw gauges, 357  
 Modulus of elasticity, 304-306, 321  
 „ of rigidity of a material, 319  
 „ of a machine, 53, 136  
 Moment of a force, 21  
 „ of momentum, 290  
 Moments, principle of, 21; applied to the lever, 22; applied to the wheel and axle, 55  
 Momentum, definition of, 260, 280  
 Motion and velocity, 259  
 „ equations of, 261  
 „ Newton's laws of, 260  
 „ of saddle and slide rest of a lathe, 176  
 „ on a curved, inclined, or "banked" track, 287  
 Motions, reversing, 343  
 „ by belt gearing, 122-124

## N

NEWTON'S laws of motion, 260  
 Nippers, example of, 45

## O

Ohm, 413

## P

PACKING, Bramah's collar, 243  
 Pantograph, 339  
 Parabola, hyperbola, ellipse, 273  
 Parallel forces, centre of, 26  
 „ „ equilibrant and resultant of, 25-28  
 „ motion, 339  
 „ „ Watt's, 338  
 Parallelogram of forces, 82  
 Pascal's law, 208  
 Passive resistance, 101  
 Path of a projected body which falls under the action of gravity, 273

Pawl and ratchet wheel, 334  
 „ reversible, 335  
 Pendulum experiments by Galileo, Huyghens and Kater, 269  
 Percentage efficiency of a machine, 63, 67  
 Pincers, 44  
 Pinion, rack and, 132  
 Pitch circle, definition of, 131  
 „ surface, definition of, 131  
 „ of rivets, 310  
 „ of teeth in wheel gearing, 132  
 Plane, inclined, with friction, 110-112  
 „ without friction, 93, 270-272  
 „ principle of work applied to the inclined, 97  
 Flanges of belts and pulleys 2  
 „ gearing, 125  
 „ inclined, 93-99  
 Planing machine, 343  
 Plates, hydraulic press for flanging boiler, 245  
 Porosity of matter, 297  
 Potential energy, definition of, 280  
 Poundal, the, 2  
 Power, definition of, 12, 412  
 „ horse, definition of, 12  
 „ „ transmitted by belts, 118  
 „ that steel shafting will transmit at various speeds, 323  
 „ units of, 12, 412  
 Press, the fly, 283  
 „ Bramah's hydraulic, 241-245  
 „ „ large, 245-247  
 „ screw, for compressing bales, 165  
 Presses, packing for hydraulic, 243  
 Pressure, atmospheric, 220  
 „ centre of, 5, 217  
 „ due to head of a liquid, 209  
 „ low and vacuum water gauges, 221  
 „ on fulcrum of a lever, 19, 23-25  
 „ on ram of a Bramah's press, 244  
 „ on sluice gate, 217  
 „ on surface in liquid, 209, 211  
 „ measure of, &c., 5  
 „ transmitted by liquids, 208  
 total or thrust, 5

- Principle of moments, 21; applied to the lever, 22, 35; to the wheel and axle, 55; to the wheel and compound axle, 73
- „ of work, 52; applied to the lever, 53; to the wheel and axle, 56; to the ordinary block and tackle, 67; to the wheel and compound axle, 73; to Weston's pulley block, 76; to the inclined plane, 97; to wheel gearing, 135
- Pulley blocks and tackle, ordinary, 65
- „ combined with lever and screw, 168
- „ combined with worm-wheel and winch barrel, 168
- „ faces for belts, shape of, 127
- „ Weston's differential, 74-77
- Pulleys, 63-65
- „ arrangement of driving and following, in different planes, 125
- „ combinations of fast and loose, 122-124
- „ tendency of belts to run on highest parts of, 127
- „ velocity ratio of, in belt gearing, 119-121
- Pump, combined plunger and bucket, 234
- „ common suction, 227-229
- „ double acting force, 234
- „ force, with air vessel, 231
- „ plunger force, 229-231
- „ rods, tension in, 229
- Pumps, centrifugal, 236
- „ continuous delivery force, without air vessels, 232
- Punching machine, portable, or hydraulic bear, 251
- Quick return motion, Whitworth's, 347
- „ „ reversing motion, 346
- „ „ elliptic wheels, 351
- R
- RACK and pinion, 132
- Radius of gyration, 282
- Railway carriages, screw coupling for, 156
- „ curves, motion on bicycle and, 287
- Ratchet, masked, 336
- „ wheel, pawl and, 334
- Ratio, velocity, of change wheels in a lathe, 176-179
- „ „ definition of, 67
- „ „ of pulleys in belt gearing, 119-121
- „ „ of two friction circular discs, 130
- „ „ in wheel gearing, 133
- Reaction, action and, 3
- „ from fulcrum of a lever, 25, 30
- Relation between twisting moment, diameter, and horse power transmitted by shafting, 224
- Repose, angle of, 106
- Resilience, definitions, 307, 318
- Resistance, electrical unit, 413
- „ limiting angle of, 107
- „ passive, or friction, 101
- „ work in overcoming a uniform, 6-9
- „ work in overcoming a variable, 8-11
- Resolution and composition of forces, 82-84; of velocities, 260
- Resolution of a force into two components at right angles, 84
- Resultant and components, 4
- „ of parallel forces, 25-28
- „ two forces at angle and any number at a point, 84
- „ pressure and thrust, 5
- Reversible pawl, 335
- „ pendulum, 269
- Q
- QUADRANT or reversing plate for change wheels, 183-187
- Quantity (motion or momentum), 260
- Quick return, cam, 334
- „ „ common, 350

- Reversing by friction cones and bevel wheels, 344  
 „ gear, bevel wheel and clutch, 345  
 „ „ friction cone, 344  
 „ „ quick return, 346  
 „ „ Whitworth, 345  
 „ motions, 343  
 „ for belt gearing, 122-124  
 „ plate for change wheels, 183-187
- Rigidity, definition of, 298  
 „ modulus of, 319, 321
- Rivets, pitch, 310  
 „ shearing stress of, 310
- Rods, tension in pump, 187  
 „ torsion of, 318
- Roller or ball bearings, 110, 183
- Roman balance, 35
- Roof truss, stresses in a, 88, 89
- Rotating body, accumulated work in a, 281
- Rotor, 5a
- Rounded screw threads, 155
- Rules and syllabus of Exams. for admission of Students to the Institution of Civil Engineers, 370
- S
- SADDLE and slide rest of a lathe, 176, 185
- Safe loads and elasticity, 302
- Safety, factors of, for materials, 302  
 „ valve, 38-40
- Sawing machine, vertical, 337
- Scalars, 5a
- Screw bench vice, 166, 167  
 „ combined with lever and pulley, 160  
 „ -coupling for railway carriages, 156  
 „ -cutting lathe, description of a, 180-188  
 „ -cutting lathe, self-acting, 182  
 „ -cutting mechanism in a lathe, 174-188  
 „ endless and worm-wheel, 168  
 „ -gauges, micrometer, 357  
 „ gear, apparatus for demonstrating efficiency of, 160  
 „ gearing, backlash in, 158  
 „ -jack, bottle, 162-164
- Screw-jack, compound hydraulic and, 249  
 „ -jack, traversing, 164  
 „ leading, of a lathe, 175, 186;  
 „ split nut for engaging, 185  
 „ -press for compressing bales, 165  
 „ or spiral, for grain, 148  
 „ pressure or thrust, 5  
 „ viewed as an inclined plane, 149
- Screws, right and left-hand, 156  
 „ single-, double-, and treble-threaded, 157  
 „ strength, durability, and efficiency of, 151
- Screw thread, 148  
 „ „ buttress, 155  
 „ „ rounded, 155  
 „ „ forming a, on a cylinder, 149  
 „ „ square, 154
- Screw threads, characteristics of, 151  
 „ „ different forms of, 151-156  
 „ „ Seller's, 154  
 „ „ Whitworth, 151-154
- Second of Time, 411
- Shafts, strength of hollow, 324  
 „ strength of solid round, 322
- Shaping machine, 350
- Shearing strength of rivets, 310
- Shears or bed of a lathe, 183, 186
- Ship's capstan, 57-59
- Silent feed, 336
- Single-riveted lap joints, 310
- Siphon, the, 222
- Slide rest of a lathe, 176, 185
- Sliding angle, 107
- Slotting machine, vertical, 341  
 „ „ Whitworth's, 349
- Sluice gate, pressure on, 217
- Snatch block, 65
- Solid shafts, strength of, 322
- Solids, immersion, 217; defn., 224
- Specific gravity, 214, 298
- Speed cones, 124
- Split nut for engaging leading screw o a lathe, 185
- Squared paper, 14-17
- Standard measuring machines, 355-366
- Starting and stopping gear, 124
- Starrett micrometer gauge, 357
- Steel, high speed, 195

Steel, specific gravity test, 203  
 Steelyard, 35-38  
 Stepped speed cones, 124  
 Straight levers acted on by inclined forces, 42  
 Strain, compressive stress and, 301  
 „ definition of, 300  
 „ shearing stress and, 317  
 „ tensile stress and, 301  
 Strength of materials, machine for testing, 40-42  
 „ of materials, ultimate, 301  
 „ of solid and hollow round shafts, 322-324  
 Stress, centrifugal, on fly-wheels, 279  
 „ definition of, 300  
 „ intensity of, 300  
 „ limiting, or ultimate strength, 301  
 „ shearing and strain, 317  
 „ total, 300  
 Stresses in chains, 316  
 „ in jib cranes, 85-89  
 „ in simple roof truss, 88, 89  
 „ tensile and compressive, 301  
 Stubs' wire gauge, 358  
 Students' Exams., Institution of Civil Engineers, 389-401, 407  
 Suction pump, common, 227-229  
 Sun and planet wheels, 330  
 Surface, definition of pitch, 131  
 „ immersed in liquid, 211  
 Swing radius, 282

## T

TABLE of melting points of metals, 300  
 „ of moduli of elasticity, 305  
 „ of power that steel shafting will transmit at various speeds, 323  
 „ of ultimate strengths of materials, 302  
 Tackle, block and, 65-68  
 Tangentometer, 364  
 Tearing strength of plates, 310  
 Teeth, pitch of, in wheel gearing, 132  
 Tenacity, definition of, 298  
 Tensile strength of materials, machine for testing, 40-42  
 „ stress and strain, 301  
 Tension in driving belts, 116

Tension in pump rods, 229  
 Test of steel, by specific gravity method, 203  
 „ specimen, work done per cubic inch in fracturing a, 309  
 Testing machine, 40-42  
 Theoretical advantage, 66-68  
 Toggle joint, 46-49  
 Tongs, Lumberer's, 43  
 Torsion of rods and wires, 318-320  
 Torque or twisting moment, definition of, 322  
 Transmission of power by belting, 118; by liquids, 208  
 Traversing screw-jack, 164  
 Treadle lathe, self-acting screw-cutting, 182  
 Triangle of forces, 82  
 Turning tools, forging and hardening, 202  
 „ tool holders, 194  
 „ with high speed steel, 195  
 Turkus, carpenter's, 44  
 Turret-lathe, 188-195  
 Twisting moment, 322

## U

ULTIMATE strength of materials 301  
 Uniform velocity, definition of, 259  
 Unit of acceleration, 260  
 „ force, 2, 412  
 „ horse-power, 12, 412  
 „ power, 12, 412  
 „ velocity, 259, 411  
 „ work, 6, 412  
 Units, C.G.S., 411, Practical, 413  
 Universal joint, Hooke's, 329  
 Useful constants—Appendix D, 414  
 „ data regarding fresh and salt water, 214  
 „ work in a machine, 52  
 Uses of squared paper, 14-17

## V

VACUUM water gauges, 221  
 Variable resistance, work done against a, 8  
 „ velocity, definition of, 2459  
 Valve, lever safety, 38-40  
 V-screw-thread, Seller's, 154  
 „ „ „ Whitworth's, 151

- Vector's, 5a  
 Velocities, composition and resolution of, 260  
     ,, graphic representation of, 260  
 Velocity, and motion, 259  
     ,, angular, 260  
     ,, attained by a body sliding down any smooth inclined plane, 270  
     ,, definition of, 260, 411  
     ,, linear, with uniform acceleration, 260  
     ,, linear, 259  
     ,, uniform and variable, 259  
     ,, unit of, 259, 411  
     ,, ratio of change wheels in a lathe, 176-179  
         ,, definition of, 67  
         ,, pulleys in belt-gearing, 119-121  
         two friction circular discs, 130  
         ,, in wheel-gearing,  
 Vessel, action of an air, 232  
     ,, force pump with,  
 Viscous fluid, 224  
 Volt, 413
- W
- WATER gauges, low pressure and vacuum, 221  
     ,, useful data regarding fresh and salt, 214  
 Watt, 413  
 Watt's parallel motion, 338  
 Weems' compound screw and hydraulic jack, 249-251  
 Weston's differential pulley block, 74-77  
 Wheel and axle, 55-57  
     ,, compound axle, 72-74  
 Wheel gearing, backlash in, 158  
     ,, in jib cranes, 144-146  
     ,, pitch of teeth in, 132  
     ,, principle of work applied to, 135  
 Wheel gearing, velocity ratio in, 133  
 Wheels, anti-friction, 109  
     ,, change, in a lathe, 176-179, 183, 184, 187; quadrant or reversing plate for, 183, 187  
     ,, fly-centrifugal stress in, 279  
     ,, pawl and ratchet, 334  
 Wheels, sun and planet, 330  
 Whitworth's millionth measuring machine, 358  
     ,, quick return motion, 347  
     ,, reversing gear, 345  
     ,, slotting machine, 349  
     ,, V-screw-threads, 151-154  
 Winch barrel, 57  
     ,, double purchase, 143-144  
     ,, single purchase, 140-142  
 Winch drum combined with pulley, worm, and worm wheel, 168  
 Windlass, Chinese, 72  
 Wire gauge, Stubs', 358  
     ,, -testing machine, Lord Kelvin's, 210  
 Wires, torsion of, 319-321  
 Work, accumulated, 280-282  
     ,, definition of, 6, 42  
     ,, diagrams of, 9  
     ,, done against variable resistances, 8, 17  
     ,, done in extending or compressing a bar within the elastic limit, 307  
     ,, done on inclines, 110-112  
     ,, done per cubic inch in fracturing a test-bar, 309  
     ,, principle of, 52  
     ,, relation between, and heat, 102  
     ,, transmitted by belts, 118  
     ,, unit of, 6, 412  
 Workshop measuring machine, 363  
 Worm-wheel, screw and worm, 168  
     ,, lifting gear, 170  
 Worssam's silent feed, 338



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